Hall effect & polarity effect on flow structure in counter-helicity spheromak merging

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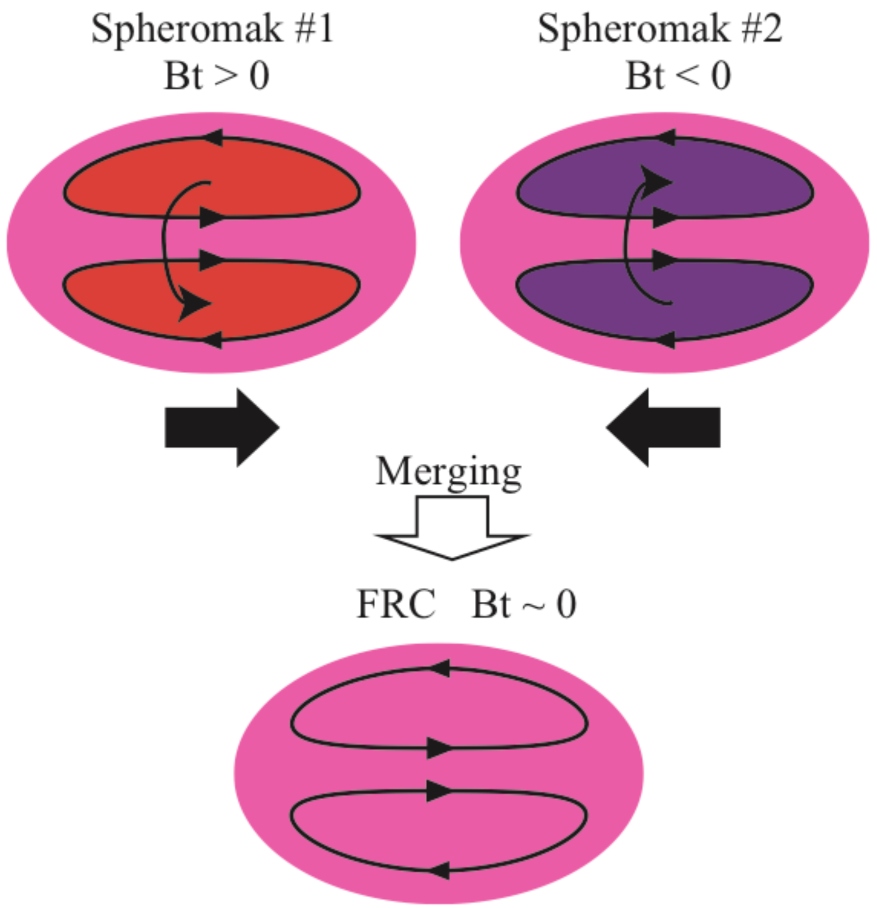
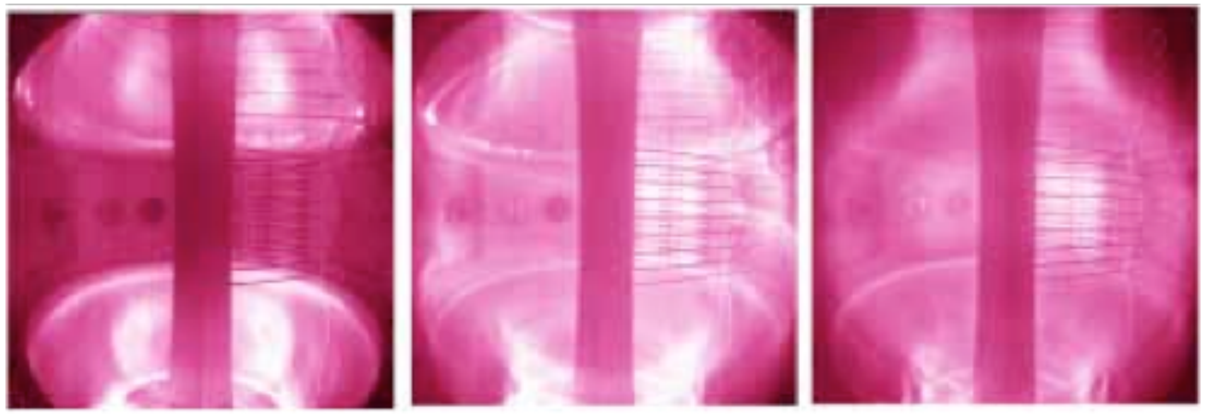
Outline

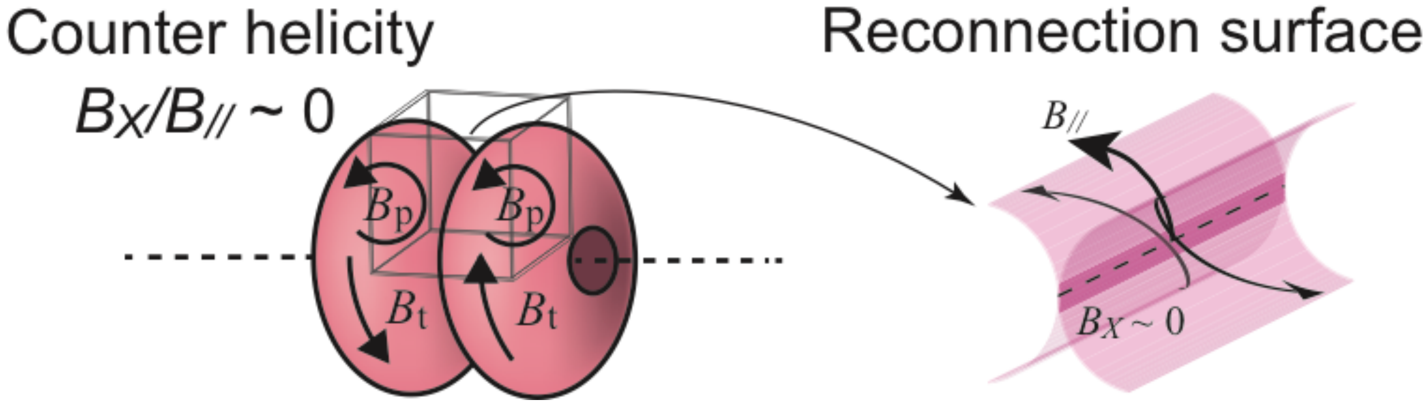
- Introduction
- Hall-MHD simulation
- Particle in Cell simulation
- Summary

Introduction & Background
FRC / Counter-helicity Spheromak Merging



(photo) ST merging in UTST





The characteristics of Counter-helicity Spheromak Merging

- Spheromak formation is easier than forming an FRC by Field-Reversed Theta Pinch method.
- Spheromaks can get large magnetic flux by a MCPG (CHI) or a flux core. → easy to get a large flux FRC

- magnetic reconnection with zero guide-field & reconnection with both poloidal and toroidal magnetic field.
- Non-MHD effects, such as Hall effect or toroidal effect on merging process are not fully understood.

Introduction & Background
Ion flow in high-beta relaxation

Two-fluid relaxation theory

ion fluid

$$\mathbf{P_i} = m_i \mathbf{u_i} + q_i \mathbf{A}$$

$$\mathbf{\Omega_i} = \nabla \times \mathbf{P_i} = m_i \boldsymbol{\omega}_\mathbf{i} + q_i \mathbf{B}$$

$$K_i = \int \mathbf{P_i} \cdot \mathbf{\Omega_i} dV$$

electron fluid

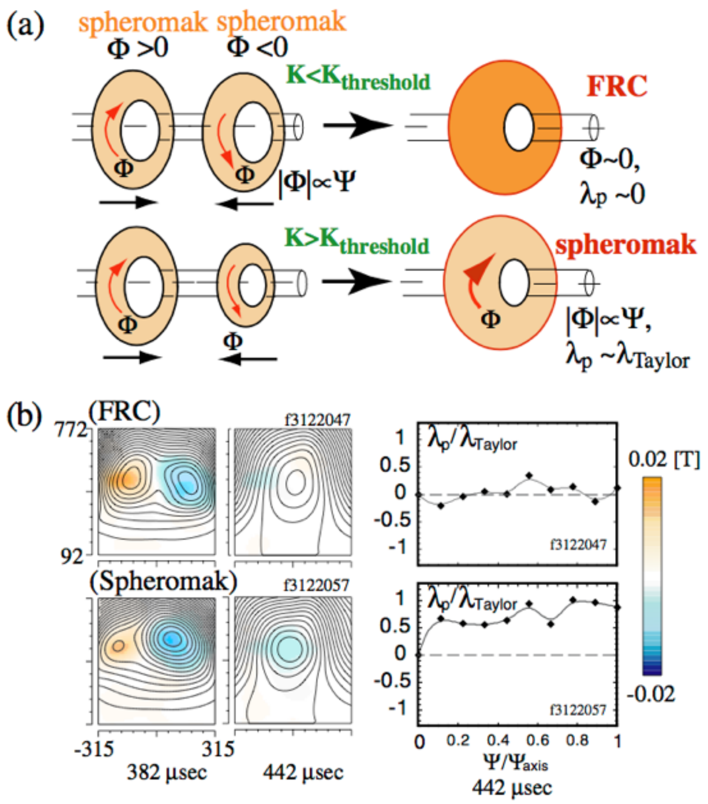
$$\mathbf{P_e} = q_e \mathbf{A}$$

$$\mathbf{\Omega_e} = \nabla \times \mathbf{P_e} = q_e \mathbf{B}$$

$$K_e = \int \mathbf{P_e} \cdot \mathbf{\Omega_e} dV = q_e^2 \int \mathbf{A} \cdot \mathbf{B} dV$$

