Asynchronous 3D HYPERS Simulations of Compact Toroids and Magnetoplasmas

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Hybrid Model

(radiation-free + kinetic ions + inertialess fluid electrons + quasineutrality)

$$0 = \frac{4\pi}{c}\left(\mathbf{j}_i + \mathbf{j}_e\right) - \nabla \times \mathbf{B}$$

$$0 = en_e\left[\mathbf{E} - \eta\left(\mathbf{j}_i + \mathbf{j}_e\right)\right] - \frac{\mathbf{j}_e \times \mathbf{B}}{c} + \nabla p_e$$

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

$$en_e = q_i n_i \,, \mathbf{j}_e = -en_e \mathbf{v}_e$$

$$p_e = n_e T_e \sim n_e^\gamma$$

Why Go Hybrid

- Full-orbit ion dynamics (high-beta, reconnection)
- Hall effect (reconnection and fast m.f. penetration)
- Ion energization (heating & acceleration)
- Ion skin depth (shocks and boundaries)
- Plasma mixing (counter streaming)

Hybrid paradigm is a natural choice for global modeling but one has to handle disparate ion and whistler scales

My Hybrid Adventure

• Wrote first hybrid codes (2D FIRE, 3D FLAME) at Cornell to study ion rings in FIREX (1993-1998)

• Applied these codes to simulate 2D θ-pinch FRC formation and 3D stability of FRCs at GA (2000-2001)

• Contributed to developing the H3D code for modeling the Earth's magnetosphere at UCSD (2005-2014)

• **A new code (HYPERS)** has changed the way hybrid simulation is conducted (2010-now)

Two Simulation Worlds

Event-
Driven

Time-
Driven

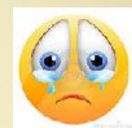
Engineering
Simulation

NO
CROSS-TALK

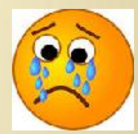
Scientific
Simulation

Three Bad Things About Using Time as a Driver

Time is NOT a measure of CHANGE

→ Idle CPU speed 

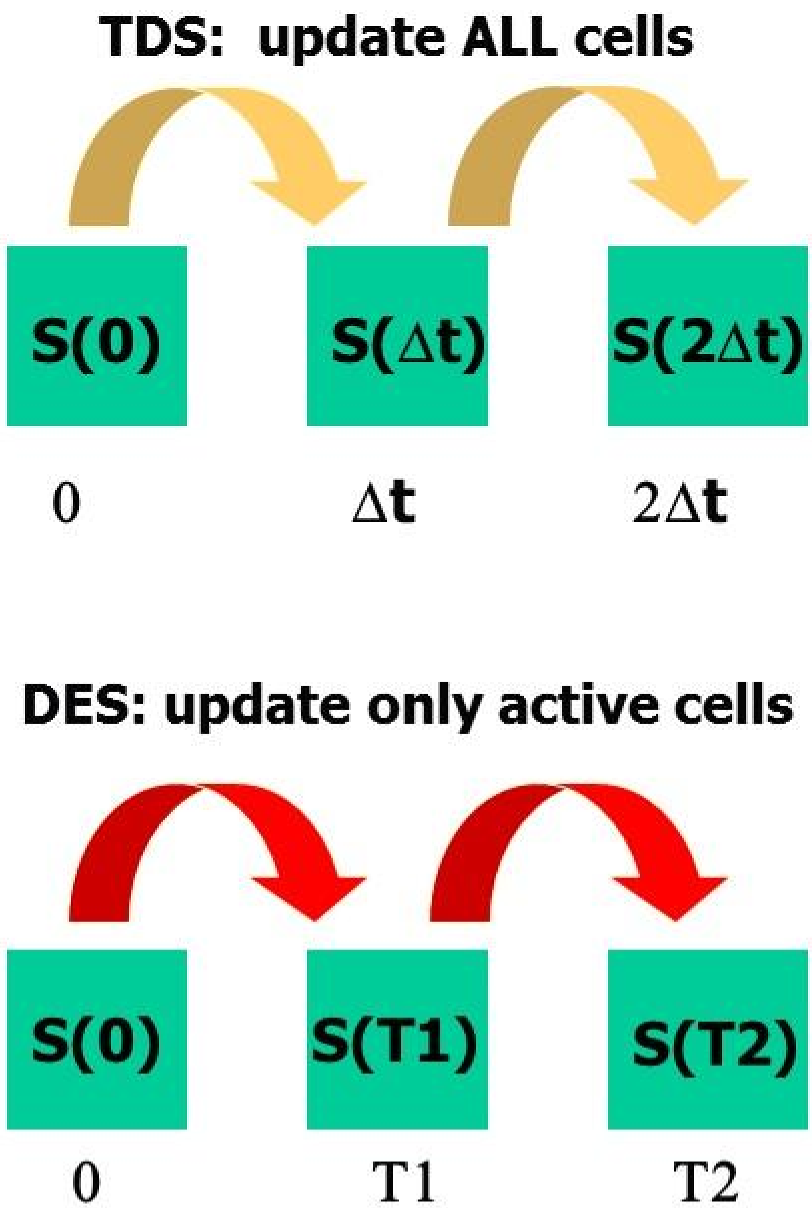
Time is NOT a ruler of ACCURACY

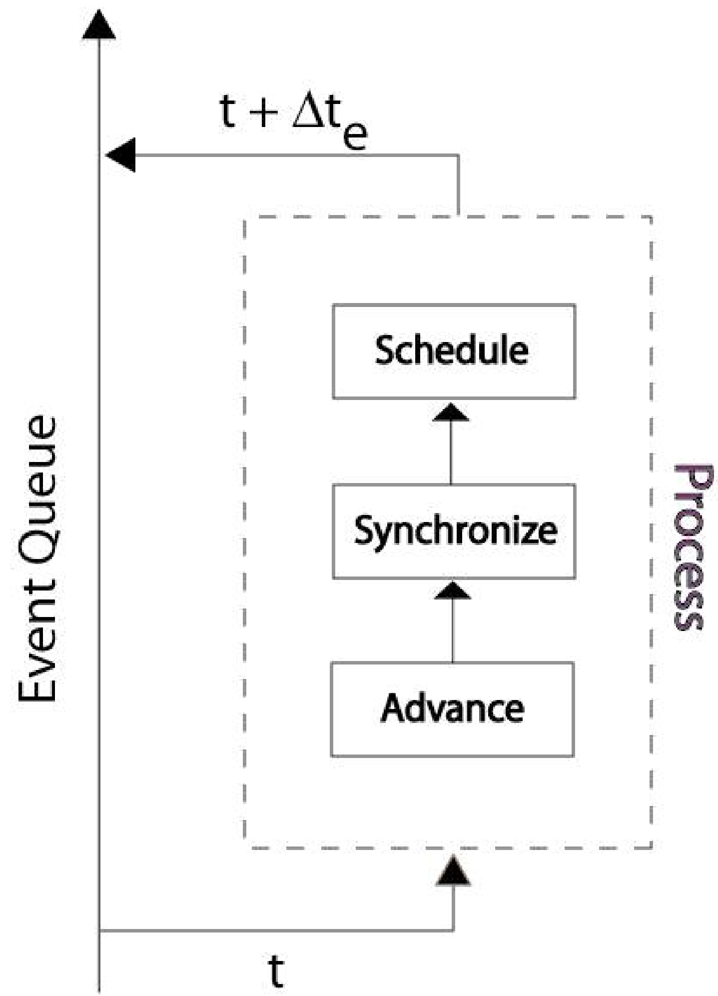
→ Missed correlations 

Time is NOT a guarantee of CAUSALITY

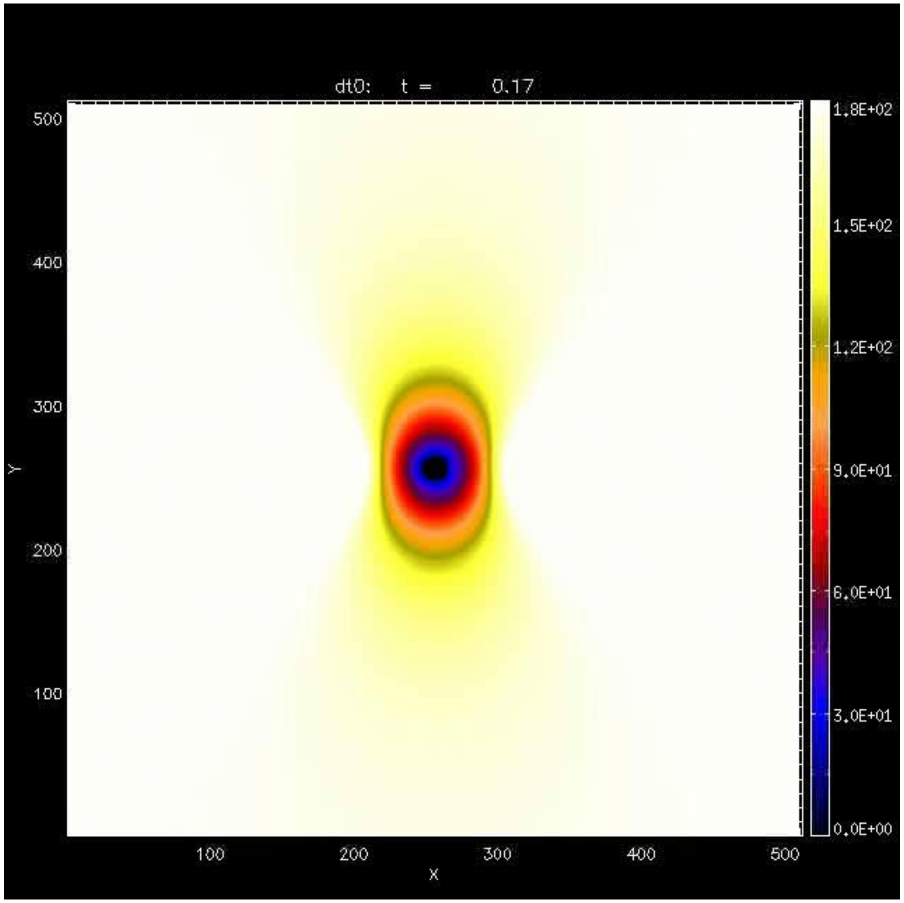
→ Unphysical results !!! 

