Hall effect & polarity effect on flow structure in counter-helicity spheromak merging

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Outline

- Introduction
- Hall-MHD simulation
- Particle in Cell simulation
- Summary
Introduction & Background
FRC / Counter-helicity Spheromak Merging

The characteristics of Counter-helicity Spheromak Merging

- Spheromak formation is easier than forming an FRC by Field-Reversed Theta Pinch method.
- Spheromaks can get large magnetic flux by a MCPG (CHI) or a flux core. → easy to get a large flux FRC
- Magnetic reconnection with zero guide-field & reconnection with both poloidal and toroidal magnetic field.
- Non-MHD effects, such as Hall effect or toroidal effect on merging process are not fully understood.
Introduction & Background
Ion flow in high-beta relaxation

Two-fluid relaxation theory

\[
\begin{align*}
\text{ion fluid} & \quad P_i = m_i u_i + q_i A \\
\text{electron fluid} & \quad P_e = q_e A \\
\Omega_i = \nabla \times P_i &= m_i \omega_i + q_i B \\
\Omega_e = \nabla \times P_e &= q_e B \\
K_i &= \int P_i \cdot \Omega_i dV \\
K_e &= \int P_e \cdot \Omega_e dV = q_e^2 \int A \cdot B dV \\
\n\n\n\n\n\n= \nabla \times (\nabla \times B) - \alpha \nabla \times B + \beta B = 0 \\
\beta_i + 0.5 V^2 = \text{constant.}
\end{align*}
\]

\[\text{E. Kawamori et al., Nucl. Fusion (2005)}\]

- FRC is high-beta, and it is considered that FRC can be a two-fluid relaxation state.
- In two-fluid relaxation, ion flow is important.
- In merging-formed FRC, strong ion flow is generated by magnetic reconnection.

\[\therefore\text{To understand non-MHD effects on counter-helicity merging process is important.}\]
Goal: To understand whole formation process of an FRC by counter-helicity spheromak merging
   - plasma merging / magnetic reconnection
   - flow formation
   - ion/electron heating
   - relaxation / self-organization
   - two-fluid relaxation

Approach:
   - experiment
   - numerical simulation
     - Hall-MHD…Hall-parameter dependency
     - PIC …electron / ion heating mechanism

Plasma merging contains two scale:
   - Large scale (torus plasma)
   - Small scale (reconnection physics)
Counter-helicity merging has two patterns, which are defined by combinations of poloidal/toroidal magnetic flux.

Poloidal field (Br) reconnection… Inflow (Vz), Current sheet (Jt < 0), Outflow (Vr)
Toroidal field (Bt) reconnection… Inflow (Vz), Current sheet (Jr), Outflow (Vt)
Reconnection plane tilts toward toroidal direction, therefore counter-helicity is different from null-helicity.

In this presentation, we report the Hall effect, toroidal effect (polarity effect) through comparing case-O & case-I merging.
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In this simulation, we set the magnetic axis are unity. Electric resistivity (\(\eta\)), and the time step solving the density continuous equation is determined by CFL condition of whistler wave speed, and the time step for solving magnetic field and fluid quantities are di.

In this code, we installed sub-cycling scheme, which suppresses grid-scale numerical oscillations. In order numerical smoothing, 4th-order spatial explicit Hall-MHD code. 4th-order spatial differentiation and 2nd-order Adams-Bashforth scheme is adopted for time-advancing.