$$\nabla \times (\nabla \times \mathbf{B}) - \alpha \nabla \times \mathbf{B} + \beta \mathbf{B} = 0$$

$$\beta_i + 0.5V^2 = \text{constant}$$



E. Kawamori et al., Nucl. Fusion (2005)

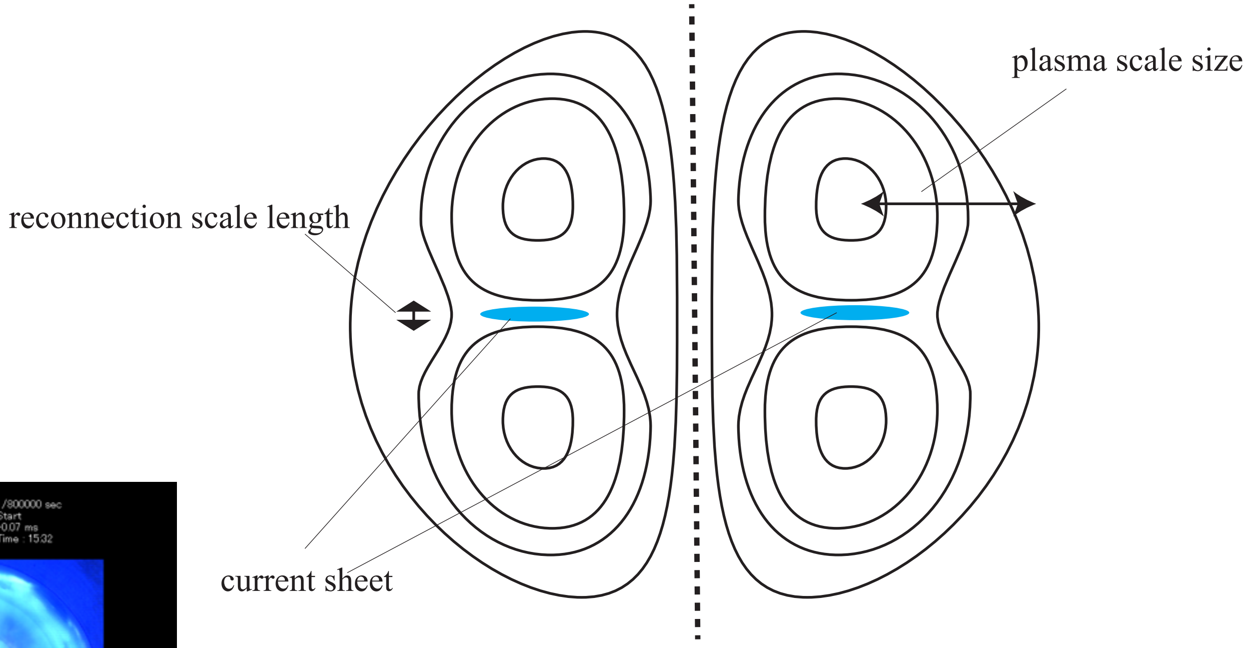
- FRC is high-beta, and it is considered that FRC can be a two-fluid relaxation state.
- In two-fluid relaxation, ion flow is important.
- In merging-formed FRC, strong ion flow is generated by magnetic reconnection.

∴To understand non-MHD effects on counter-helicity merging process is important.

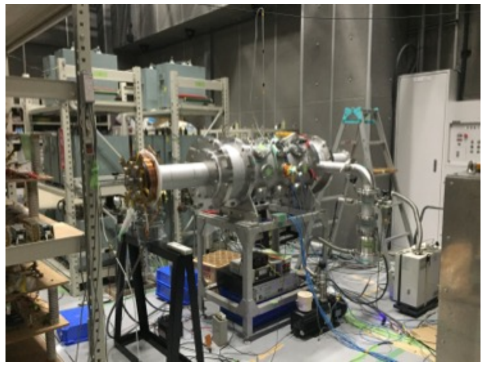
Introduction & Background
Objective of the work

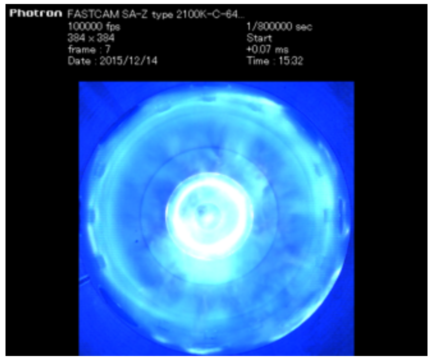
Goal: To understand whole formation process of an FRC by counter-helicity spheromak merging

- plasma merging / magnetic reconnection
  - flow formation
  - ion/electron heating
- relaxation / self-organization
  - two-fluid relaxation



Approach:
- experiment
- numerical simulation
  - Hall-MHD…Hall-parameter dependency
  - PIC …electron / ion heating mechanism