Self-Adaptive Integration: EMAPS

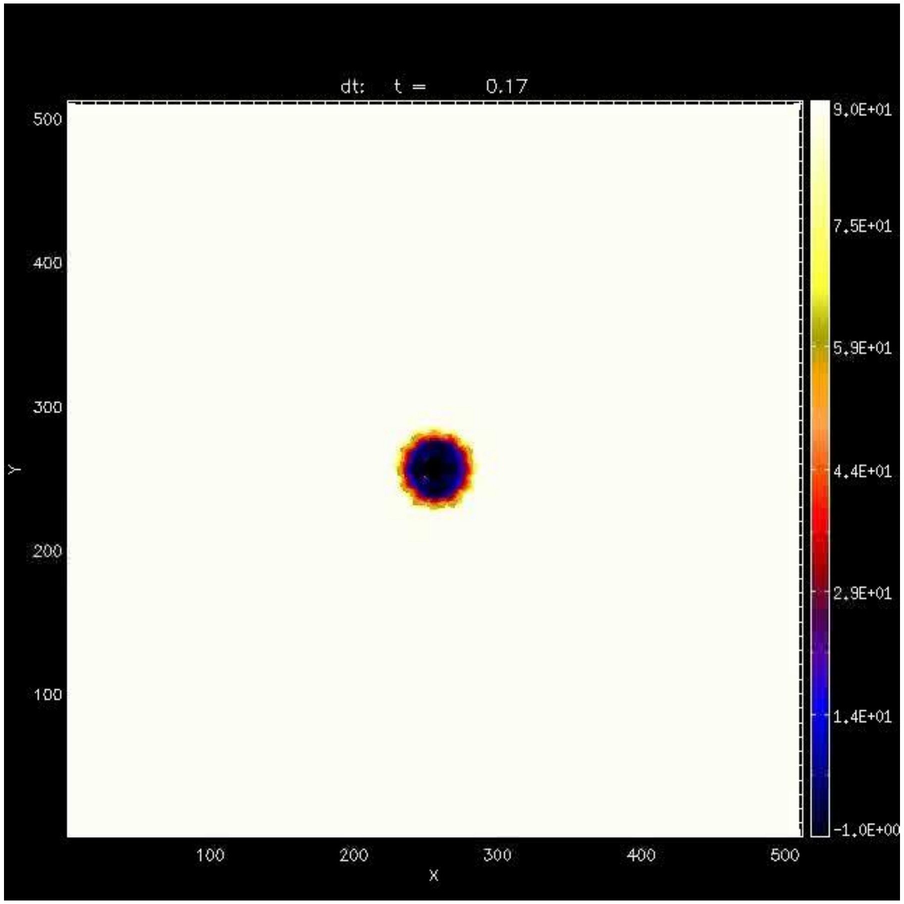




PIC Δt



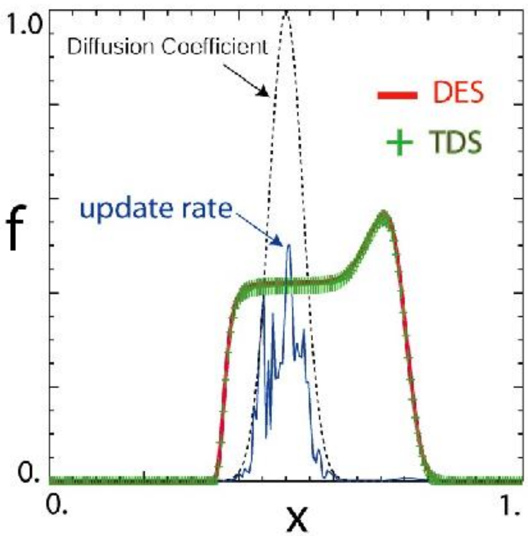
Field Δt



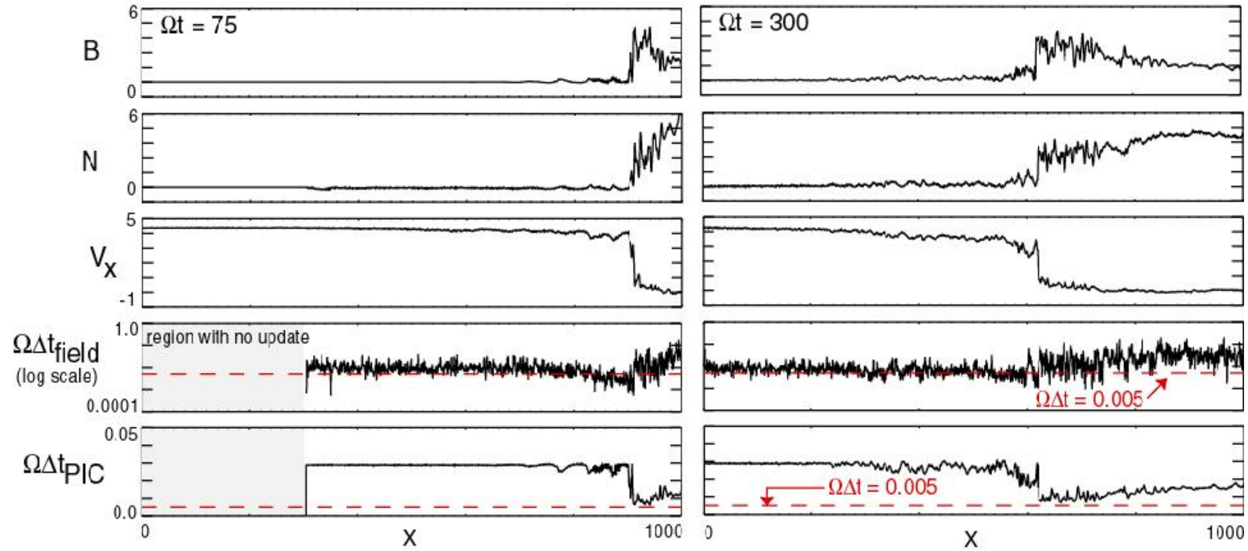
Local timesteps self-adapt in time

History of EMAPS: 2006-2013

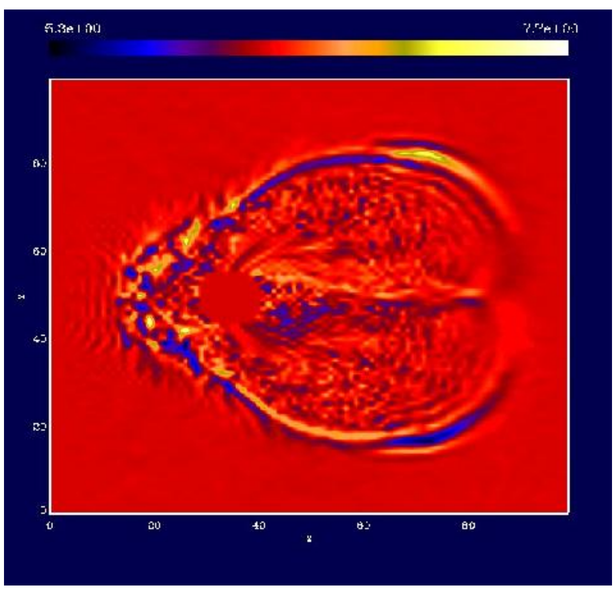
2006: 1D PDEs



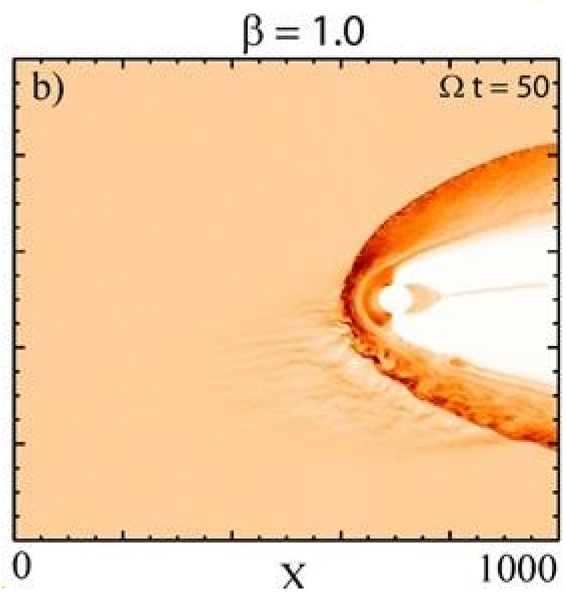
2008: 1D Hybrid



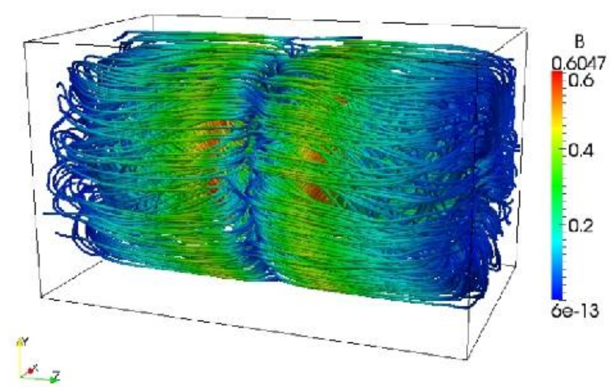
2010: 2D-Hybrid (serial)



2012: 2D HYPERS



2013: 3D HYPERS



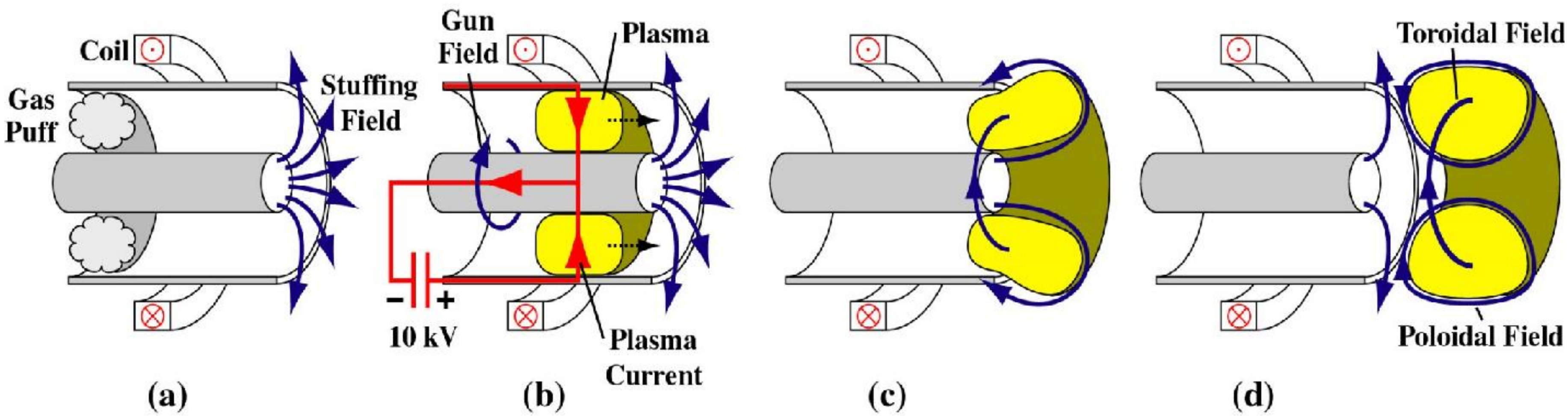
HYPERS: a "3G" Hybrid Code

- **EMAPS integration: robustness and speed**
- Compile-configurable (2D/3D) Cartesian grid
- Staircase walls with masked cells
- **Staggered (Yee) grid: exact preservation of divB=0**
- Local and periodic boundary conditions, injection
- Multiple ion species
- Phenomenological resistivity models
- **Momentum/energy conserving ion-ion collisions**
- Object-oriented programming (C++/Fortran)
- Scalable (>100K cpus), flexible domain decomposition
- Post-processing/visualization (IDL/ParaView)
- **Current setups**: spheromaks, FRCs, astrophysics-in-lab experiments, turbulence, space plasma flows with obstacles ("Earth", "Moon"), etc.

3D Spheromak Merging[1,2]
(2013)

with
[1]H. Karimabadi (SciberQuest, Inc)
[2]M. Brown (Swarthmore College)

SSX: "MHD Wind Tunnel"

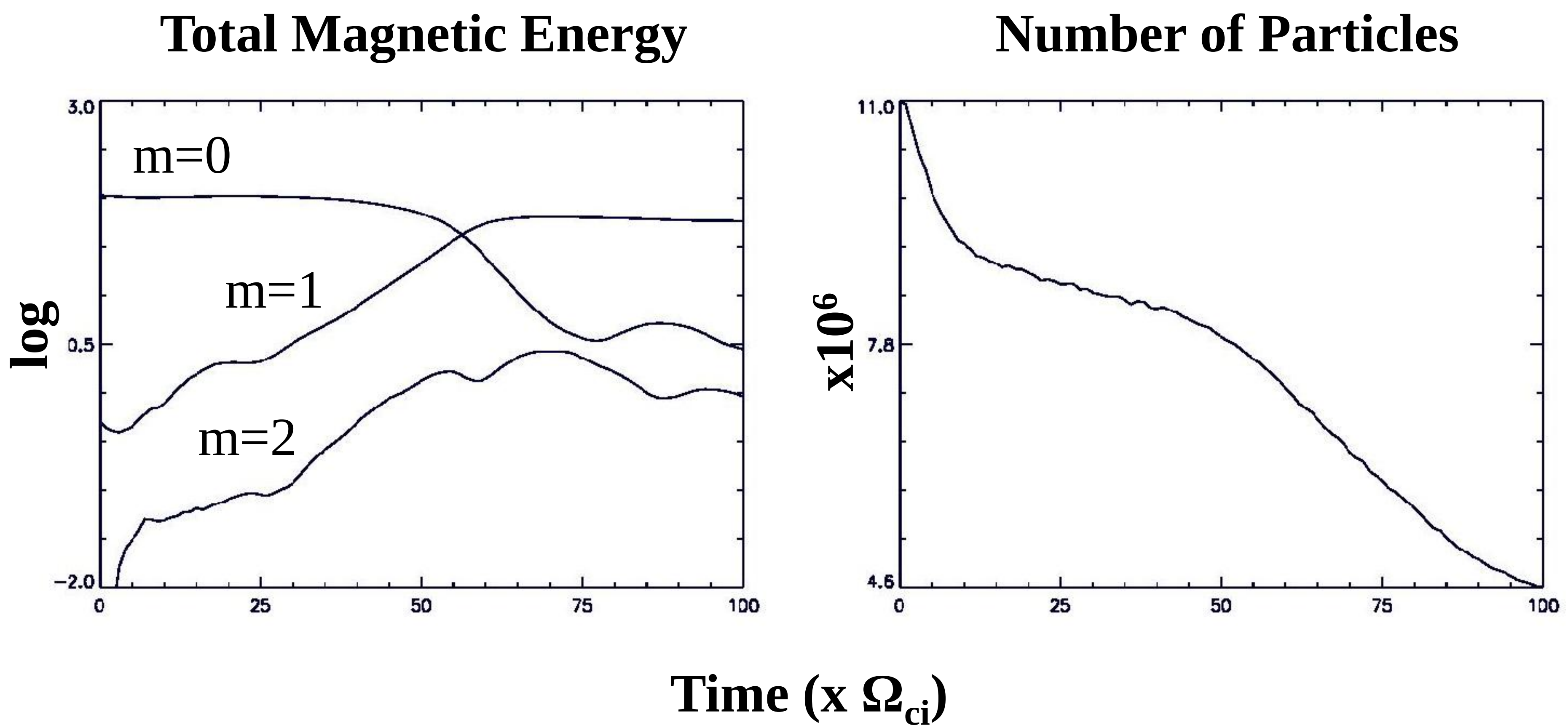


Ion Density (protons)	10^{14} -10^{15} cm^{-3}
Temperature (T_e,T_i)	20 - 80 eV
Magnetic Field	0.1 Tesla
Ion gyroradius	0.5 cm
Alfvén speed	100 km/s
S (Lundquist number)	>1000
Plasma β	10-100%
Poloidal flux	3-4 mWb

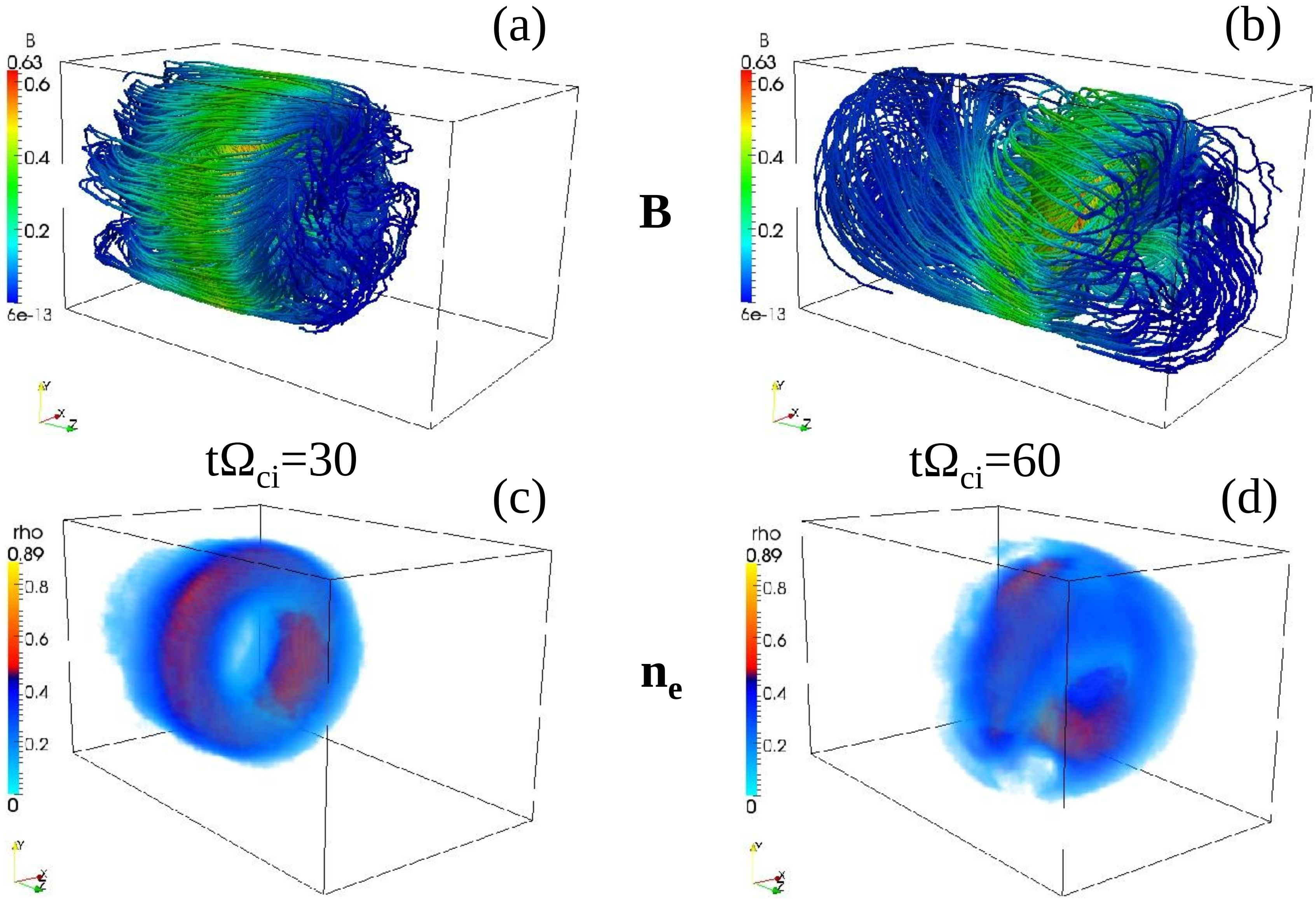
Spheromak simulation setup

- Initialized with quasi-axisymmetric spheromaks
- Initial flow speed ~ 75 km/sec
- Absorbing walls, S > 1000, no ion-ion collisions
- Perfectly conducting flux limiter: L=80cm, R=8cm
- Case 1: single spheromak dynamics
- Cases 2-3: colliding counter-helicity spheromaks