**Basic equations**

\[
\begin{align*}
\frac{\partial \rho}{\partial t} &= -\nabla \cdot (\rho \mathbf{v}) \\
\frac{\partial p}{\partial t} &= -\nabla \cdot (p \mathbf{v}) - (\gamma - 1)p(\nabla \cdot \mathbf{v}) \\
&\quad + (\gamma - 1)(\eta j^2 + \nu \left(\frac{4}{3}(\nabla \cdot \mathbf{v})^2 + |\nabla \times \mathbf{v}|^2\right)) \\
\frac{\partial (\rho \mathbf{v})}{\partial t} &= -\nabla \cdot \rho \mathbf{v} \mathbf{v} - \nabla p + \mathbf{j} \times \mathbf{B} \\
&\quad + \nu \left(\frac{4}{3}(\nabla \cdot \mathbf{v}) - \nabla \times \nabla \times \mathbf{v}\right) \\
\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \mathbf{j} - \frac{\mathbf{j} \times \mathbf{B}}{\rho})
\end{align*}
\]

**Hall term (di:ion skin depth)**

\[
\begin{align*}
\frac{\partial u^n}{\partial t} &= f^n(u) \\
\quad u^{n+1} &= u^n + \frac{\Delta t}{2}(3f^n - f^{n-1})
\end{align*}
\]

**Spacial difference: 4th-order central difference**

Numerical viscosity: 4th-order smoothing

\[
F_i = (1 - \alpha)F_i + \alpha \frac{-F_{i-2} + 4F_{i-1} + 4F_{i+1} - F_{i+2}}{6}
\]

**Parameters:**

(Nr,Nz)=(512,4032), Rm=2000, Re=2000 (Rw~20cm, B~100mT, Te~30eV)

**FIG. 2.**

**Toroidal magnetic field (Bt)**

**Thermal pressure (P)**
Hall-MHD plasma merging simulation
Polarity effect & Hall effect on merging speed

merging rate = (reconnected flux)/(total flux)

\[ \alpha = \frac{\Psi_{com}}{\Psi_{max}} \]

Case-O: Hall effect does **not enhance** merging speed.
Case-I: Hall effect **enhances** merging speed.
Merging speed is different between case-O and case-I. Polarity effect appears.

→ Merging speed is determined by global pressure balance of inflow region / outflow region.
Hall-MHD plasma merging simulation
Hall effect on current sheet structure

- Current sheet length become short.
- Radial current sheet ($J_r$) extend to downstream region.

...Difference between toroidal current sheet and poloidal current sheet becomes large.
Hall-MHD plasma merging simulation
Polarity effect & Hall effect on flow structure

Hall effect on flow formation by merging:
- Outflow (V_r) is biased to single direction. Case-O is negative, and case-I is positive.
- Toroidal flow is negative near the X-point, and positive flow arises at the downstream region.
Hall-MHD plasma merging simulation
role of Hall effect on flow formation

1. Hall effect moves X-point along the electron flow (electron current) at the reconnection point.
2. Reconnection outflow $V_r$ is strong toward the opposite direction from the X-point motion.
3. The strong radial flow and the large ion inertia generates Hall current ($J_r$) at the downstream region.
4. The Hall current ($J_r$) generates the strong positive toroidal flow at the downstream region.

$\therefore$ Hall effect in reconnection + outflow damping $\rightarrow$ flow generation at the downstream region
Hall-MHD plasma merging simulation
Hall effect on structure of energy conversion (ion acceleration) region

\[
\mathbf{E} \cdot \mathbf{j} = \eta j^2 + (\mathbf{j} \times \mathbf{B}) \cdot \mathbf{v}
\]

- Magnetic dissipation to thermal energy
- (\(JxB\)) \(\cdot \) \(V_{pol}\)

**MHD**

- Flow acceleration region of poloidal flow and toroidal flow are different.
- Poloidal (\(V_r, V_z\)): MHD...inside the current sheet / Hall-MHD...near the reconnection separatrix
- Toroidal (\(V_t\)): Strong acceleration region at the downstream region by Hall current.