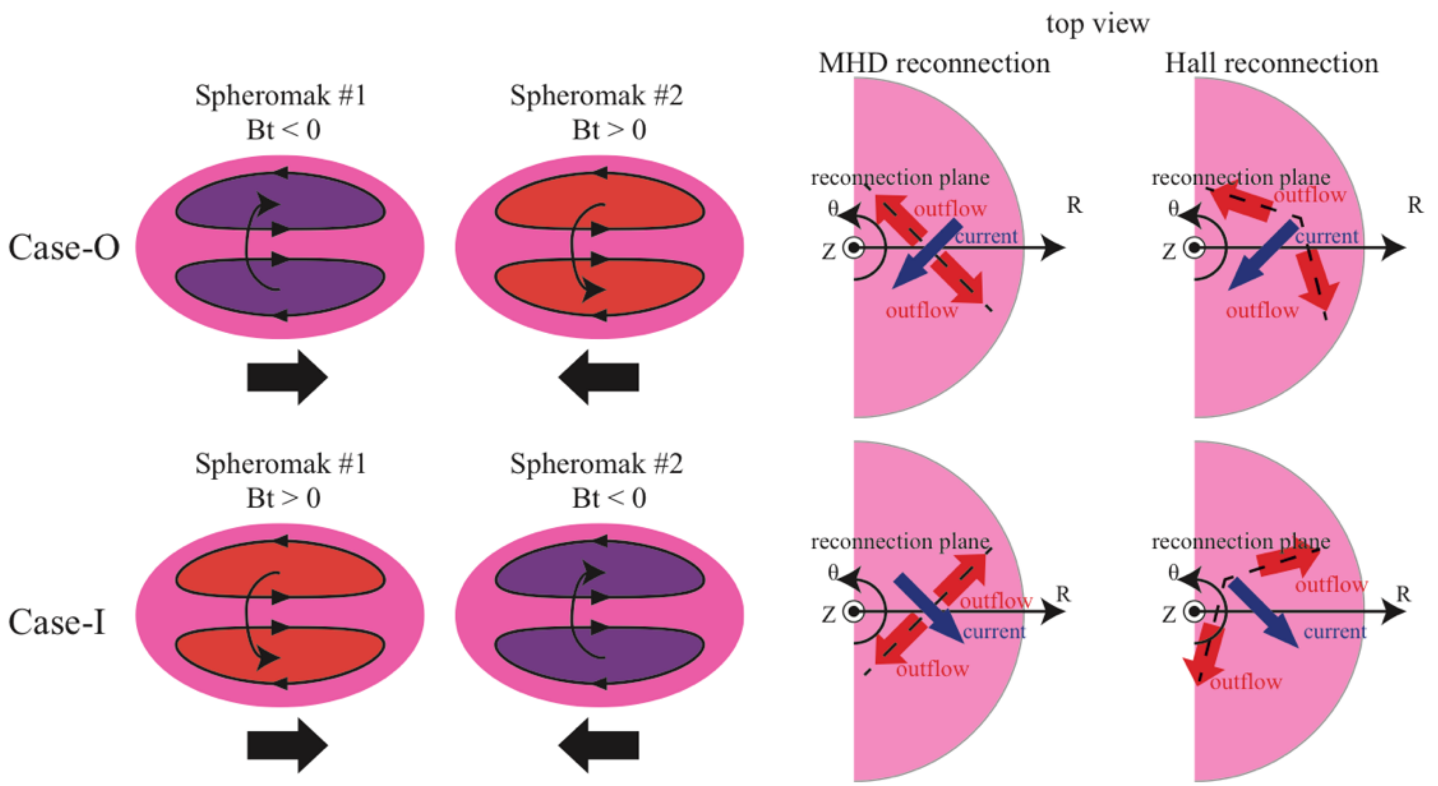


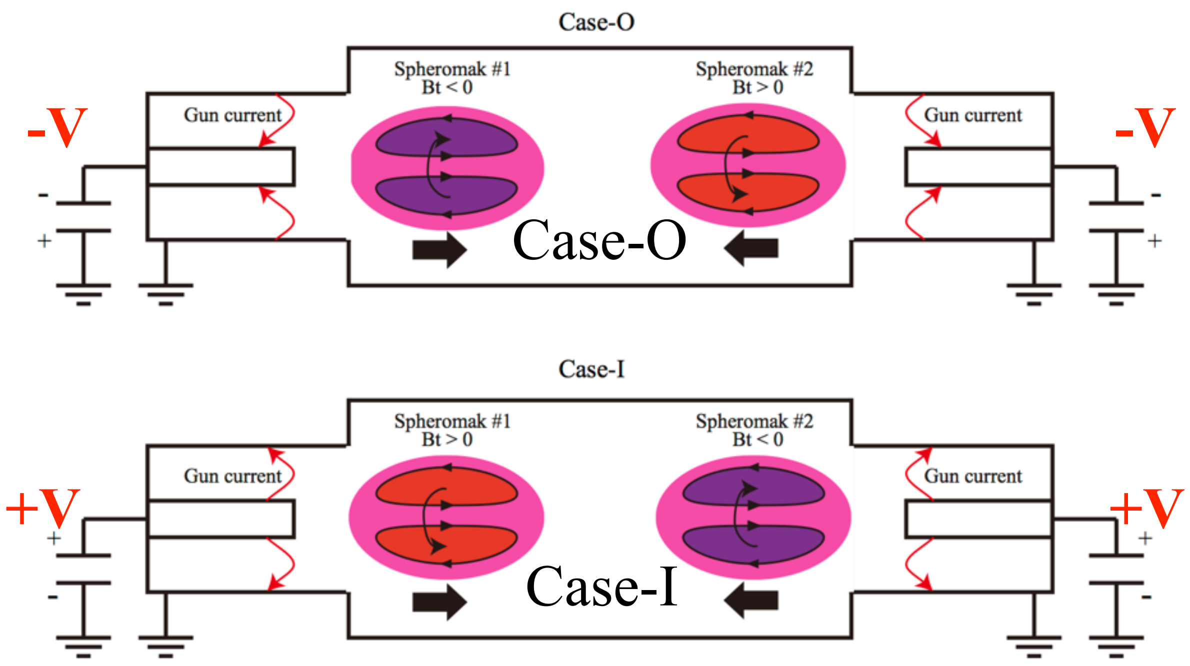
Plasma merging contains two scale:
- Large scale (torus plasma)
- Small scale (reconnection physics)

Introduction & Background
Polarity of counter-helicity spheromak merging : Case-O & Case-I

2H1-IX

Counter-helicity merging has two patterns, which are defined by combinations of poloidal/toroidal magnetic flux.





Spheromaks are created by MCPG at both ends,

- Case-O: inner electrode has negative voltage
- Case-I: inner electrode has positive voltage

Poloidal field (Br) reconnection… Inflow (Vz), Current sheet (Jt < 0), Outflow (Vr)

Toroidal field (Bt) reconnection… Inflow (Vz), Currehs sheet (Jr), Outflow (Vt)

Reconnection plane tilts toward toroidal direction, therefore counter-helicity is different from null-helicity.
In this presentation, we report the Hall effect, toroidal effect (polarity effect) through comparing case-O & case-I merging.

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- Hall-MHD simulation
- Particle in Cell simulation
- Summary

Hall-MHD plasma merging simulation
Basic equations & numerical scheme

2H1-IX

Basic equations

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot(\rho\mathbf{v}) \qquad (1)$$

$$\begin{aligned}\frac{\partial p}{\partial t} &= -\nabla\cdot(p\mathbf{v}) - (\gamma-1)p(\nabla\cdot\mathbf{v}) \\ &+ (\gamma-1)\big(\eta j^2 + \nu(\frac{4}{3}(\nabla\cdot\mathbf{v})^2 + |\nabla\times\mathbf{v}|^2)\big) \qquad (2)\end{aligned}$$

$$\begin{aligned}\frac{\partial(\rho\mathbf{v})}{\partial t} &= -\nabla\cdot\rho\mathbf{v}\mathbf{v} - \nabla p + \mathbf{j}\times\mathbf{B} \\ &+ \nu(\frac{4}{3}\nabla(\nabla\cdot\mathbf{v}) - \nabla\times\nabla\times\mathbf{v}) \qquad (3)\end{aligned}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla\times(\mathbf{v}\times\mathbf{B} - \eta\mathbf{j} - \boxed{d_i\frac{\mathbf{j}\times\mathbf{B}}{\rho}}) \qquad (4)$$

Hall term (di:ion skin depth)

Time advancing: 2nd order Adams-Bashforth scheme

$$\frac{\partial u^n}{\partial t} = f^n(u)$$

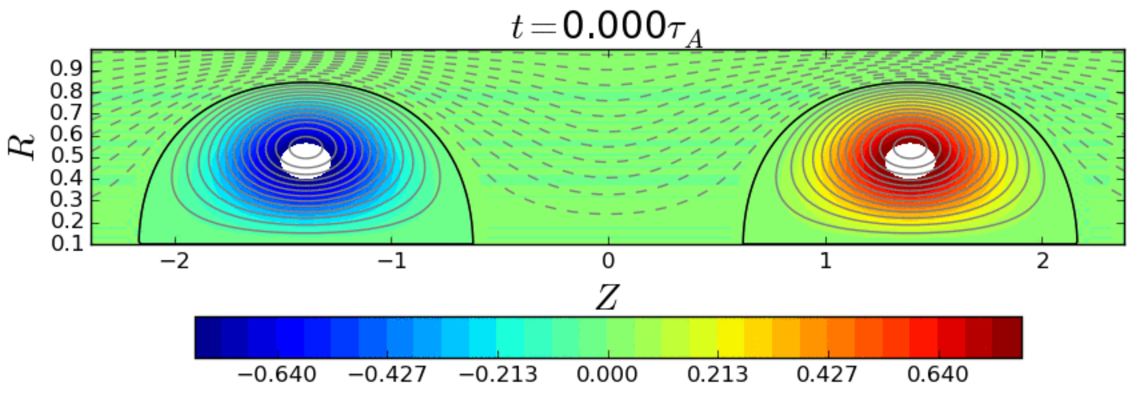
$$u^{n+1} = u^n + \frac{\Delta t}{2}(3f^n - f^{n-1})$$

Spacial difference: 4th-order central difference

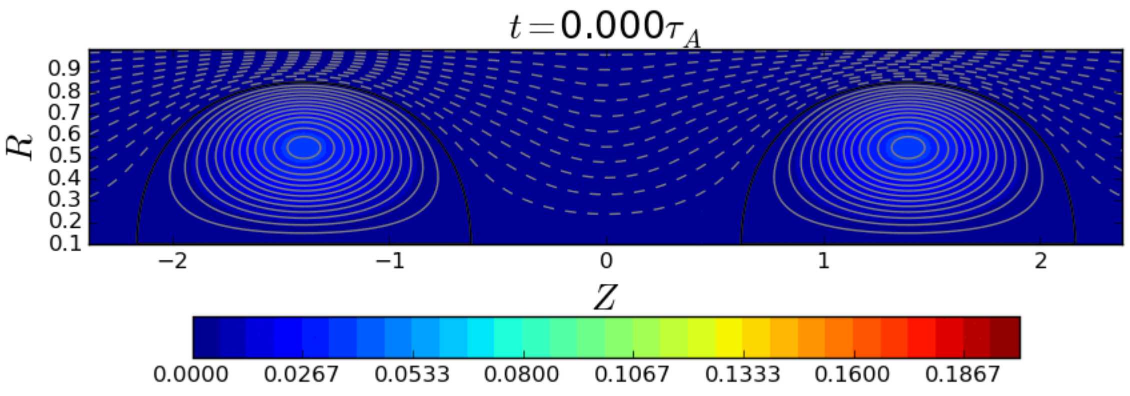
Numerical viscosity: 4th-order smoothing

$$F_i = (1-\alpha)F_i + \alpha\frac{-F_{i-2} + 4F_{i-1} + 4F_{i+1} - F_{i+2}}{6}$$

Parameters:
(Nr,Nz)=(512,4032), Rm=2000, Re=2000
(Rw~20cm, B~100mT, Te~30eV)



Toroidal magnetic field (Bt)



Thermal pressure (P)

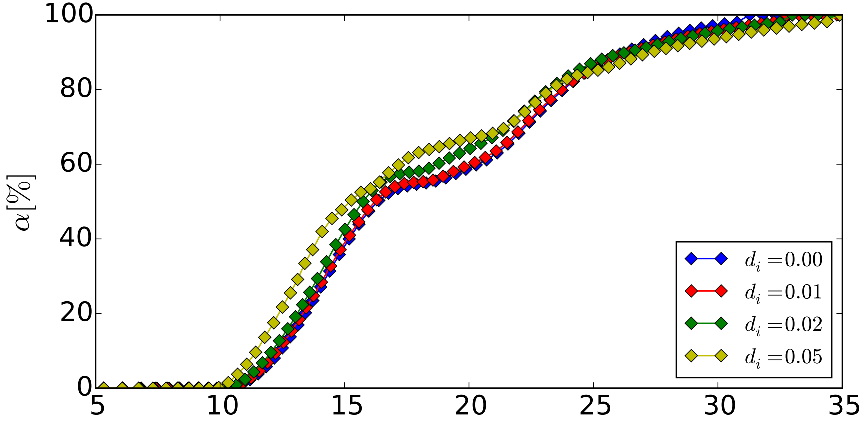
Hall-MHD plasma merging simulation
Polarity effect & Hall effect on merging speed