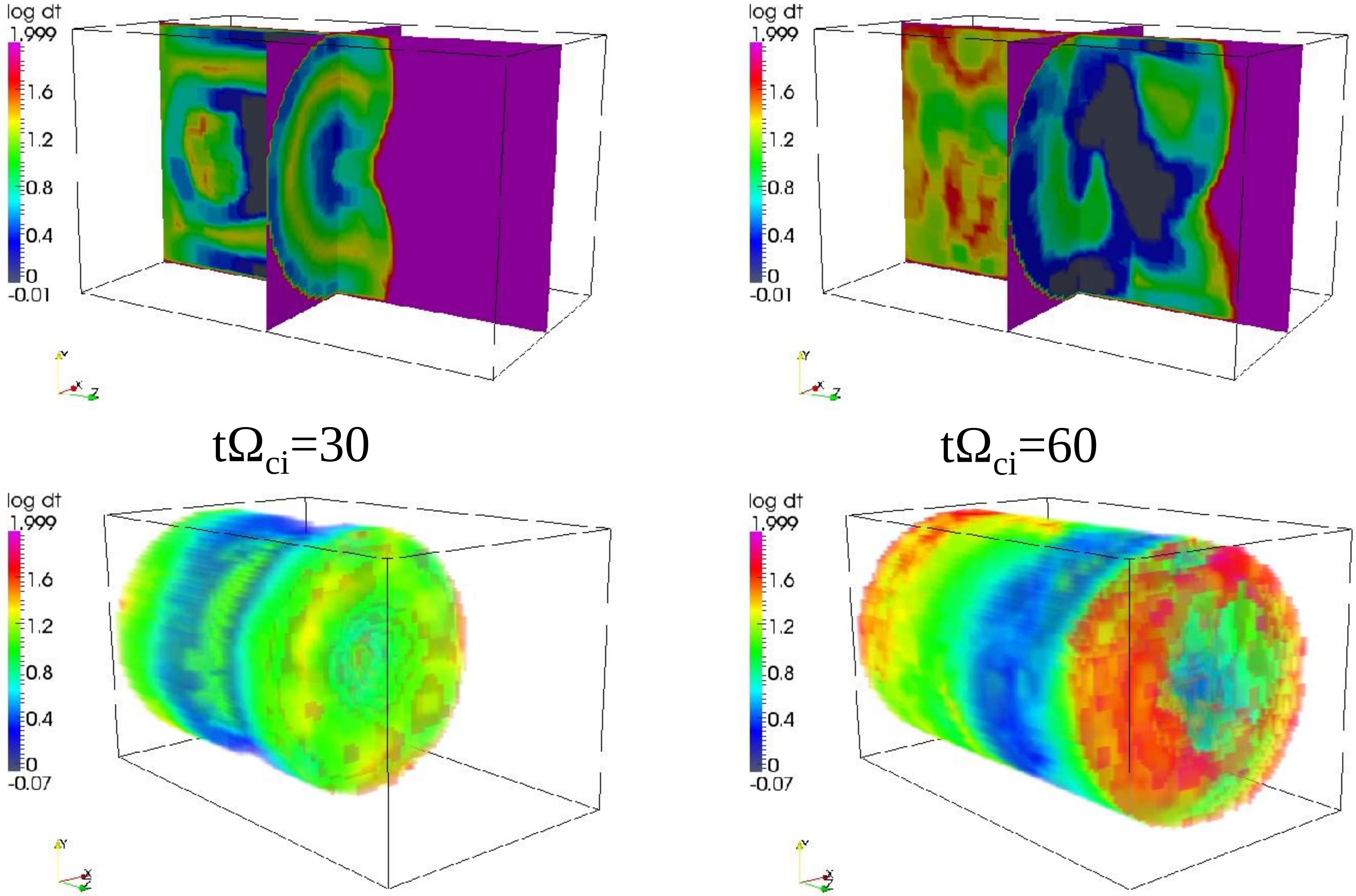
Case 1: Single spheromak



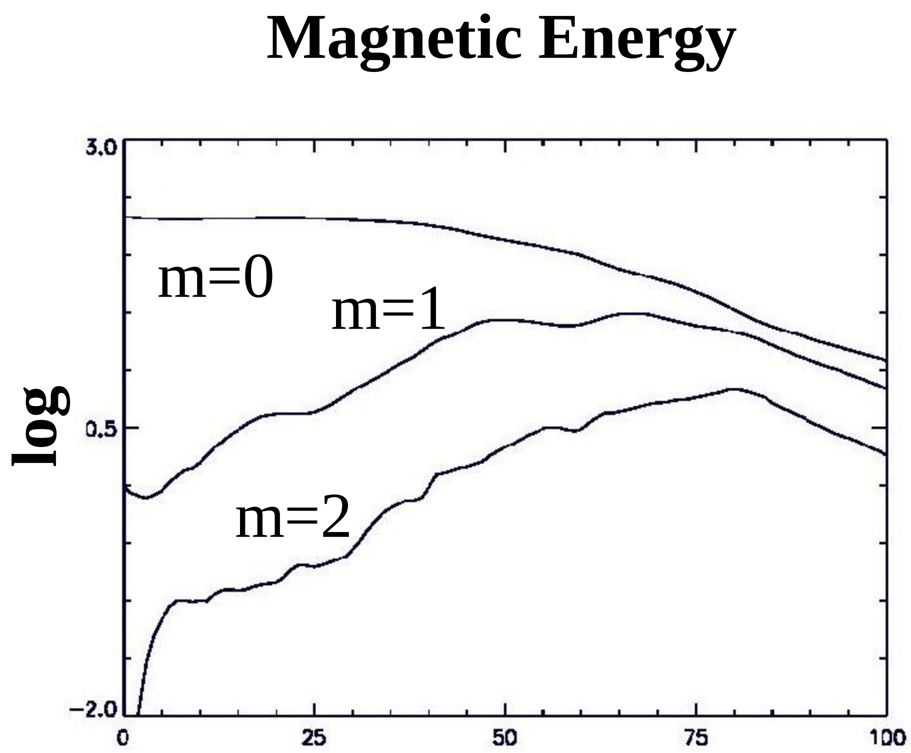
Single spheromak

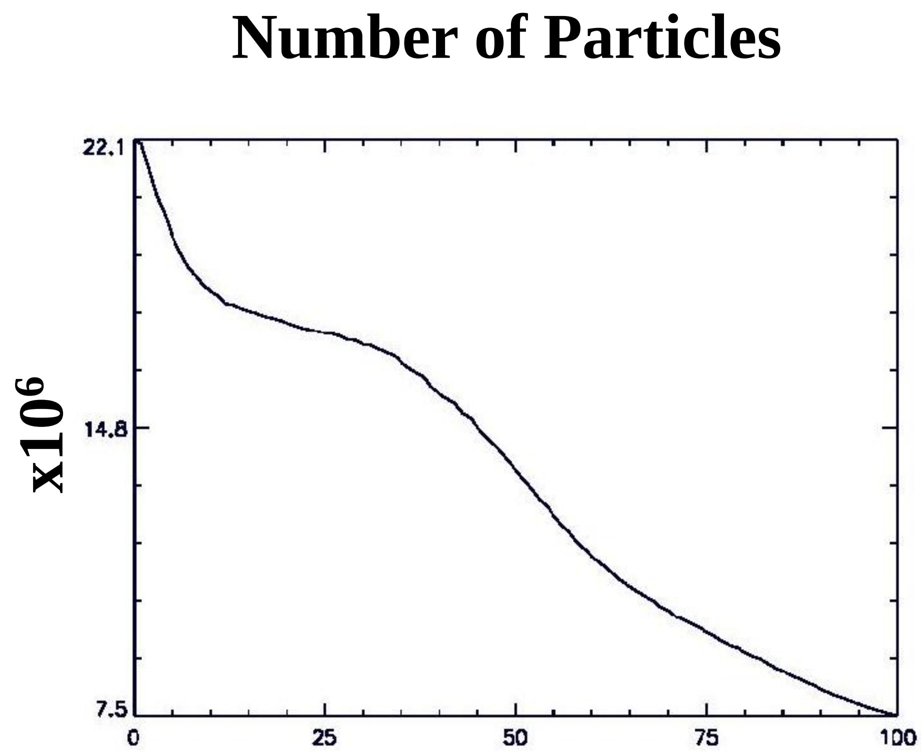


Single spheromak: field-Δt



Case 2:Two “tilting” spheromaks

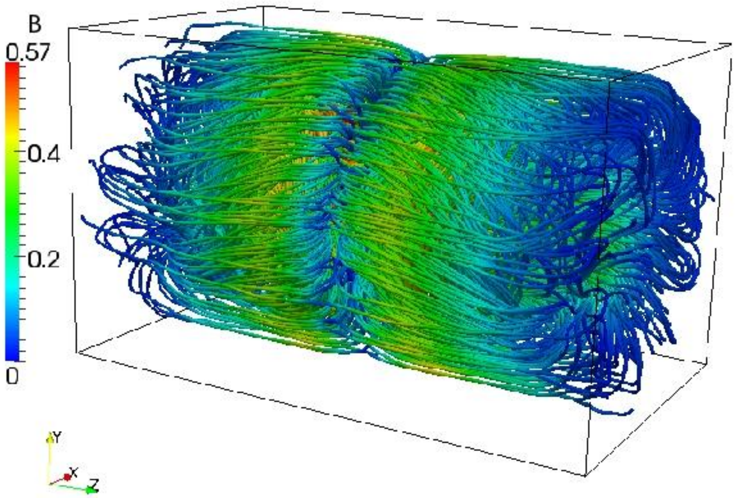




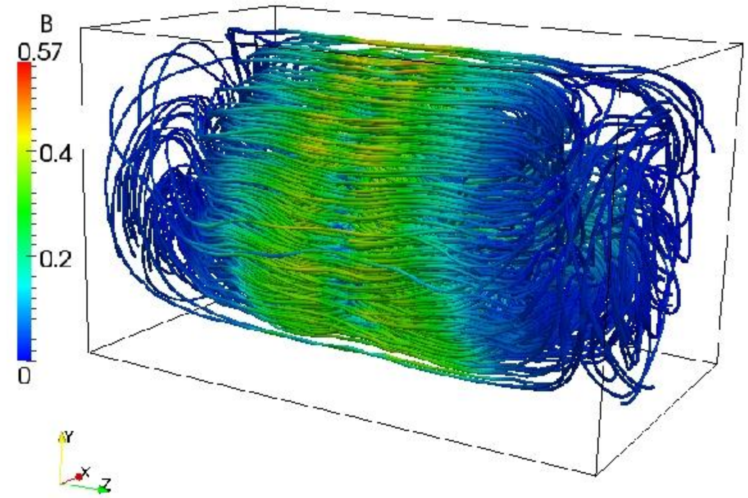
Time (x Ω_{ci})