**Hall-MHD**

- Weak (di=0.01)
- Strong (di=0.05)
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Particle-In Cell plasma merging simulation
Investigation ion & electron heating in counter-helicity merging

\[
\frac{d(\gamma_k \mathbf{v}_k)}{dt} = \frac{q_k}{m_k} (\mathbf{E} + \mathbf{v}_k \times \mathbf{B})
\]

\[
\frac{dx_k}{dt} = \mathbf{v}_k
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}
\]

\[
\frac{\partial \mathbf{E}}{\partial t} = -\nabla \times \mathbf{B} - \mathbf{j}
\]

\[
\nabla \cdot \mathbf{B} = 0
\]

\[
\nabla \cdot \mathbf{E} = \rho
\]

Spatial scale: electron skin depth \((c/\omega_e)\)

Using PIC simulation, we investigate heating mechanism near reconnection region.
Particle-In Cell plasma merging simulation
Ion & electron heating pattern on counter-helicity merging

**2H1-IX**

**Te, \(\perp\)**
- Perpendicular heating is dominant in the downstream region.
- Small parallel heating near the current sheet.

**Te, //**
- Parallel heating is greater than perpendicular.

**Ti, \(\perp\)**

**Ti, //**
- Perpendicular heating is dominant in the downstream region.

**electron heating:**
- Parallel heating is greater than perpendicular.

**ion heating:**
- Perpendicular heating is dominant in the downstream region.
- Small parallel heating near the current sheet.
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Objective of my work:
- non-MHD effect (mainly Hall effect) on counter-helicity spheromak merging & formation process of an FRC

2D Hall-MHD simulation on counter-helicity spheromak merging:
- Hall effect changes global structure of magnetic field (X-point position), and thermal pressure distribution.
- Hall effect affect on merging speed through changing magnetic field / thermal pressure distribution.
- Hall effect changes flow and current sheet structure.

2D Particle-In Cell simulation on counter-helicity flux tube merging:
- Strong ion heating is observed at the downstream region.

Future Work:
- 3D Hall effect on merging / relaxation process of counter-helicity merging
- Detailed analysis of ion/electron heating mechanism in the PIC simulations.
Hall-MHD plasma merging simulation
Sub-cycling method

Hall term & whistler wave
\[ \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left( d_i \frac{\mathbf{j} \times \mathbf{B}}{\rho} \right) \]

dispersion relation
\[ \omega = d_i \frac{v_A}{\sqrt{\rho}} k^2 \]
wave velocity
\[ v_{ph} = \frac{\omega}{k} \sim \frac{d_i v_A}{\Delta x \sqrt{\rho}} \]

Sub-cycling method

CFL condition
\[ \Delta t < \frac{\Delta x}{v_{ph}} \]

MHD
\[ \frac{\Delta x}{v_A} \]

Hall-MHD
\[ \left( \frac{\Delta x}{d_i v_A} \right)^2 \]

Hall term strongly restrict the \( \Delta t \).

CFL: Alfven wave
\[ \frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) \] (1)
\[ \frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) - (\gamma - 1) p (\nabla \cdot \mathbf{v}) + (\gamma - 1) (\eta j^2 + \nu \frac{4}{3} (\nabla \cdot \mathbf{v})^2 + |\nabla \times \mathbf{v}|^2) \] (2)
\[ \frac{\partial (\rho \mathbf{v})}{\partial t} = -\nabla \cdot (\rho \mathbf{v} \mathbf{v}) - \nabla p + \mathbf{j} \times \mathbf{B} + \nu \frac{4}{3} (\nabla \cdot \mathbf{v}) - \nabla \times \nabla \times \mathbf{v} \] (3)
\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \mathbf{j} - d_i \frac{\mathbf{j} \times \mathbf{B}}{\rho}) \] (4)

CFL: Whistler wave
Hall-MHD plasma merging simulation
Overview of merging (MHD)

2H1-IX

Counter-helicity merging:
• Toroidal magnetic field cancels out by merging.
• Thermal pressure increase by merging.
• Strong toroidal sheared-flow arise by merging.

Parameters:
(Nr,Nz)=(512,4032), Rm=2000, Re=2000
(Rw~20cm, B~100mT, Te~30eV)