2H1-IX

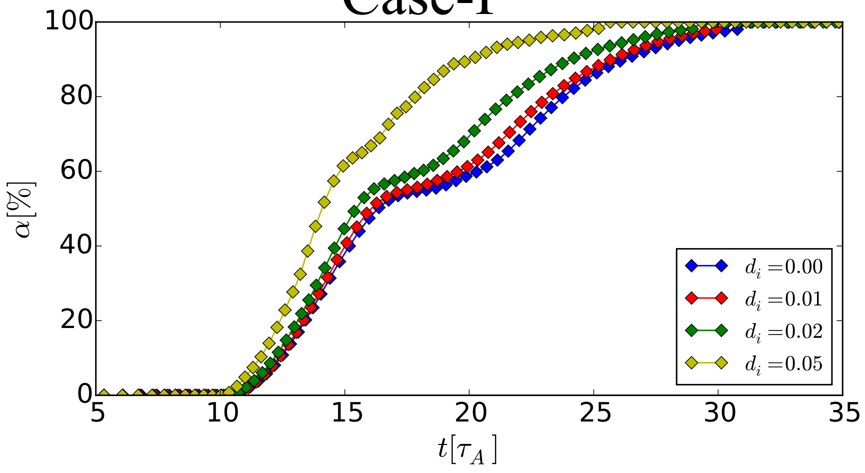
merging rate = (reconnected flux)/(total flux)

$$\alpha = \frac{\Psi_{com}}{\Psi_{max}}$$

Case-O

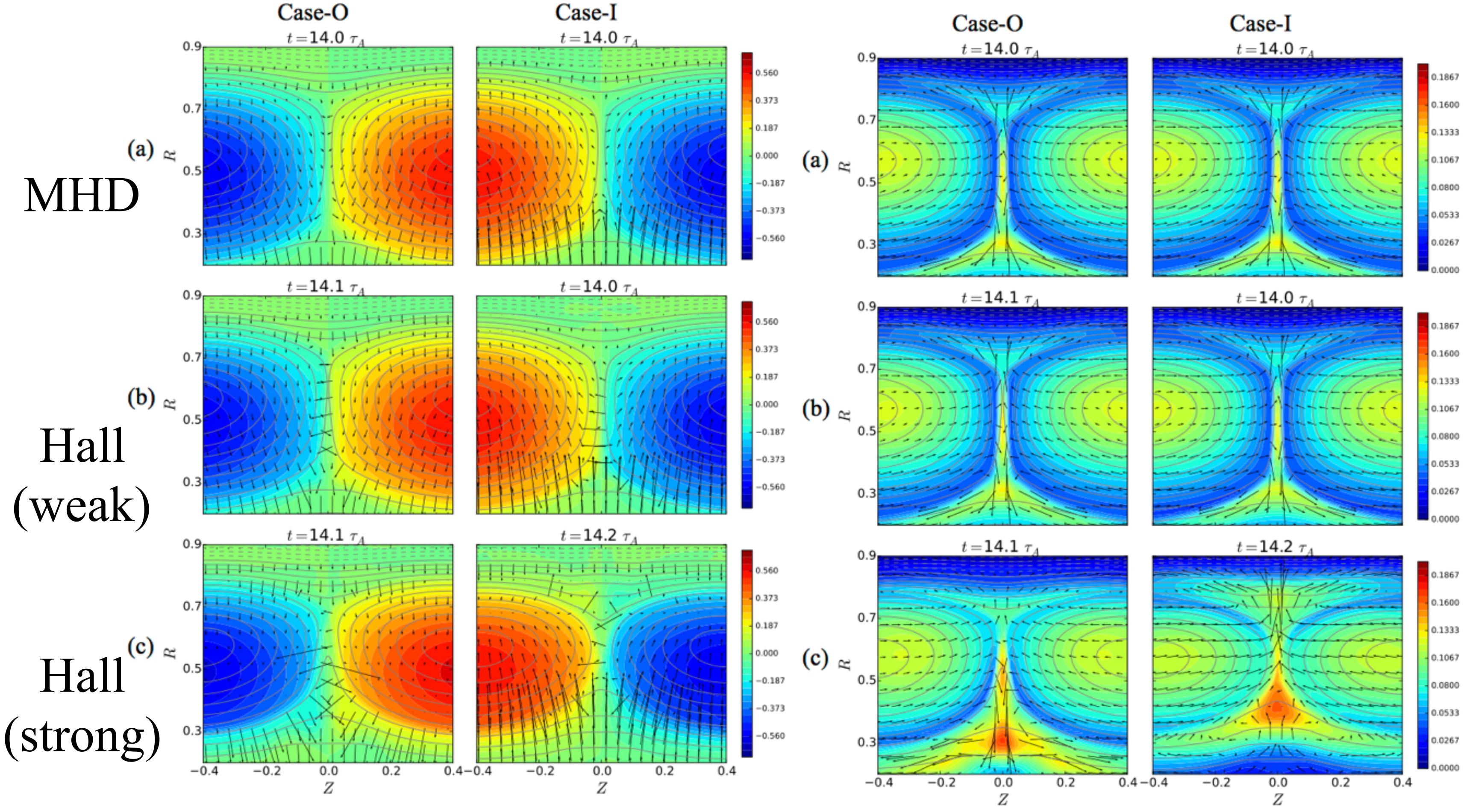


Case-I



Toroidal field Bt (vector: E_{pol})

thermal pressure P



(a) di=0, (b)di=0.01, (c)di=0.05

Case-O: Hall effect does not enhance merging speed.

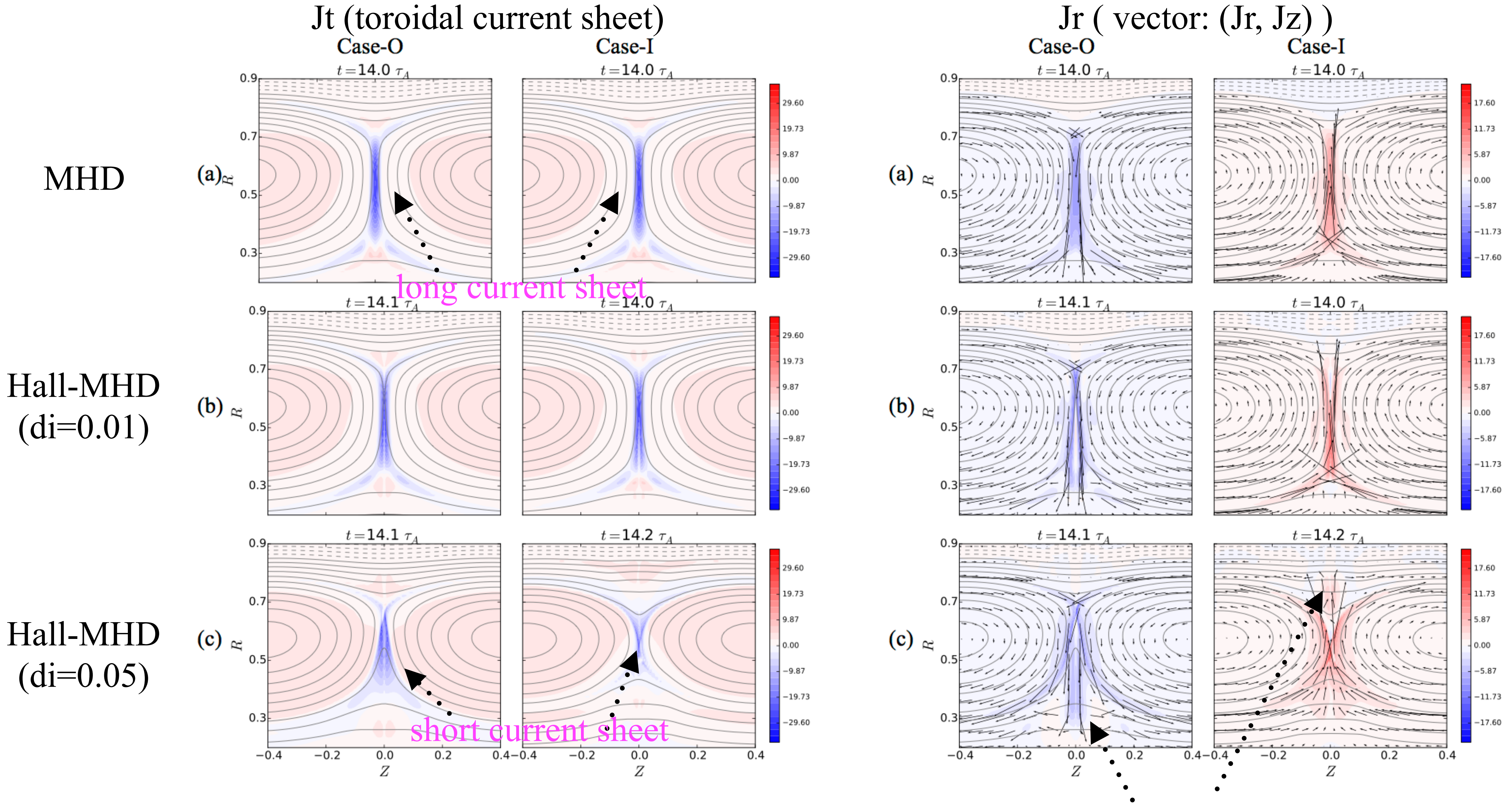
Case-I: Hall effect enhances merging speed.

Merging speed is different between case-O and case-I. Polarity effect appears.

→ Merging speed is determined by global pressure balance of inflow region / outflow region.