Final result: asymmetric FRC

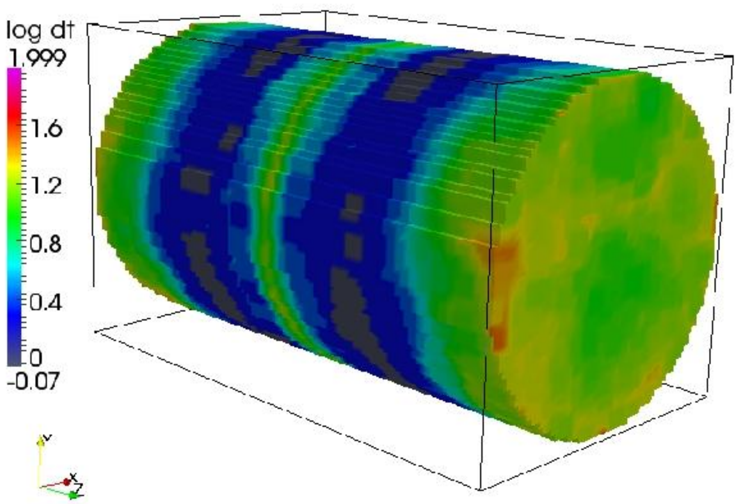
Two “tilting” spheromaks



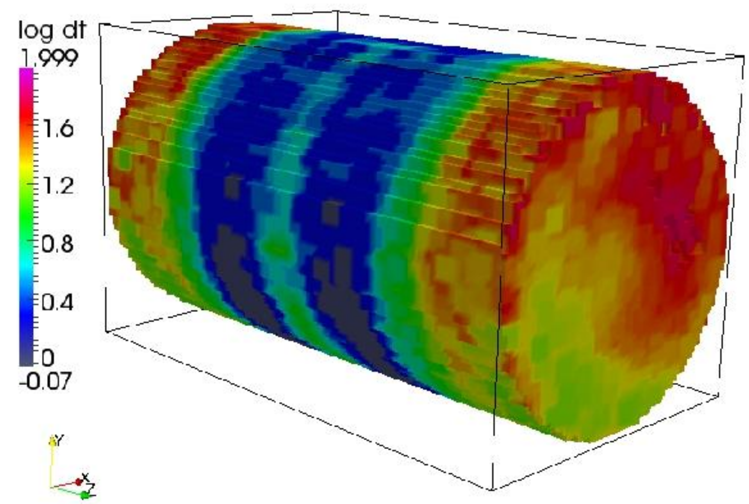
B



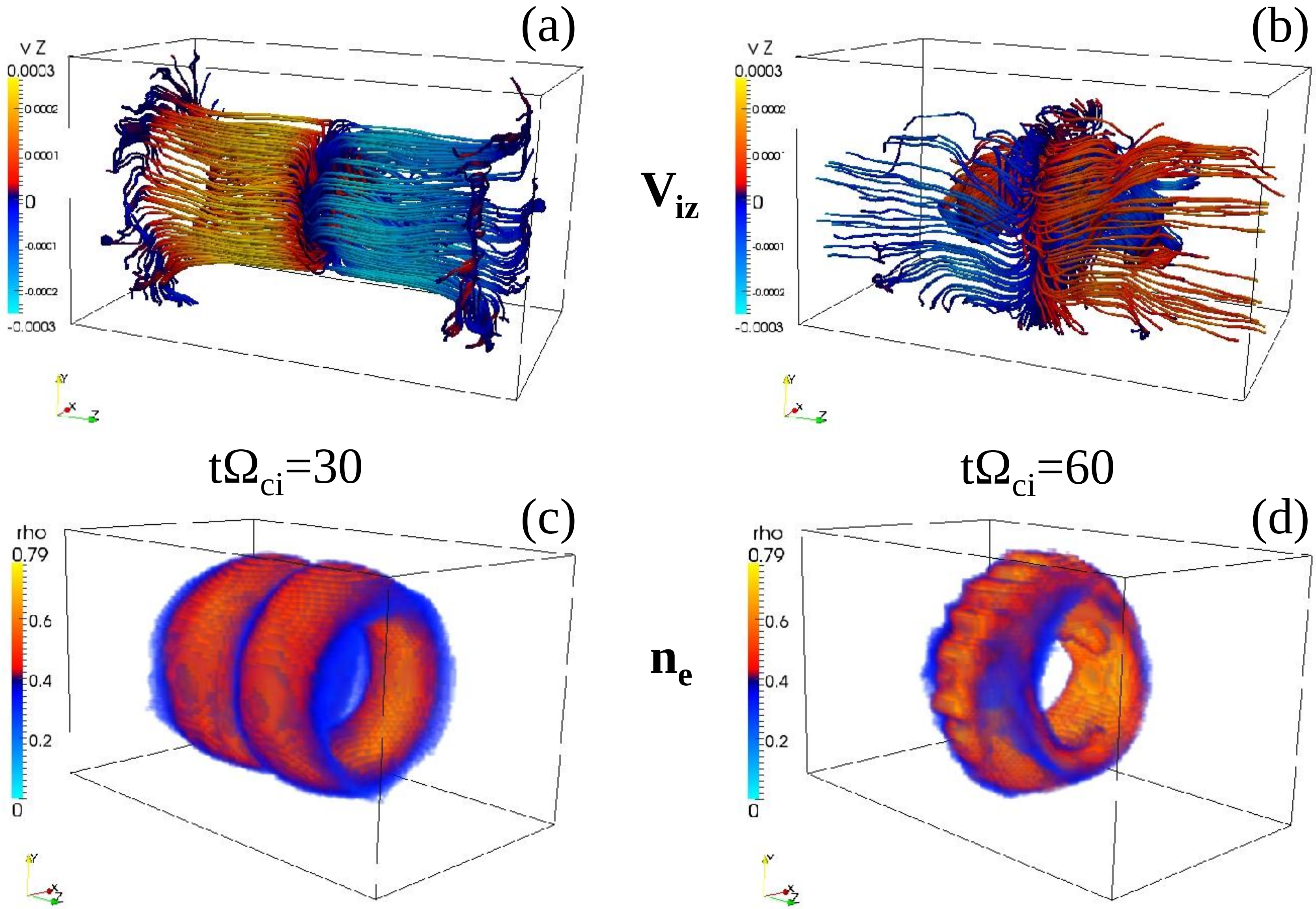
$t\Omega_{ci}=30$ $t\Omega_{ci}=60$



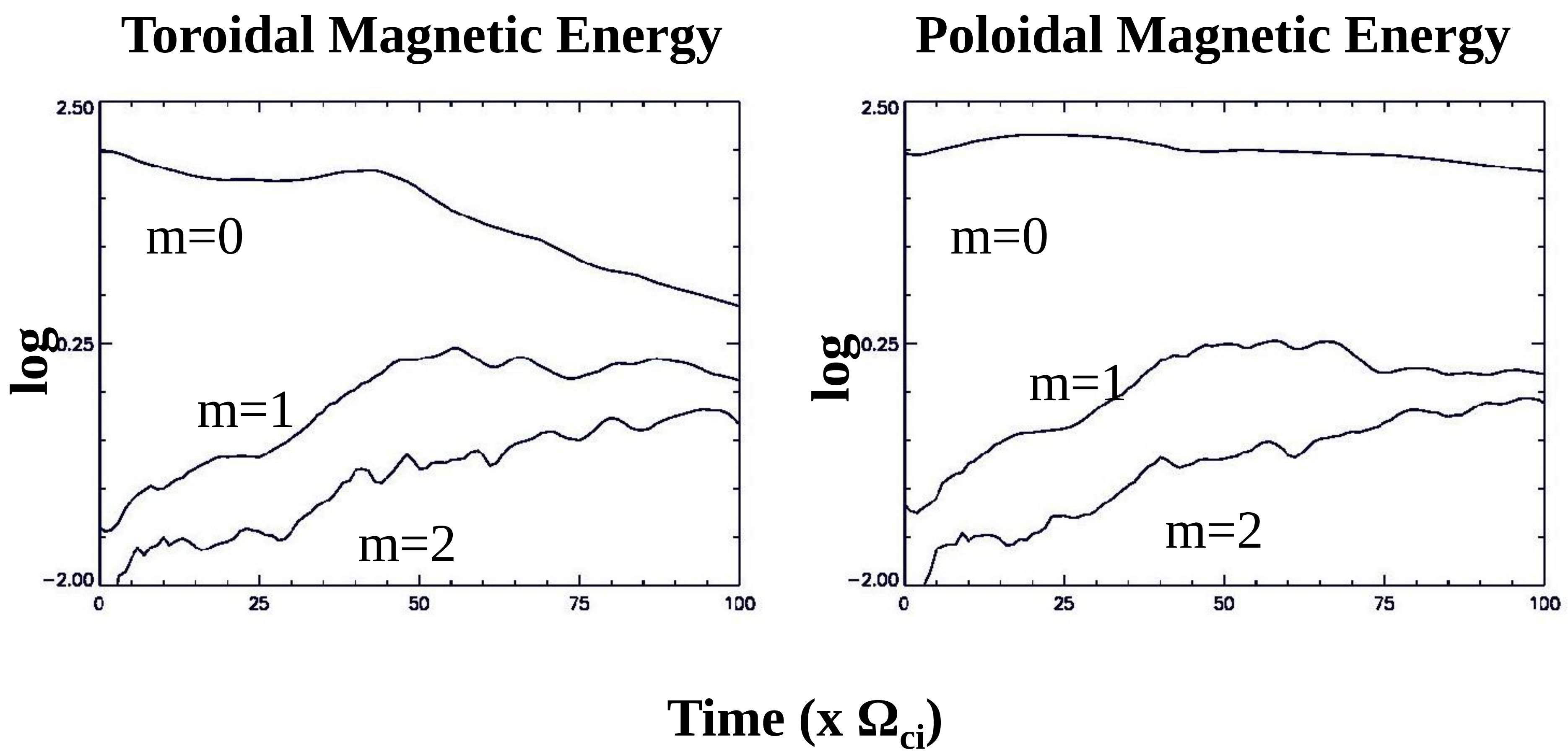
Δt



Two “tilting” spheromaks

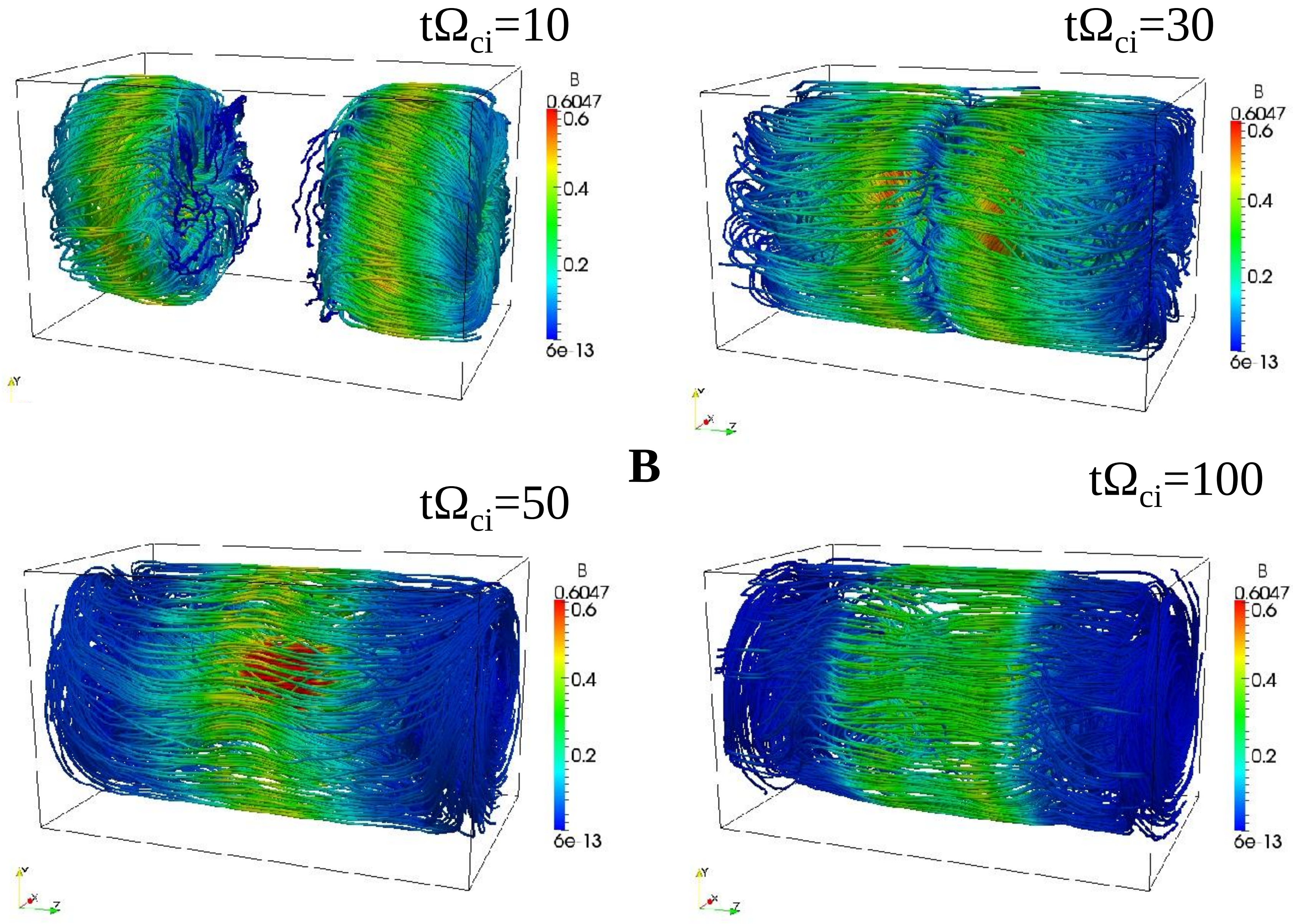


Case 3:Two "stable" spheromaks

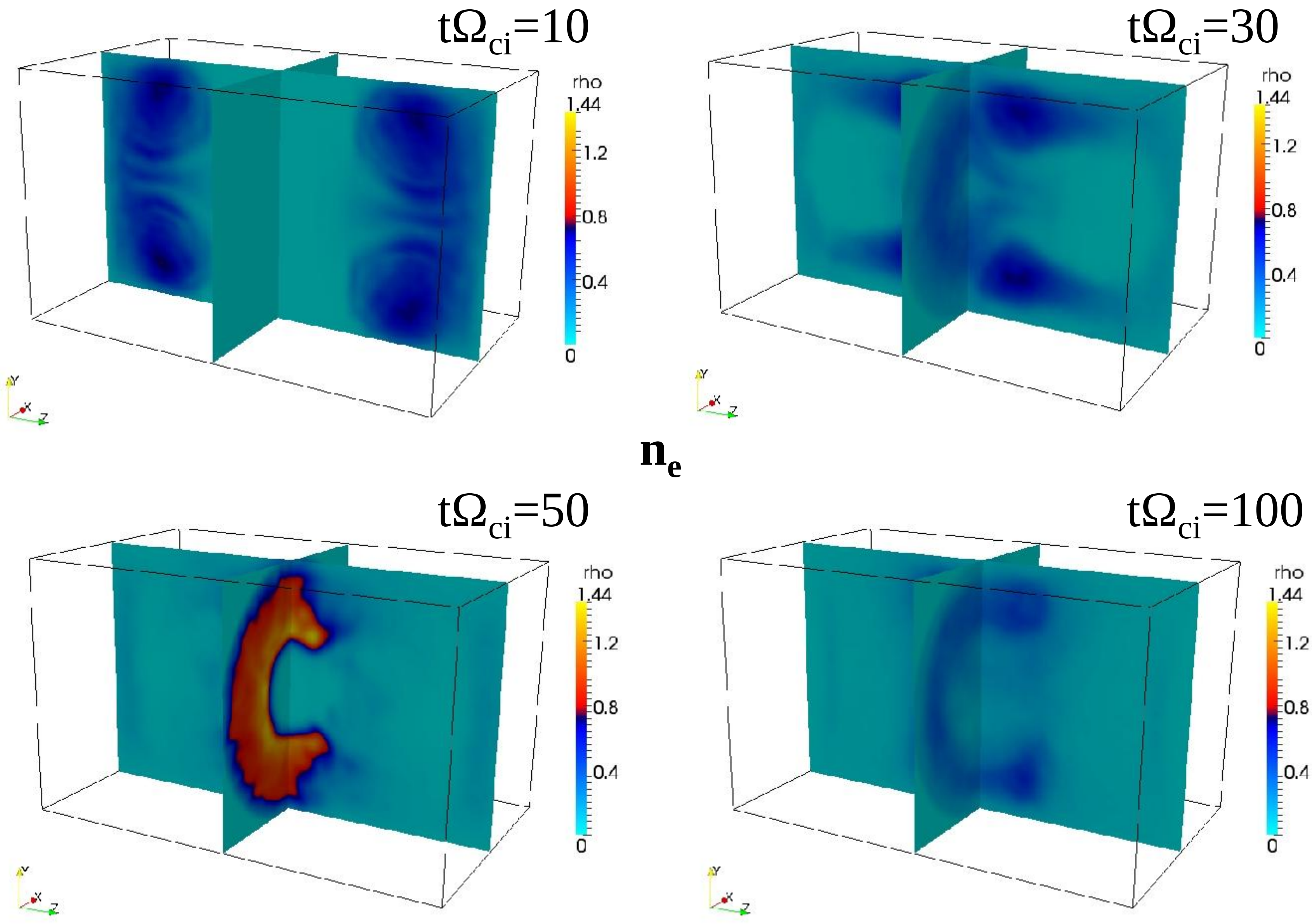


Final result: stable, symmetric FRC

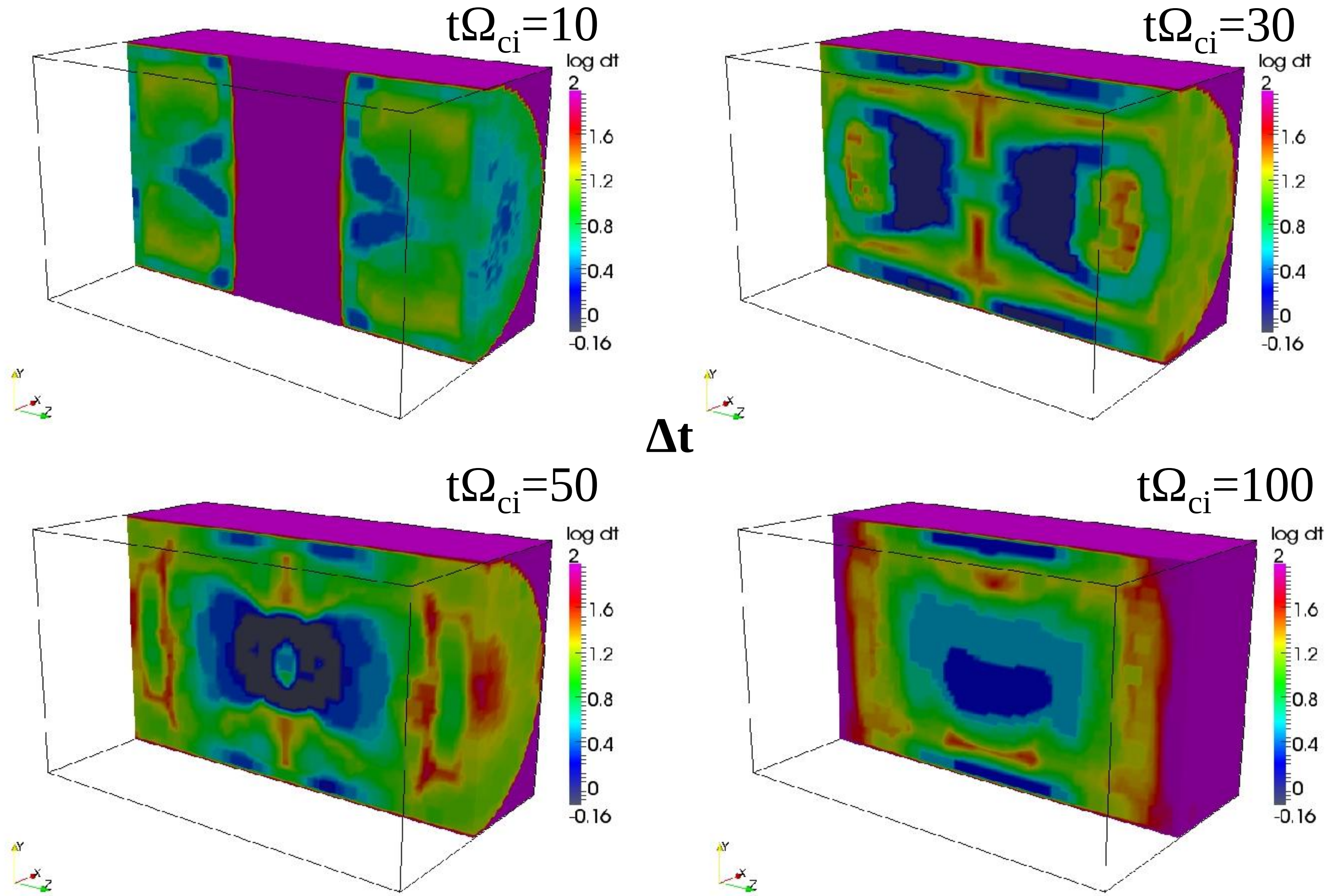
Two "stable" spheromaks



Two “stable” spheromaks



Two “stable” spheromaks

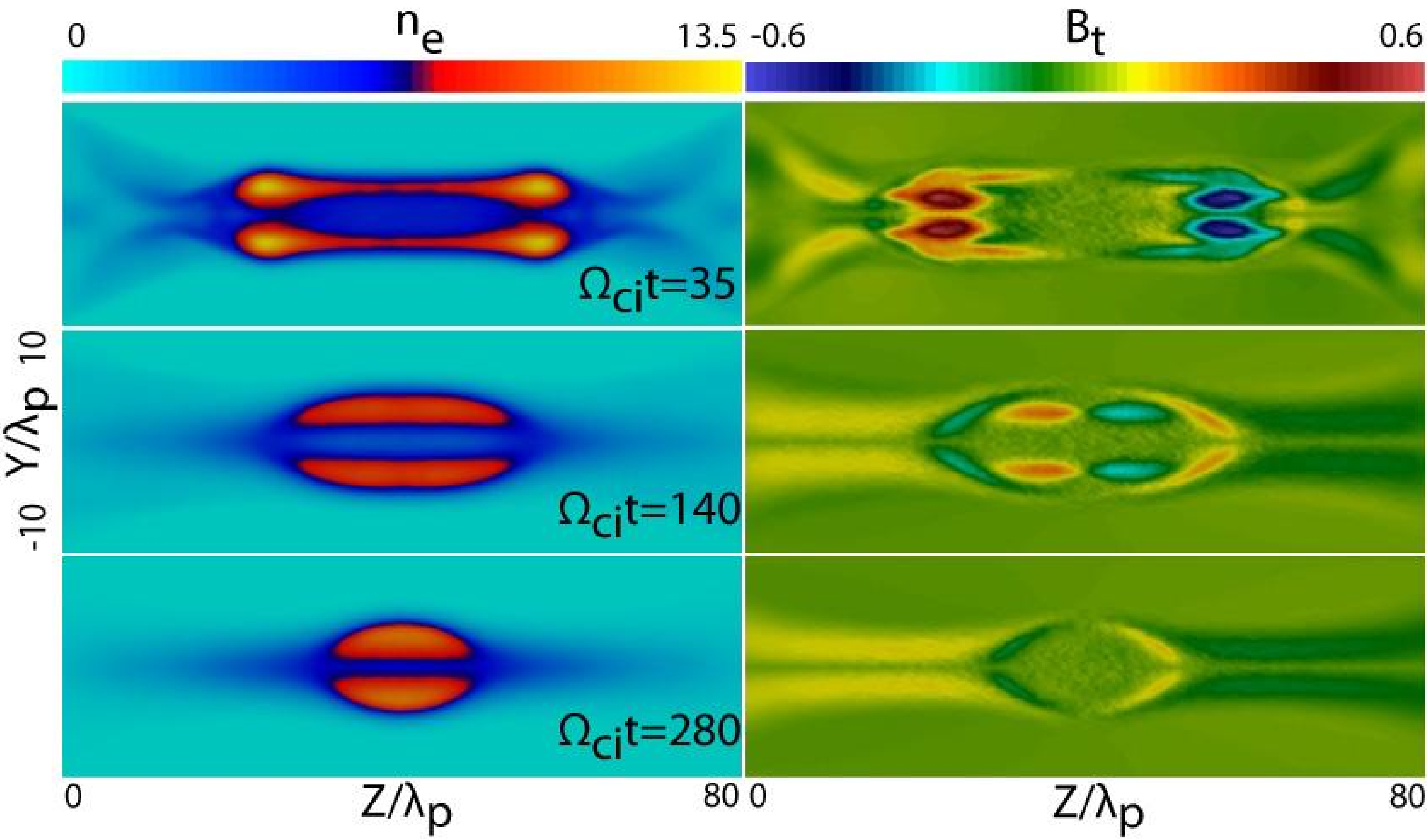


3D FRC θ-Pinch Formation
(2014)

FRC simulation setup

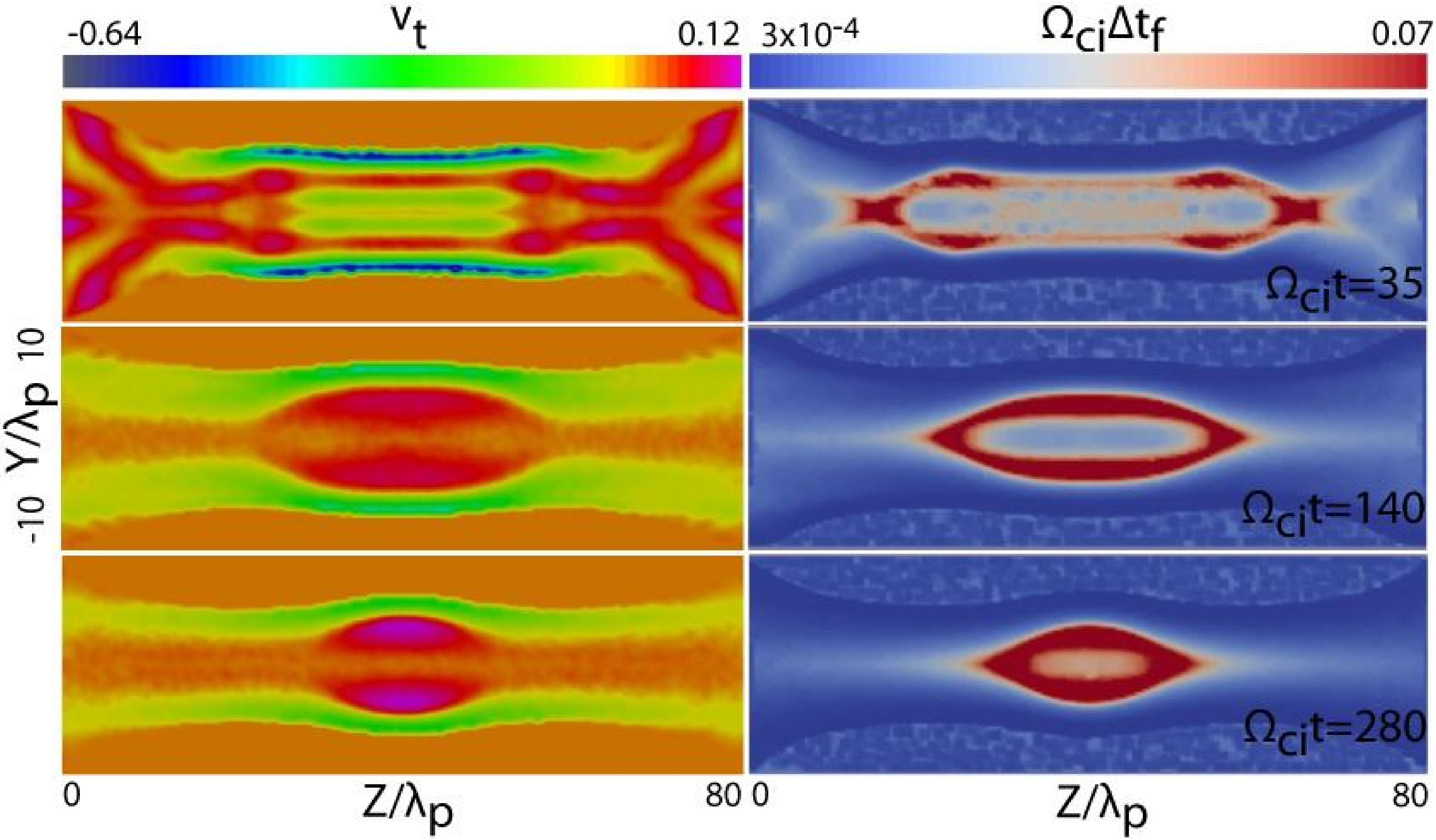
- Deuterium plasma: L=250 cm, R =30cm
- Chodura resistivity ($0.1 \leq c_{ch} \leq 0.2$)
- -0.6kG ≤Bz (on axis)≤ -0.2kG, Bz (wall) ≈ 1.7kG
- $n_{max} \approx 5 \times 10^{14}$ cm^{-3}, $2 \leq S^*/E \leq 4$
- Grid Nx x Ny x Nz = 80x80x240 cells
- Ion-ion collisions
- $\Delta t_{coil} \approx 4$ μs

Stable FRC: Density and Toroidal Field



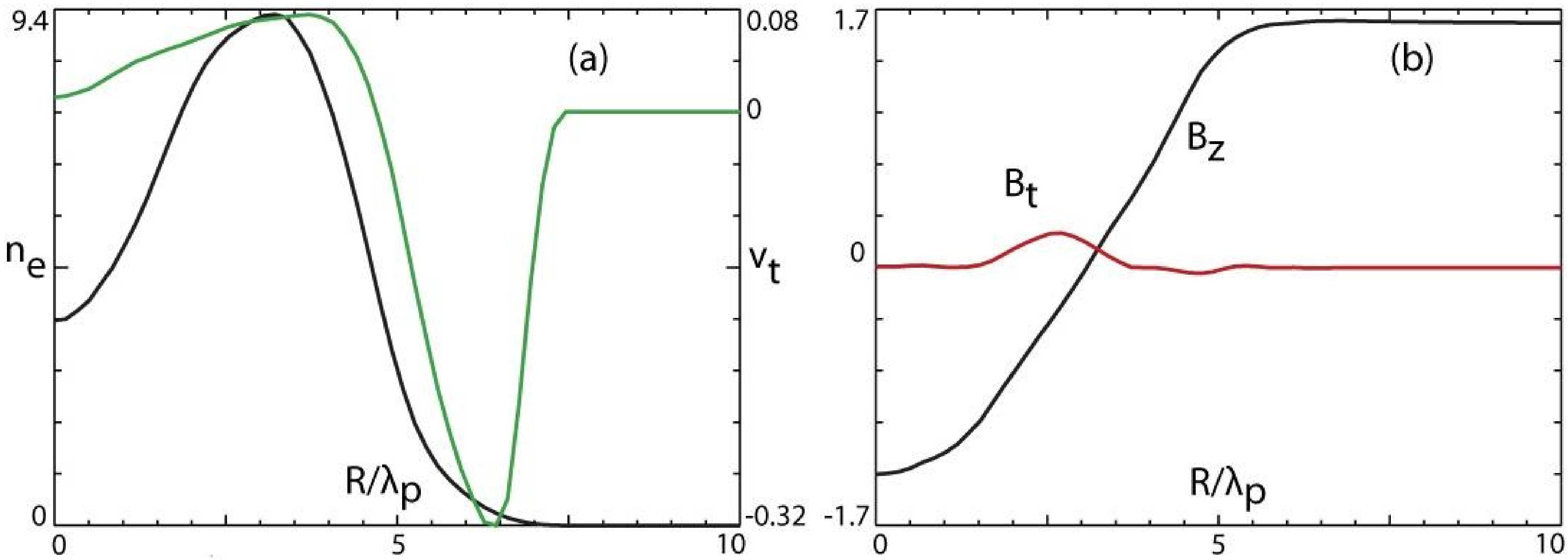
Y.A. Omelchenko, Phys. Rev. E. 92, 023105, 2015