Hall-MHD plasma merging simulation
Hall effect on current sheet structure

2H1-IX



long current sheet

short current sheet

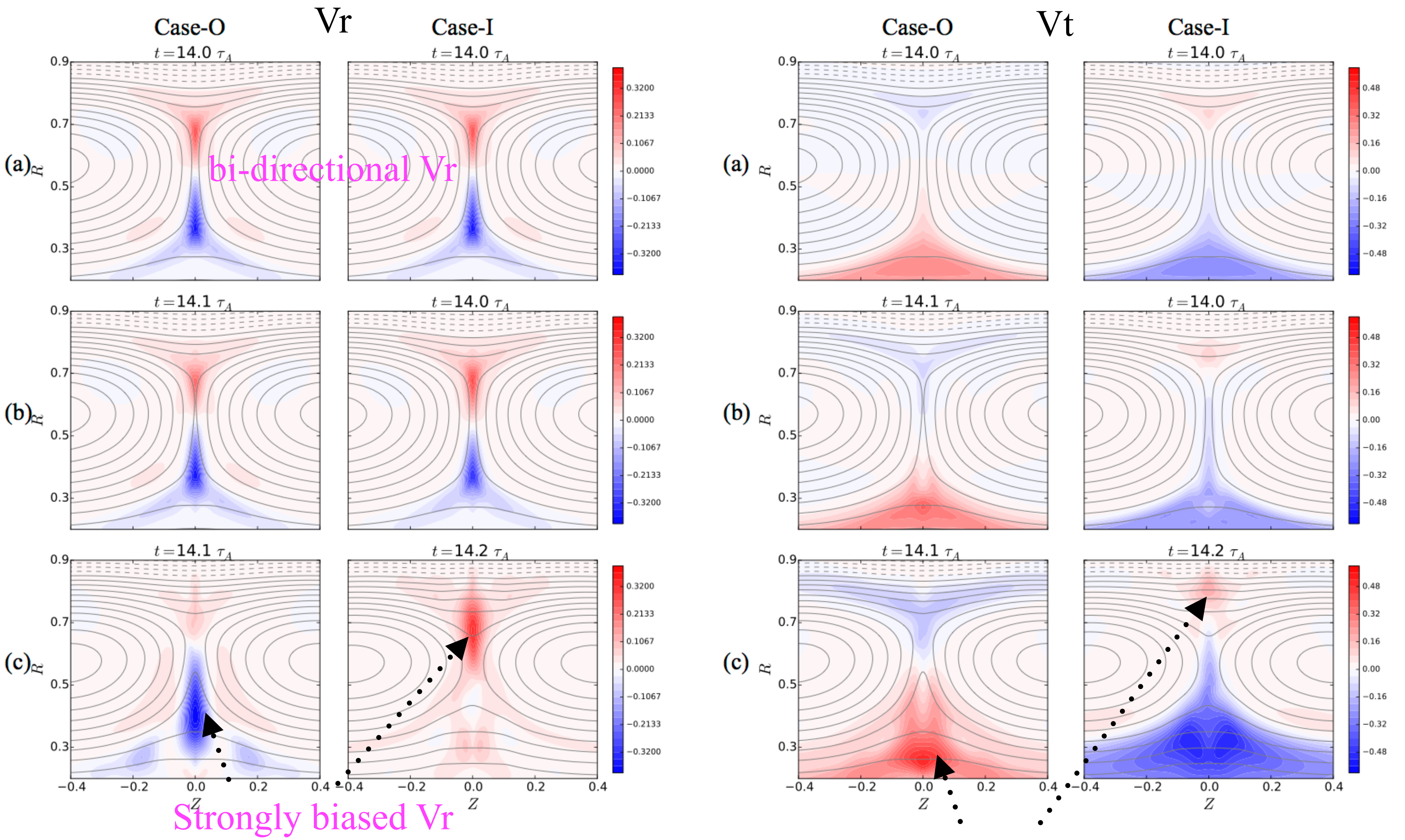
Jr extend to downstream region.

Hall effect on current sheet structure:

- Current sheet length become short.
- Radial current sheet (Jr) extend to downstream region.

…Difference between toroidal current sheet and poloidal current sheet becomes large.

Hall-MHD plasma merging simulation
Polarity effect & Hall effect on flow structure



MHD

Hall-MHD (di=0.01)

Hall-MHD (di=0.05)

bi-directional Vr

Strongly biased Vr

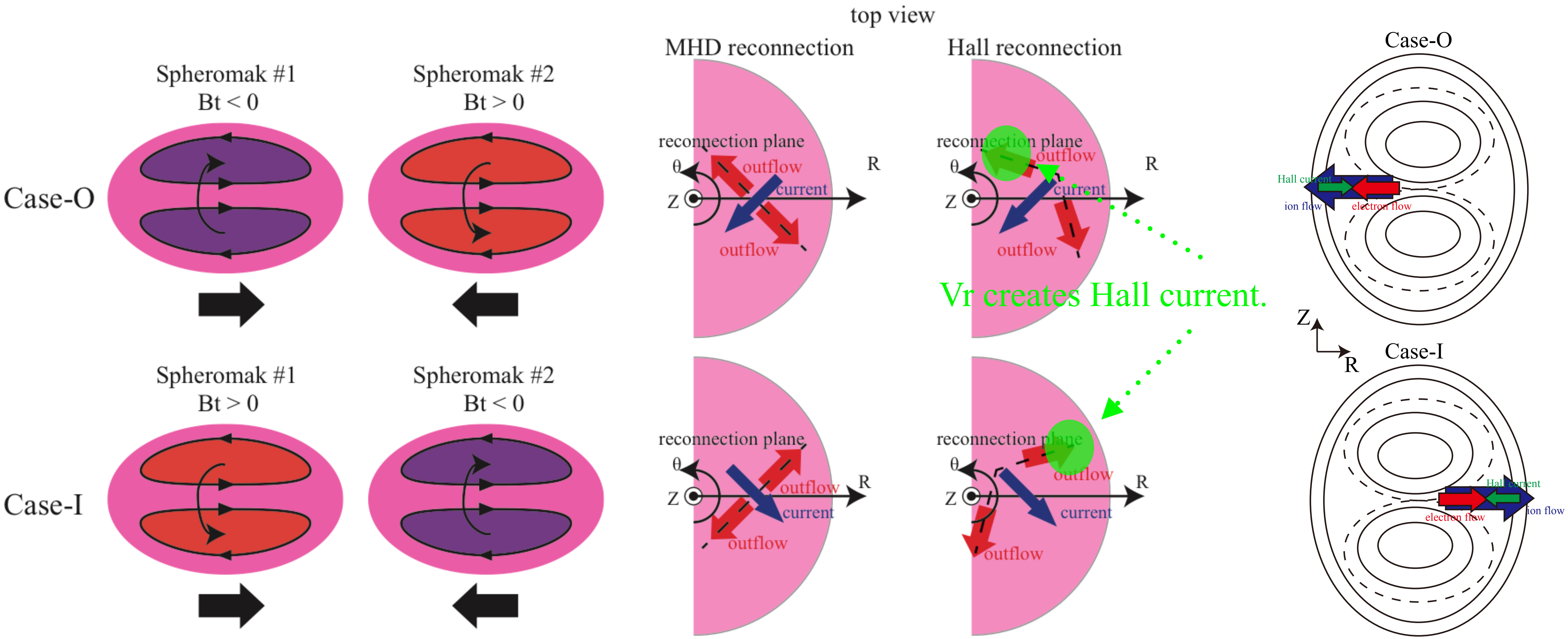
Positive toroidal flow at the downstream region.

Hall effect on flow formation by merging:

- Outflow (Vr) is biased to single direction. Case-O is negative, and case-I is positive.
- Toroidal flow is negative near the X-point, and positive flow arises at the downstream region.

Hall-MHD plasma merging simulation
role of Hall effect on flow formation

2H1-IX



Vr creates Hall current.

1. Hall effect moves X-point along the electron flow (electron current) at the reconnection point.
2. Reconnection outflow Vr is strong toward the opposite direction from the X-point motion.
3. The strong radial flow and the large ion inertia generates Hall current (Jr) at the downstream region.
4. The Hall current (Jr) generates the strong positive toroidal flow at the downstream region.