Stable FRC: Rotation and Timestep



Y.A. Omelchenko, Phys. Rev. E. 92, 023105, 2015

Stable FRC: Radial Profiles



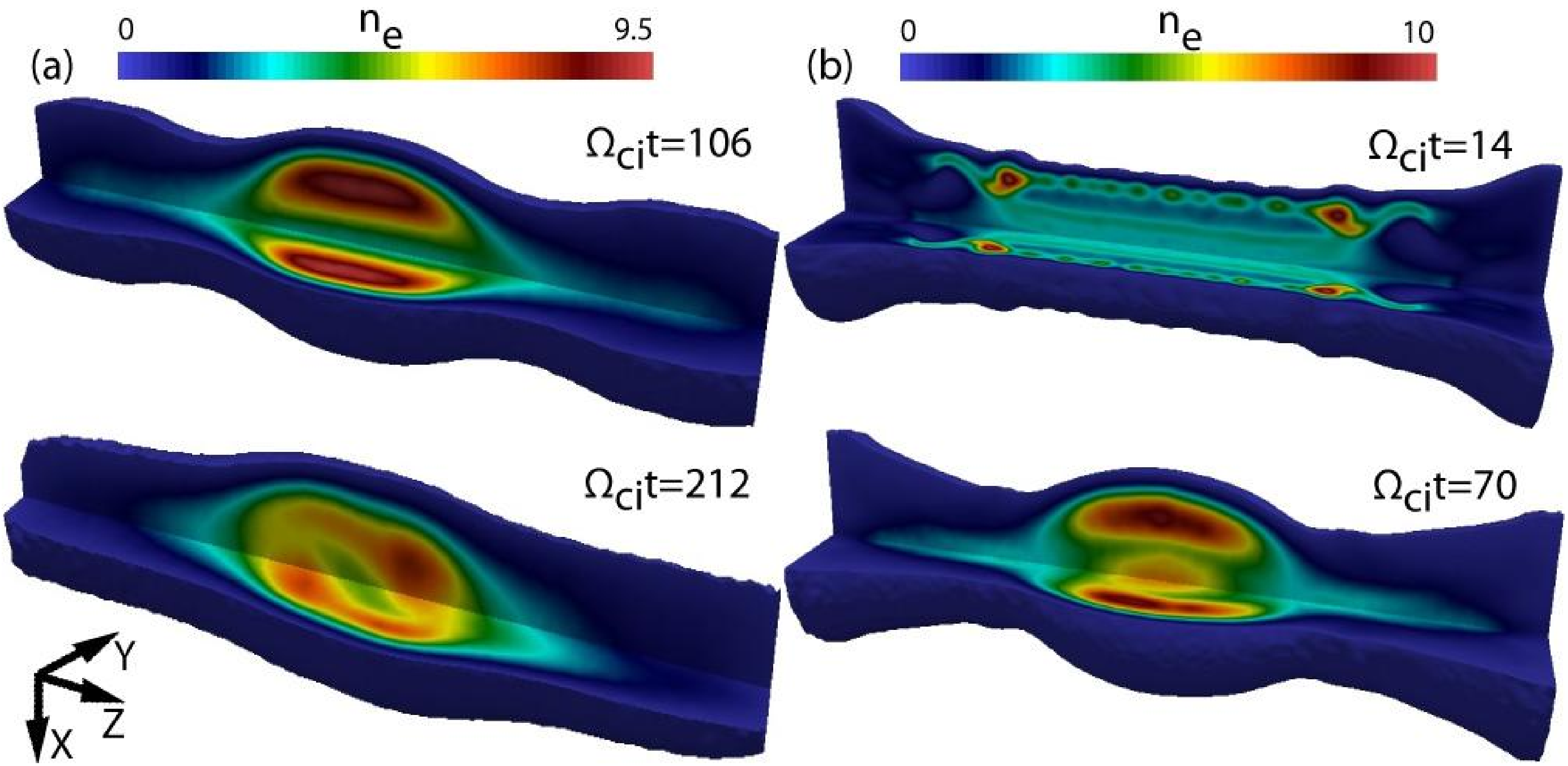
Ion-ion collisions contribute to establishing equilibrium profiles

Y.A. Omelchenko, Phys. Rev. E. 92, 023105, 2015

Unstable FRCs

Tilt instability
$S^*/E=4$

Magneto-Taylor instability
(high bias Bz, small resistivity)

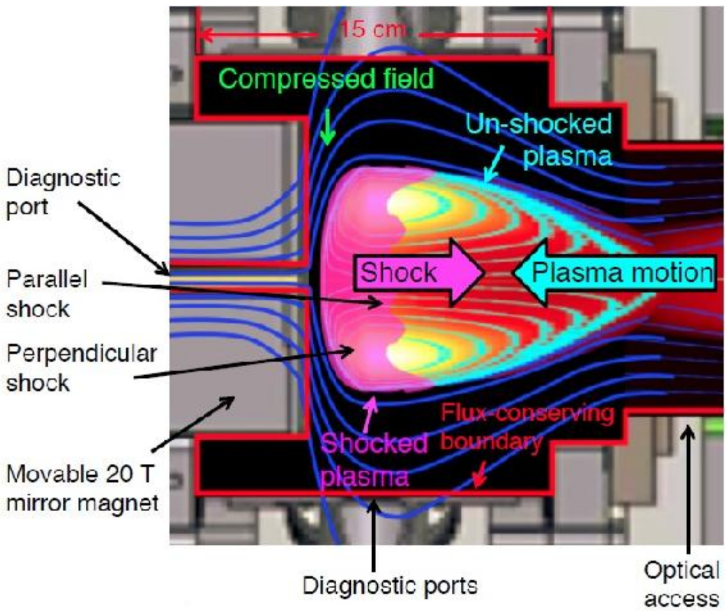


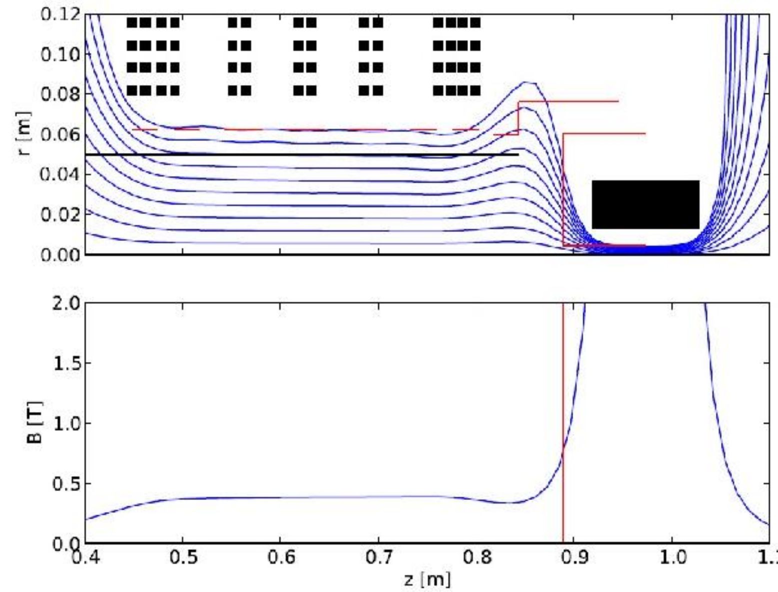
Y.A. Omelchenko, Phys. Rev. E. 92, 023105, 2015

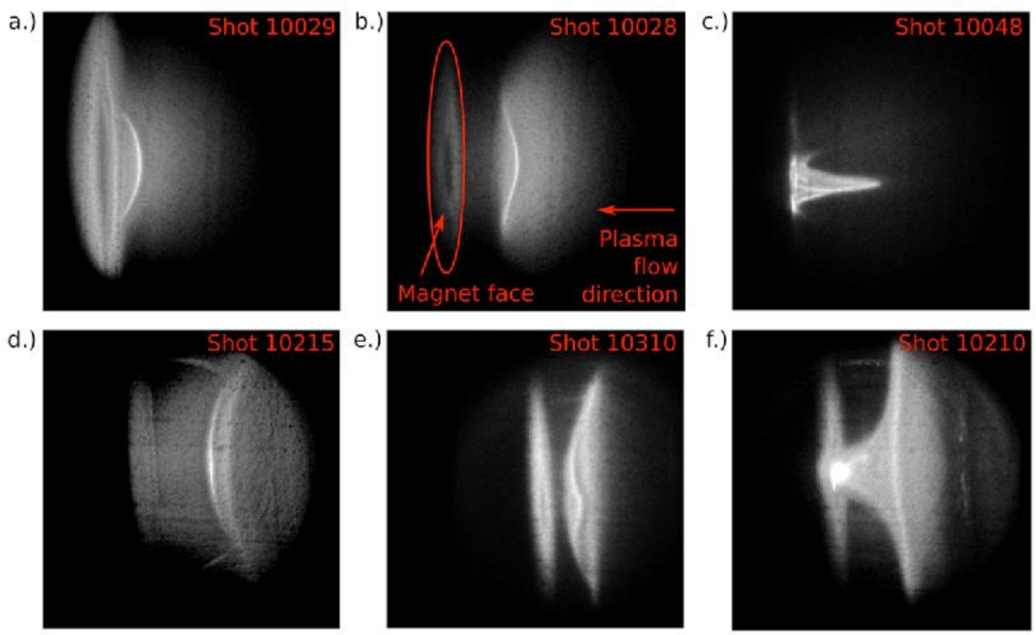
MSX Experiment[1,2] (2015)

with
[1]T. Weber (LANL)
[2]R. Smith (Tri Alpha, Inc)

MSX Experiment at LANL







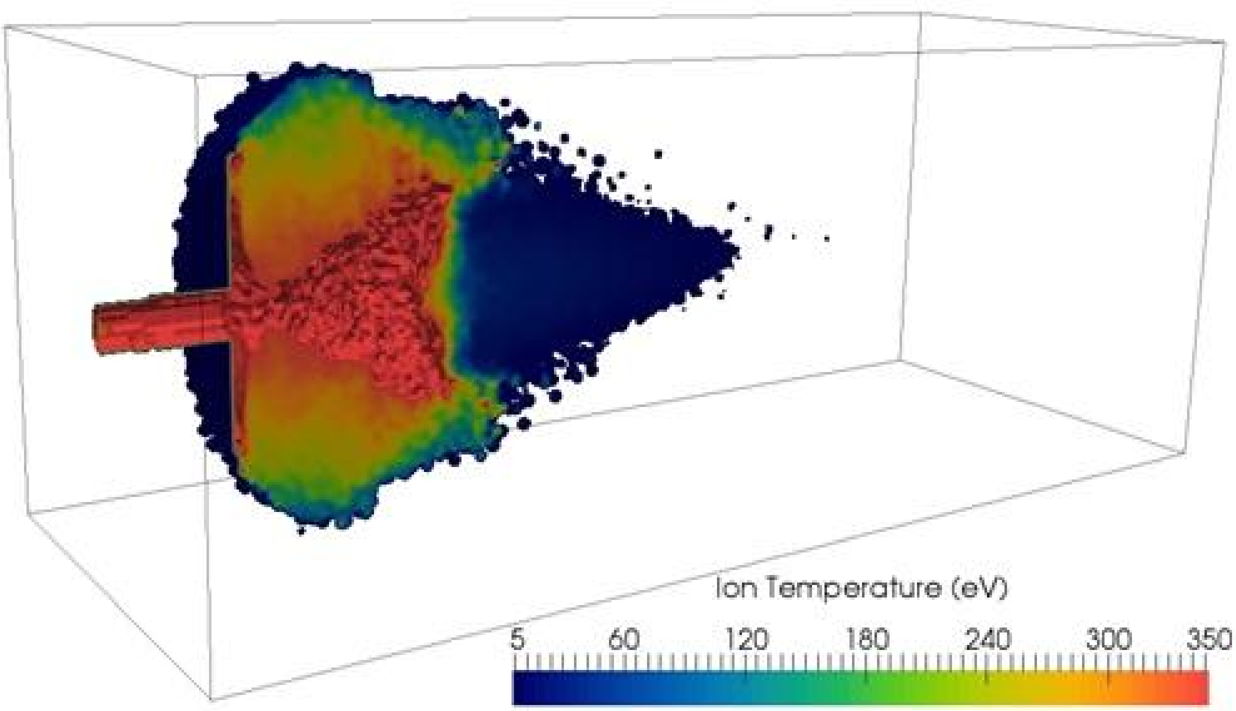
MSX Simulation Setup

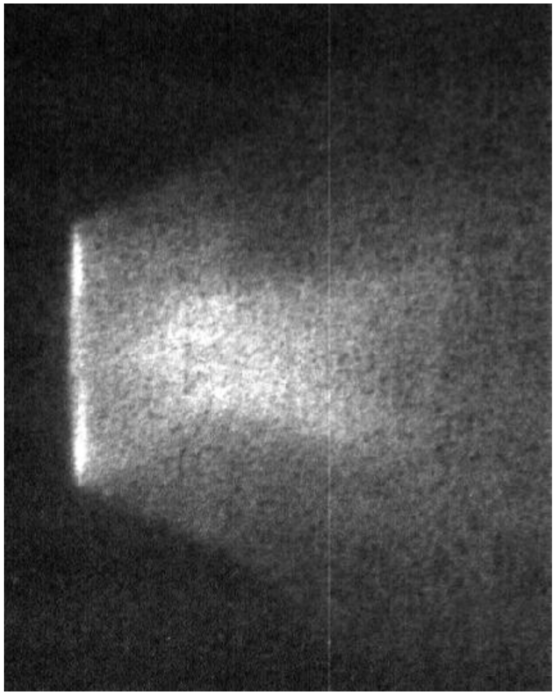
- Simulation box: L=38.4cm, R =7.8cm, λ_0 = 0.6 cm
- Deuterium FRC plasma with $T_i \approx$ 65 eV
- Bz (on axis) ≈ -0.4T, Bz (wall) ≈ 0.6T
- $n_0 \approx 10^{22}$ m^{-3}, V_{FRC} = 240km/s, $R_0 \approx$ 2.4cm, $L_s/2 \approx$ 6cm
- Grid Nx x Ny x Nz = 104x104x256 cells
- Ion-ion collisions
- $V_{A0} \approx$ 150 km/s, $\Delta t_{PP} \approx$ 0.1μs

Data Comparison: Example

Ion temperature (simulation)

Experimental snapshot

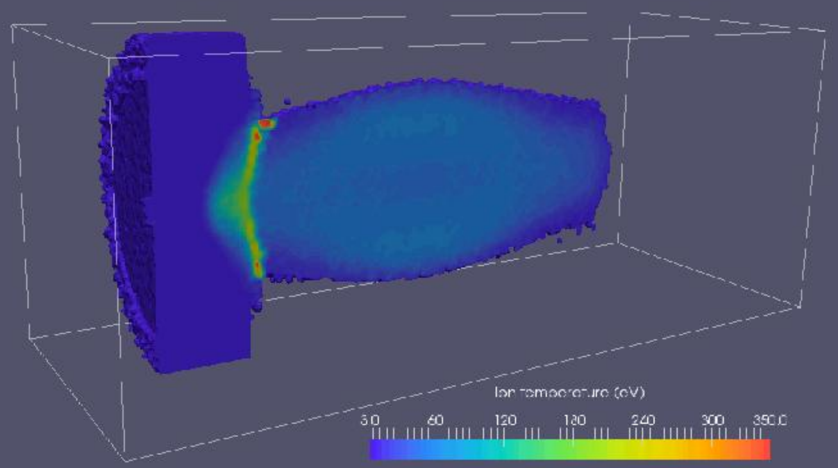




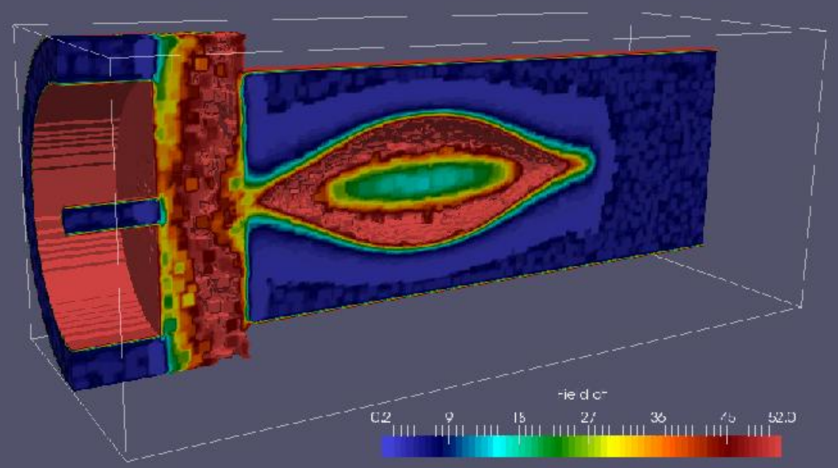
Key plasma signatures are reproduced in HYPERS simulations

HYPERS-MSX: Plasma-Filled Mirror

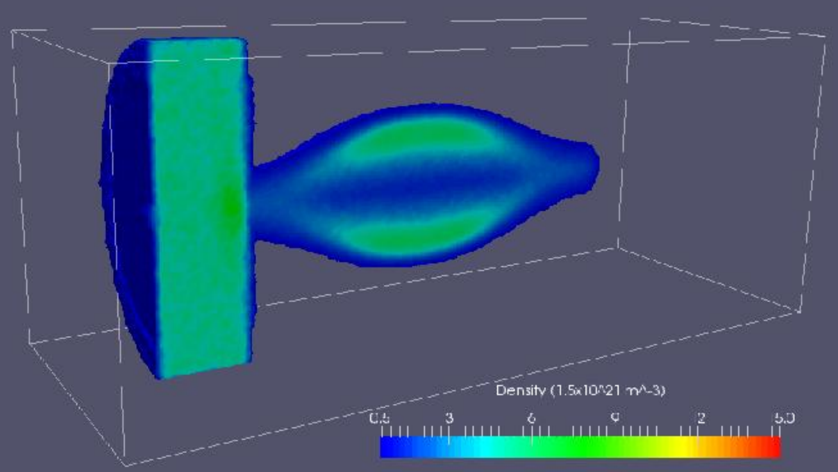
Ion temperature



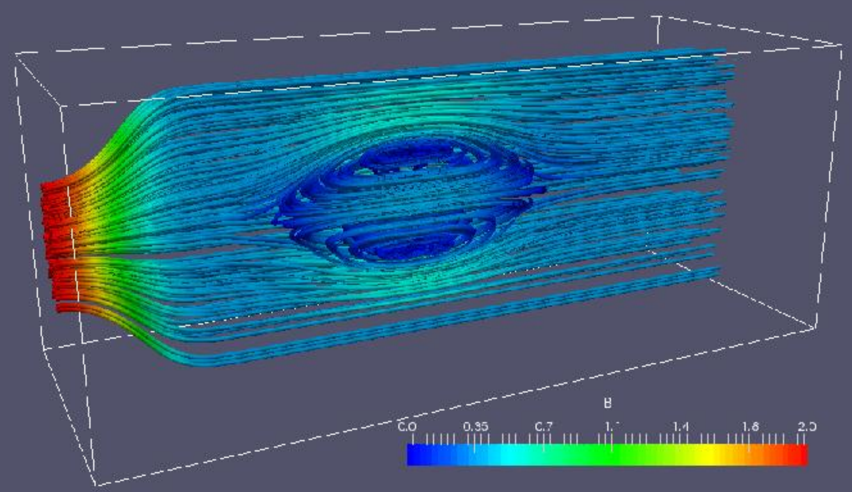
Field timestep



Plasma density



Magnetic field



3D Turbulence Simulations[1] (2015)

with
[1]V. Roytershteyn (Space Science Institute)

Why Kinetic Simulations of Turbulence?

- MHD provides a good description, but several key features of turbulence in weakly collisional plasmas are outside MHD:

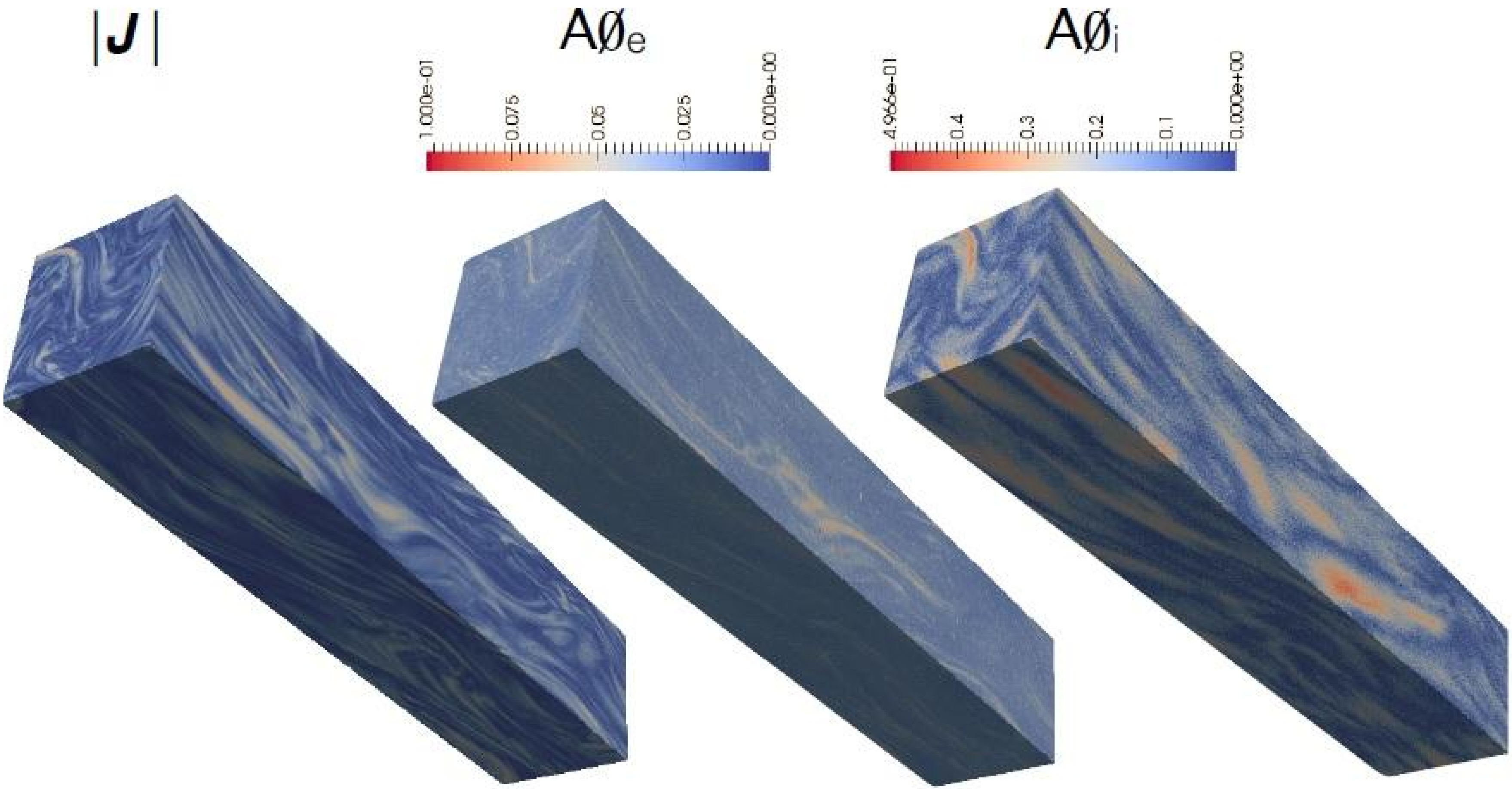
  1. Dissipation of turbulence
  2. Role of instabilities
  3. Magnetic reconnection

- These processes are associated with kinetic (both ion & electron) effects

- **Our work: large-scale hybrid and fully kinetic PIC simulations**

Full PIC Simulations of Driven Turbulence

anisotropic box, $\delta B/B_0$=0.2, β=1, T_i/T_e=1, m_i/m_e=50



Turbulence Simulation Setup

- Simulation box: 64x64x256 λ_i , ω_{pi}/ω_{ci}=2000
- Collisionless proton plasma, $\beta_i = \beta_e = 0.5$
- Grid Nx x Ny x Nz = 256x256x512 cells
- Pump wave: N=4 (harmonics):

$$\boldsymbol{j}_{ext}(\boldsymbol{x},t) = \frac{c}{4\pi}(\boldsymbol{e}_m \times \boldsymbol{k}_m)\sum_{m=1}^{N}[B_m^s(t)\cos(\boldsymbol{k}_m\cdot\boldsymbol{x}) + B_m^c(t)\sin(\boldsymbol{k}_m\cdot\boldsymbol{x})]$$

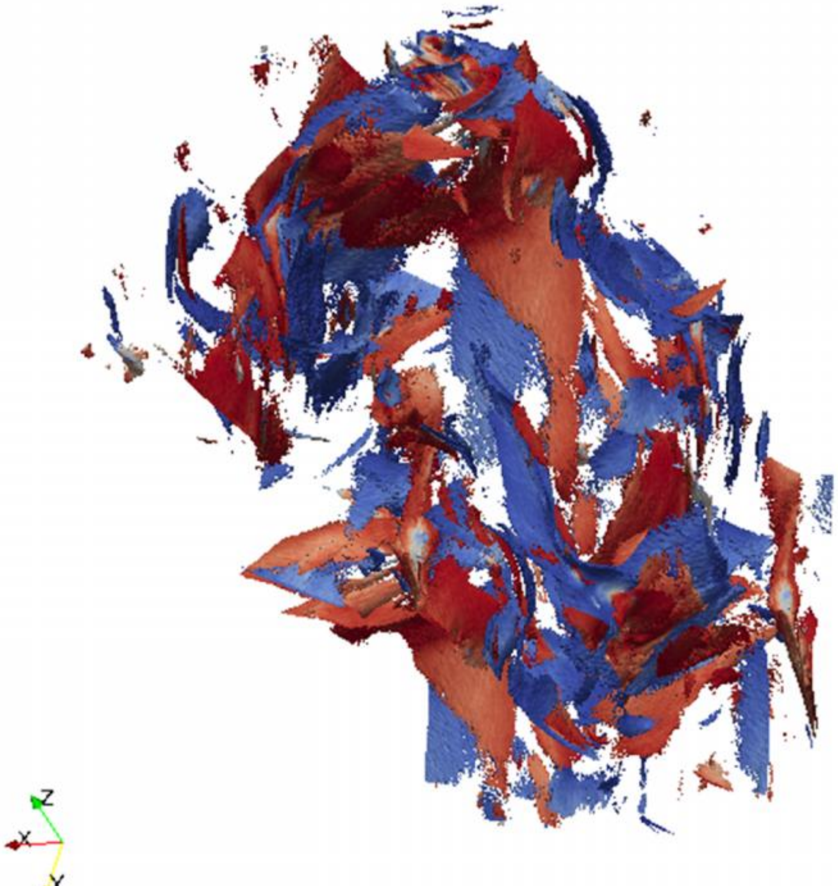
$$\boldsymbol{j}_e = \boldsymbol{J} - \boldsymbol{j}_{ext} - \boldsymbol{j}_i$$

$$\boldsymbol{E} = \frac{j_e \times \boldsymbol{B}}{en_e c} - \frac{\nabla P_e}{en_e} + \eta(\mathrm{J} - \mathrm{j}_{ext})$$

$$\boldsymbol{J} = \frac{c}{4\pi}\nabla \times \boldsymbol{B}$$

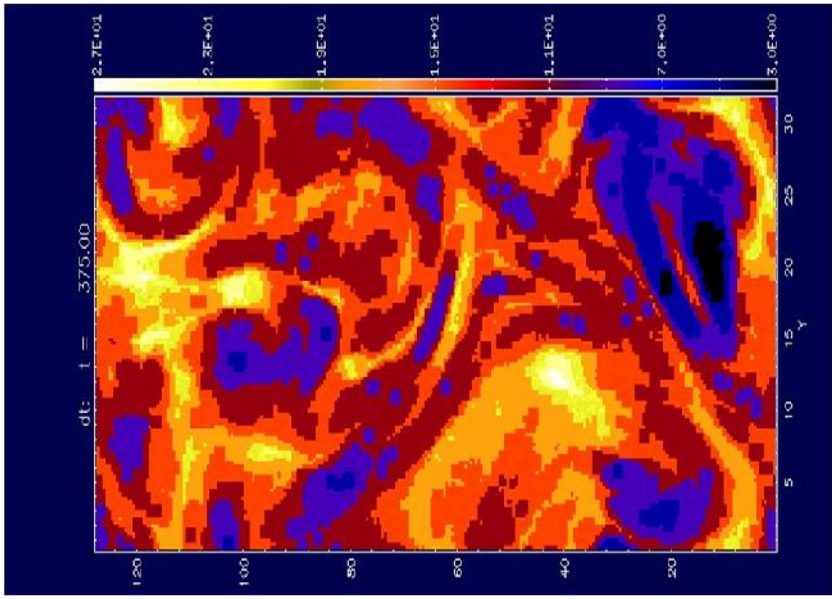
Hybrid Simulation of Large-Scale Plasma Turbulence with HYPERS (2015)

- Each particle is pushed with its own (adaptive) time step.
- Local time increments for fields adapt to dynamic physical time scales
- divB=0 is identically preserved