∴ Hall effect in reconnection + outflow damping → flow generation at the downstream region

Hall-MHD plasma merging simulation
Hall effect on structure of energy conversion (ion acceleration) region

2H1-IX

to thermal energy

$$\mathbf{E}\cdot\mathbf{j} = \eta j^2 + (\mathbf{j}\times\mathbf{B})\cdot\mathbf{v}$$

magnetic dissipation

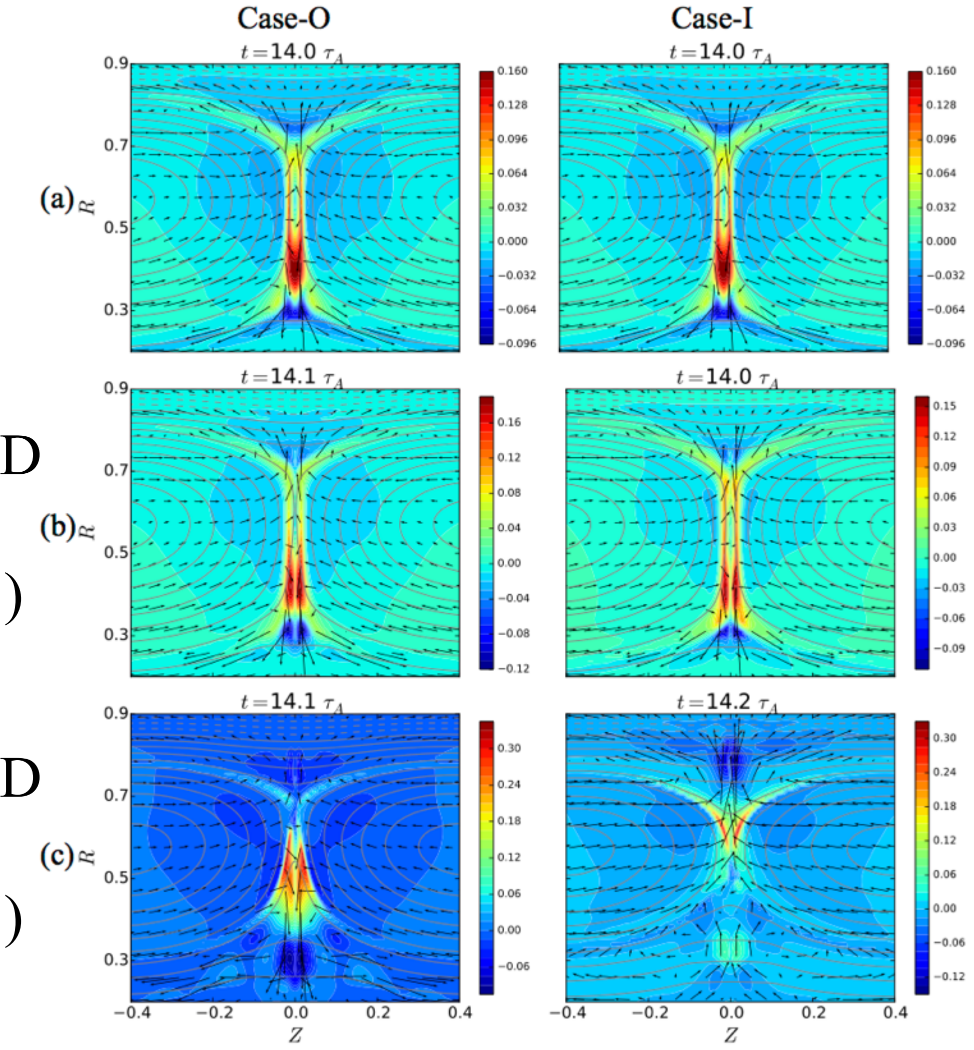
to kinetic energy

(JxB)・V_{pol}

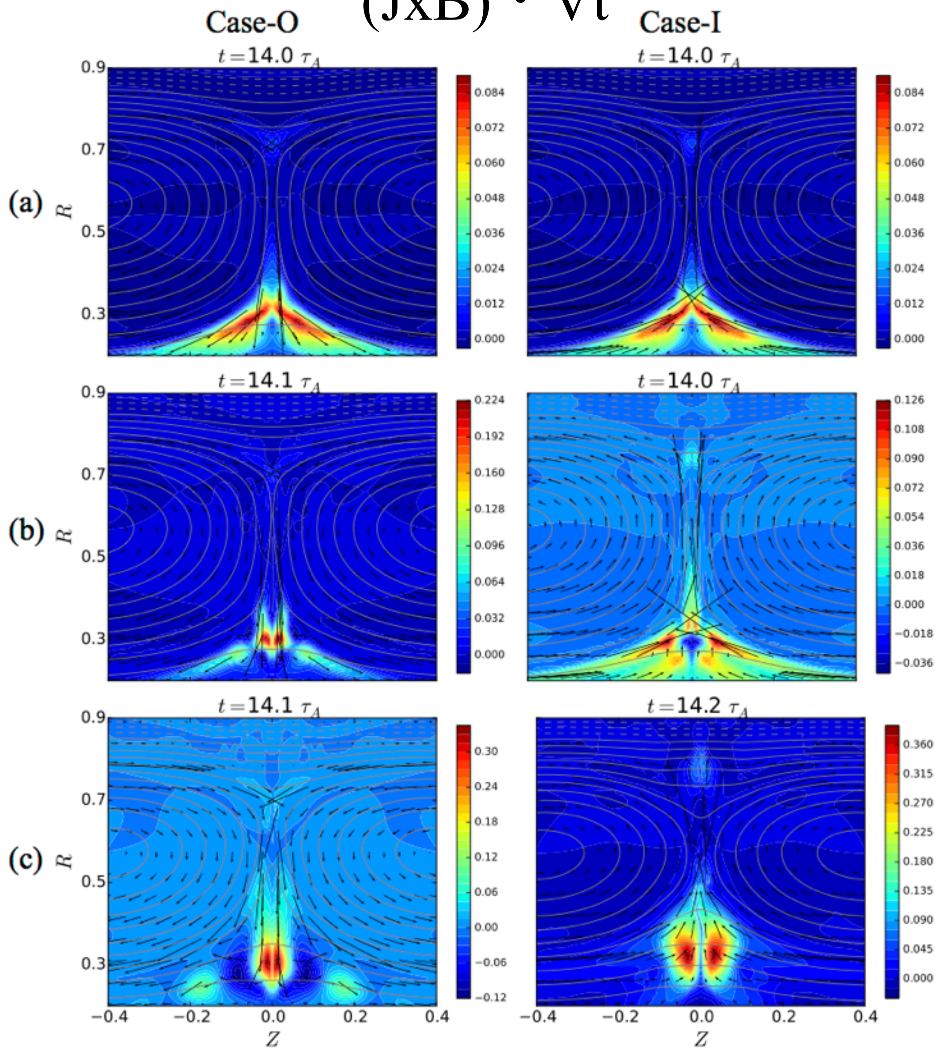
MHD

Hall-MHD
weak
(di=0.01)

Hall-MHD
strong
(di=0.05)



(JxB)・Vt



- Flow acceleration region of poloidal flow and toroidal flow are different.
- Poloidal (Vr,Vz): MHD…inside the current sheet / Hall-MHD…near the reconnection separatrix
- Toroidal (Vt): Strong acceleration region at the downstream region by Hall current.

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Particle-In Cell plasma merging simulation
Investigation ion & electron heating in counter-helicity merging

$$\frac{d(\gamma_k \mathbf{v}_k)}{dt} = \frac{q_k}{m_k}(\mathbf{E} + \mathbf{v}_k \times \mathbf{B})$$

$$\frac{d\mathbf{x}_k}{dt} = \mathbf{v}_k$$

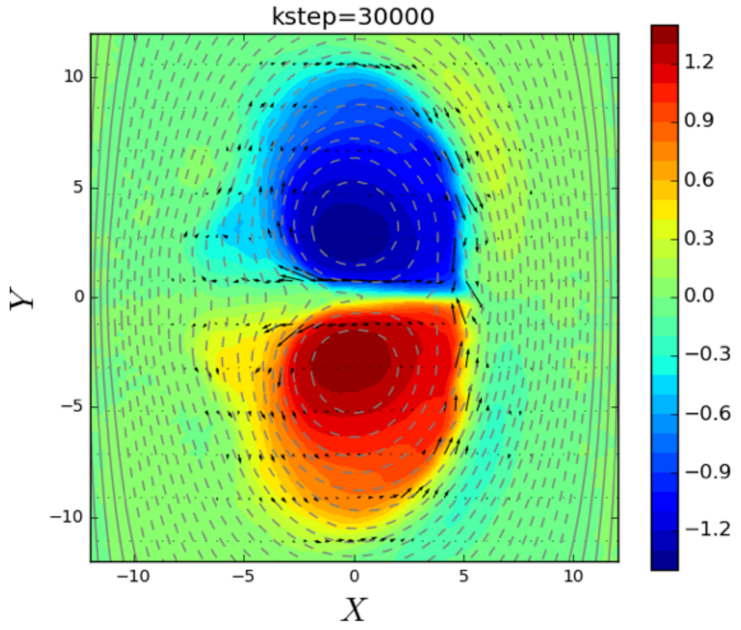
$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$\frac{\partial \mathbf{E}}{\partial t} = -\nabla \times \mathbf{B} - \mathbf{j}$$

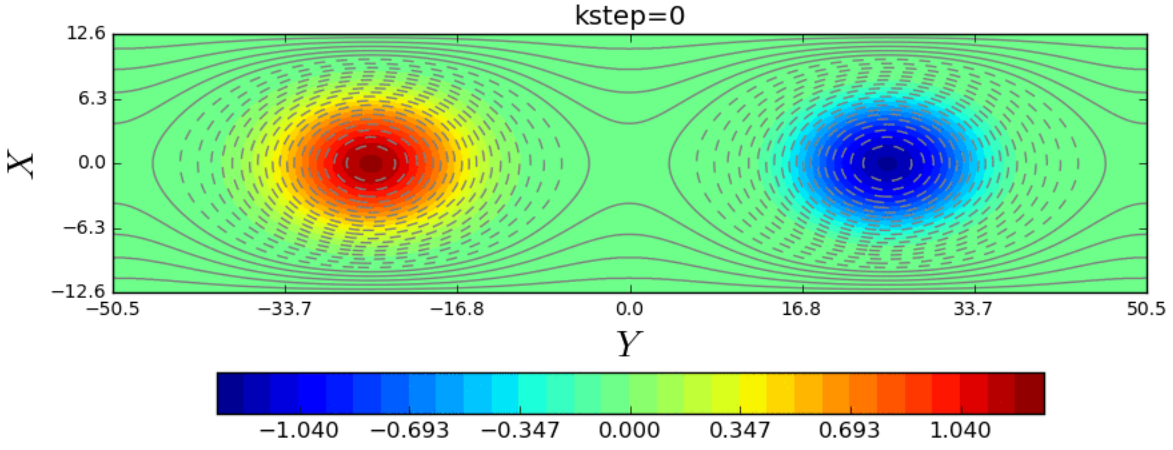
$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = \rho$$

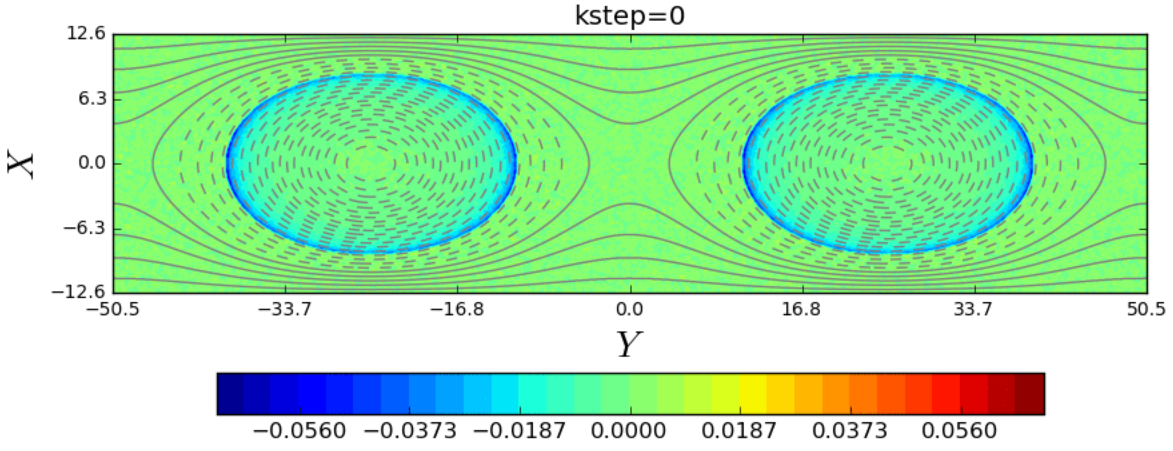
Spatial scale: electron skin depth (c/ωe)



Toroidal magnetic field (Bz)



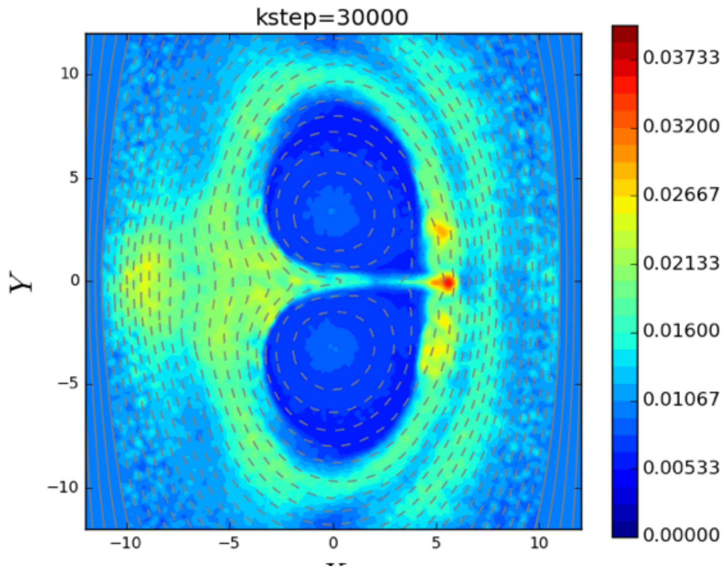
Ion Toroidal flow (Viz)



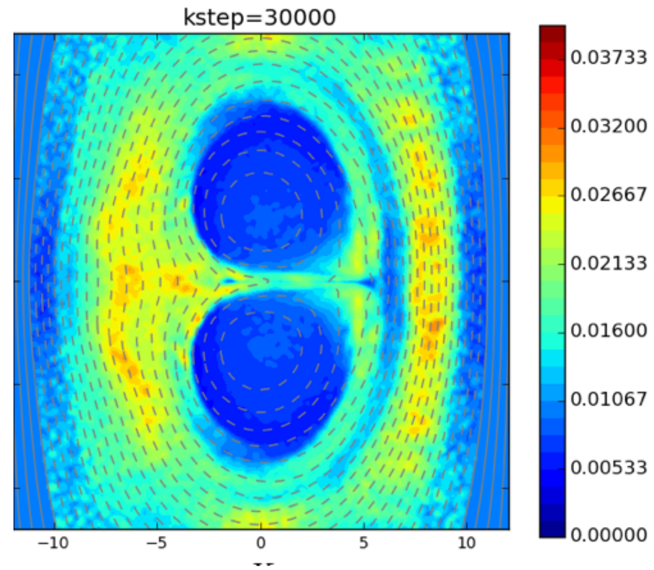
Using PIC simulation, we investigate heating mechanism near reconnection region.

Particle-In Cell plasma merging simulation
Ion & electron heating pattern on counter-helicity merging

Te, ⊥



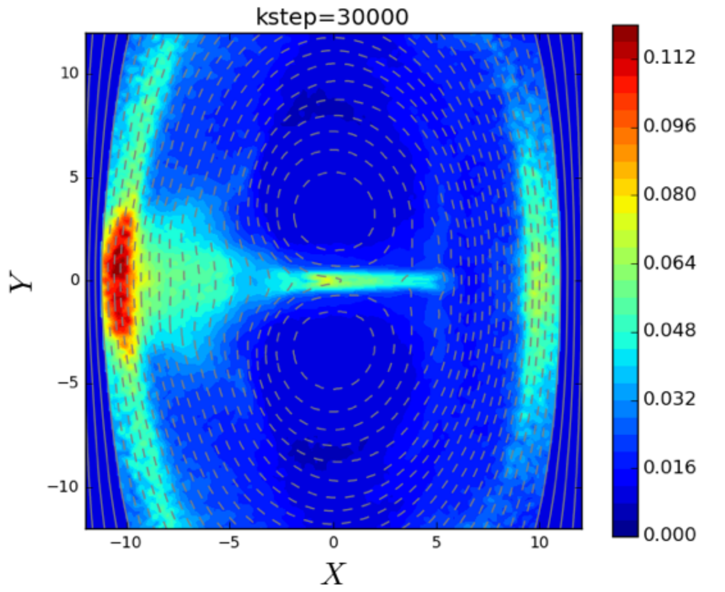
Te, //



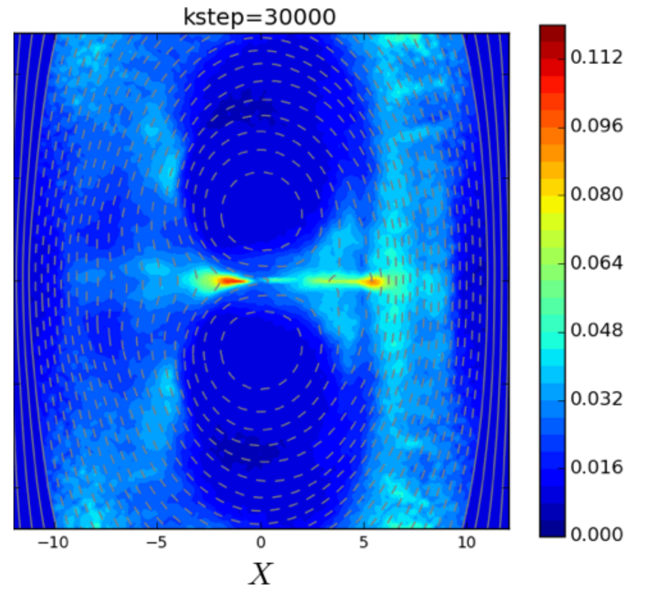
electron heating:

- Parallel heating is greater than perpendicular.

Ti, ⊥



Ti, //



ion heating:

- Perpendicular heating is dominant in the downstream region.
- Small parallel heating near the current sheet.

Outline

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Summary & Future work

Objective of my work:

- non-MHD effect (mainly Hall effect) on counter-helicity spheromak merging & formation process of an FRC

2D Hall-MHD simulation on counter-helicity spheromak merging:

- Hall effect changes global structure of magnetic field (X-point position), and thermal pressure distribution.
- Hall effect affect on merging speed through changing magnetic field / thermal pressure distribution.
- Hall effect changes flow and current sheet structure.

2D Particle-In Cell simulation on counter-helicity flux tube merging:

- Strong ion heating is observed at the downstream region.

Future Work:

- 3D Hall effect on merging / relaxation process of counter-helicity merging
- Detailed analysis of ion/electron heating mechanism in the PIC simulations.

Hall-MHD plasma merging simulation
Sub-cycling method

Hall term & whistler wave

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left(d_i \frac{\mathbf{j} \times \mathbf{B}}{\rho}\right)$$

dispersion relation

$$\omega = d_i \frac{v_A}{\sqrt{\rho}} k^2$$

wave velocity

$$v_{ph} = \frac{\omega}{k} \sim \frac{d_i}{\Delta x} \frac{v_A}{\sqrt{\rho}}$$

CFL condition

$$\Delta t < \frac{\Delta x}{v_{ph}}$$

MHD

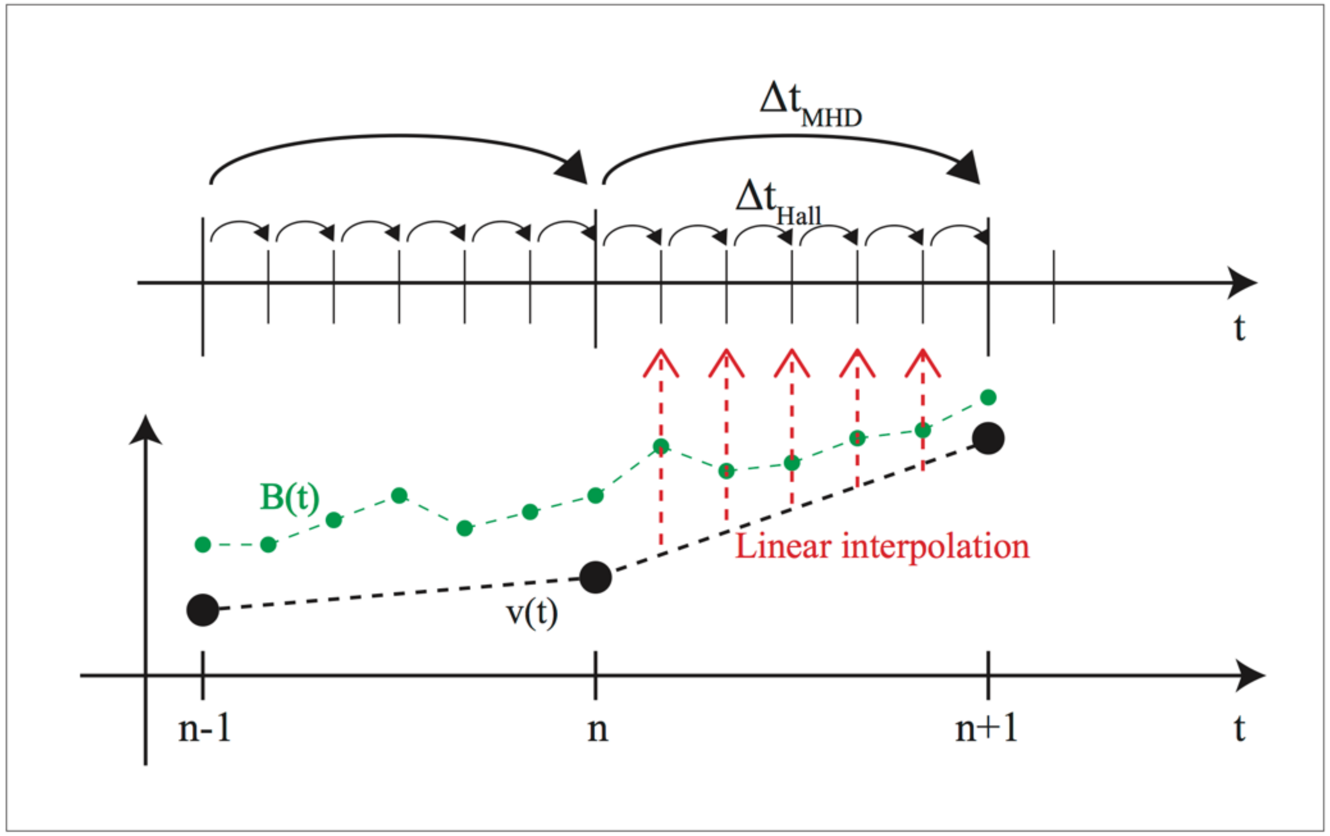
$$\frac{\Delta x}{v_A}$$

Hall-MHD

$$\frac{(\Delta x)^2}{d_i v_A}$$

Hall term strongly restrict the Δt .

Sub-cycling method



CFL: Alfven wave

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) \tag{1}$$

$$\frac{\partial p}{\partial t} = -\nabla \cdot (p\mathbf{v}) - (\gamma - 1) p (\nabla \cdot \mathbf{v}) + (\gamma - 1)\Big(\eta j^2 + \nu\big(\frac{4}{3}(\nabla \cdot \mathbf{v})^2 + |\nabla \times \mathbf{v}|^2\big)\Big) \tag{2}$$

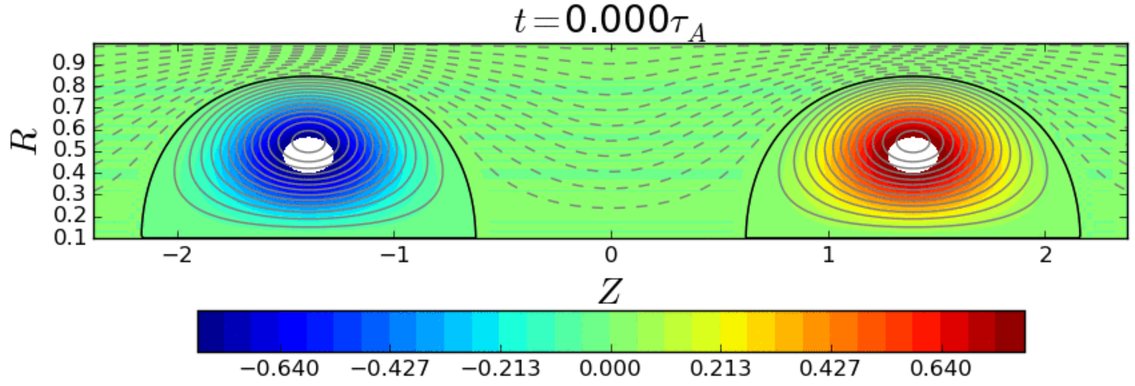
$$\frac{\partial (\rho \mathbf{v})}{\partial t} = -\nabla \cdot \rho \mathbf{v}\mathbf{v} - \nabla p + \mathbf{j} \times \mathbf{B} + \nu\left(\frac{4}{3}\nabla(\nabla \cdot \mathbf{v}) - \nabla \times \nabla \times \mathbf{v}\right) \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{v} \times \mathbf{B} - \eta \mathbf{j} - d_i \frac{\mathbf{j} \times \mathbf{B}}{\rho}\right) \tag{4}$$

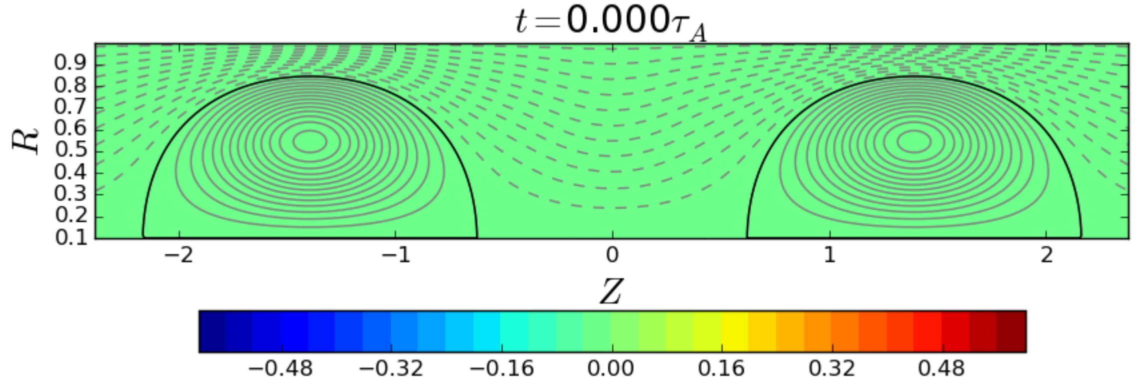
CFL: Whistler wave

Hall-MHD plasma merging simulation
Overview of merging (MHD)

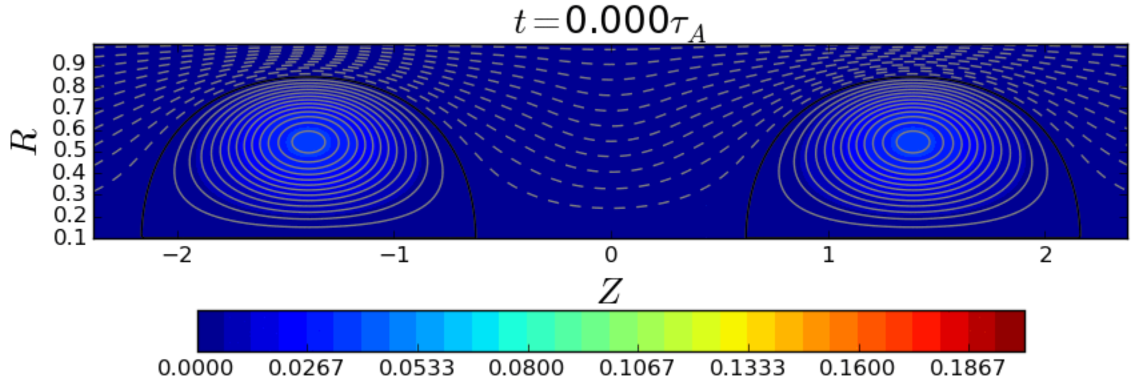
Toroidal magnetic field (Bt)



Toroidal flow (Vt)



Thermal pressure (P)



Counter-helicity merging:

- Toroidal magnetic field cancels out by merging.
- Thermal pressure increase by merging.
- Strong toroidal sheared-flow arise by merging.

Parameters:
(Nr,Nz)=(512,4032), Rm=2000, Re=2000
(Rw~20cm, B~100mT, Te~30eV)