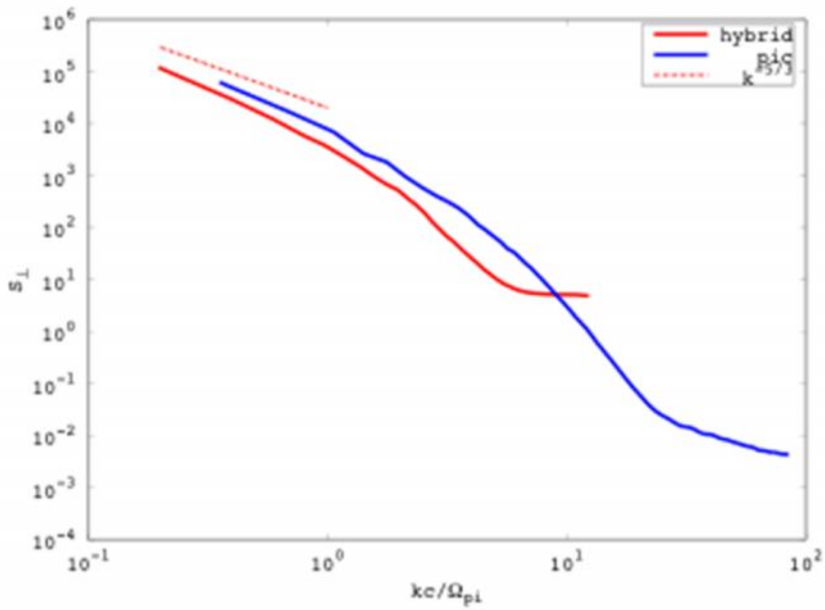
Current structures in a hybrid simulation

Time steps for fields in a turbulence simulation vary by an order of magnitude

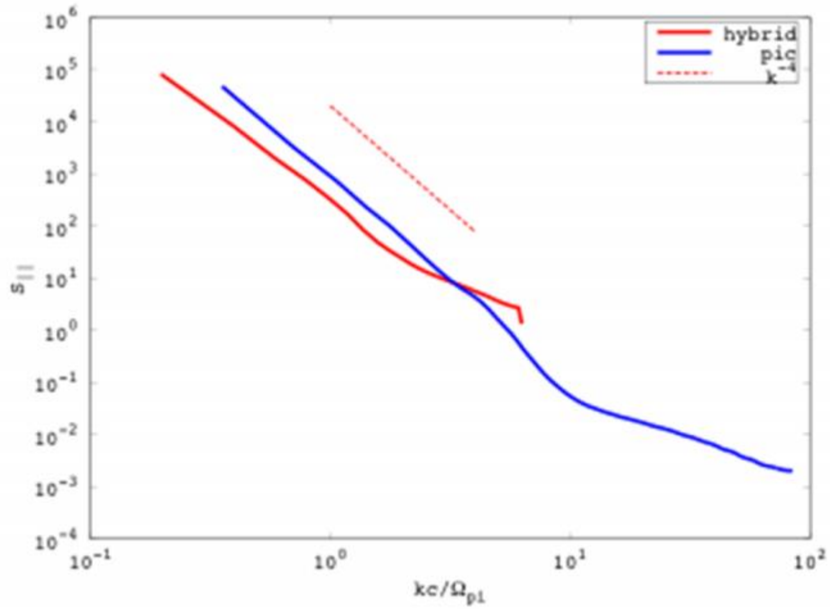


Hybrid + Full PIC : Kinetic Coverage of a Very Large Range of Scales

Perpendicular B-spectra



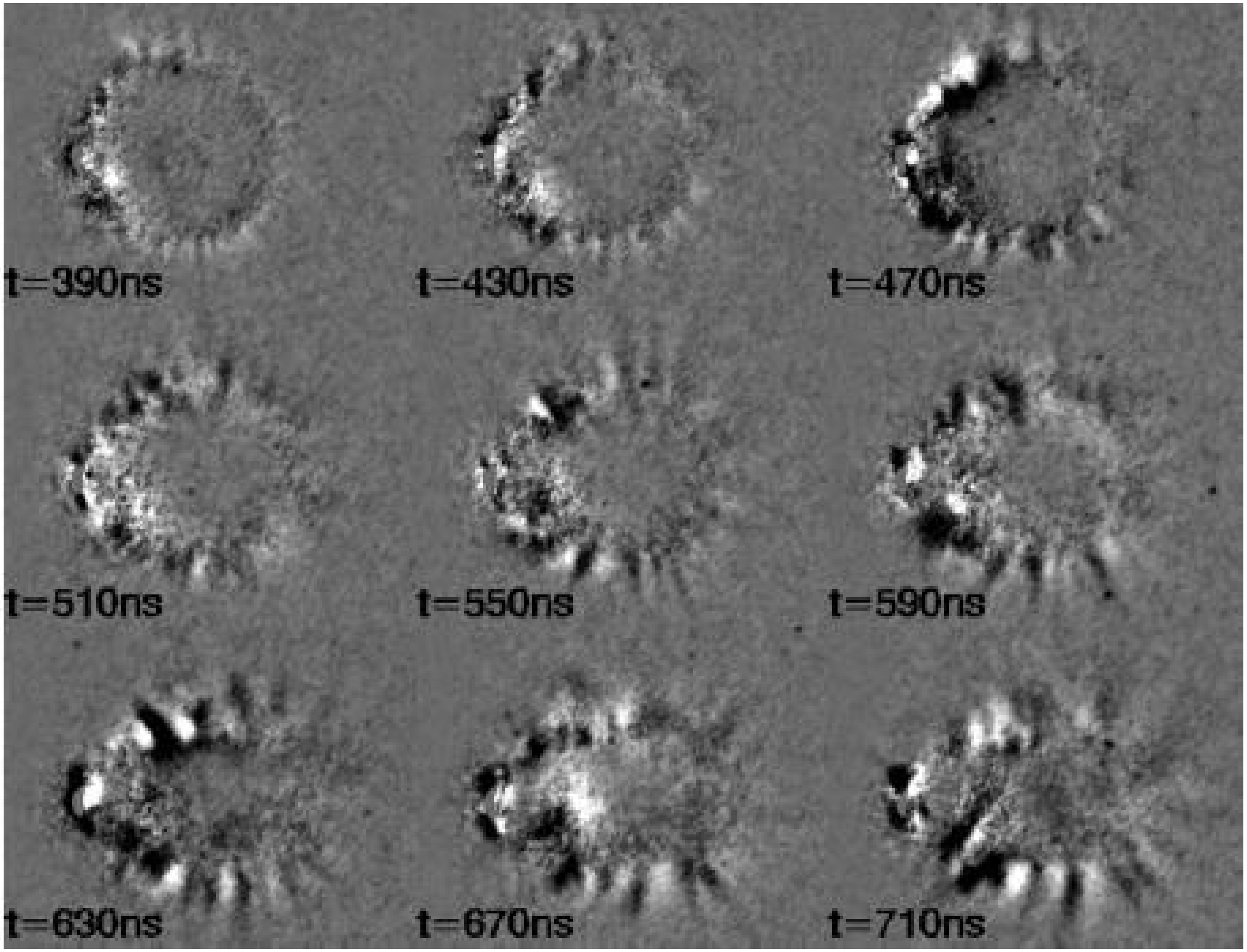
Parallel B-spectra



LAPD Experiment Simulation (2016)

LAPD Experiment

A. Colette and W. Gekelman, Phys. Plasmas 18, 055705 (2011)

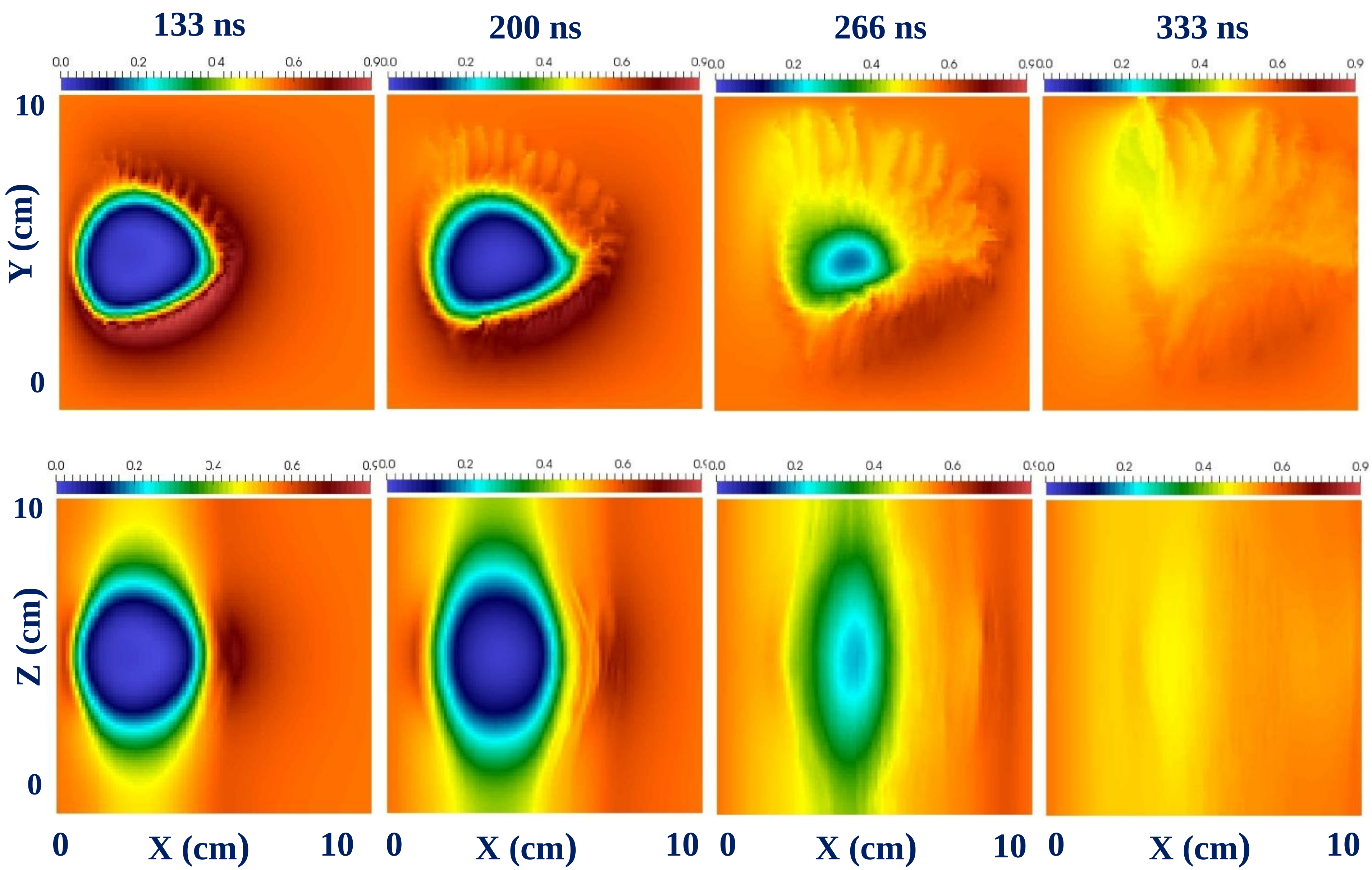


Photographs of expanding laser-produced plasma (XY)

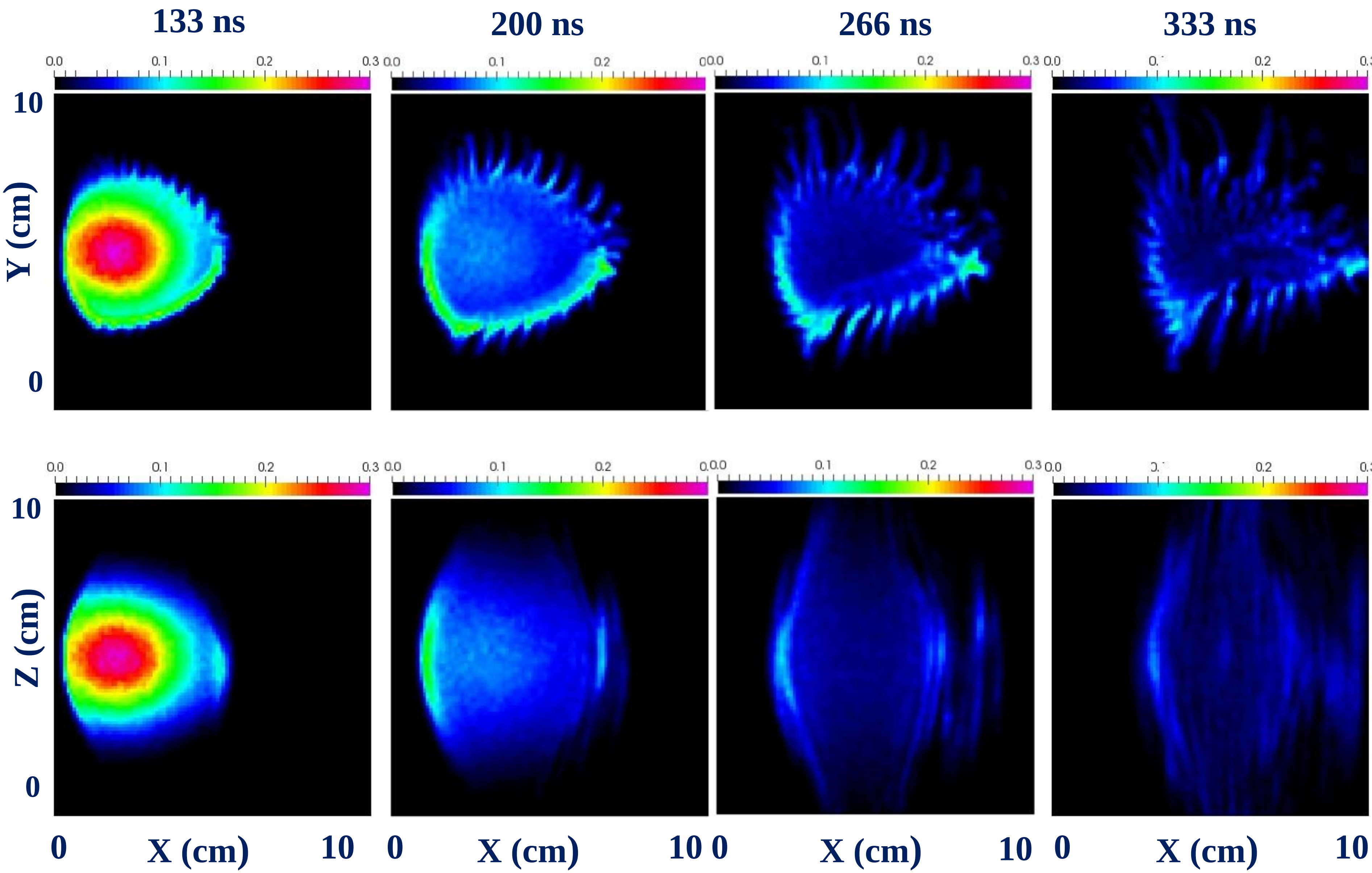
LAPD Simulation Setup

- Simulation box: 10x10x10 cm, λ_0 = 1cm
- C^{+4} plasma with energy E=2KeV
- Bz = 0.6T
- $n_0 \approx 5x10^{20}$ m^{-3}, T_e = 5eV
- Grid Nx x Ny x Nz = 100x100x100 cells
- NO ion-ion collisions
- $r_{Li} \approx$ 4 cm

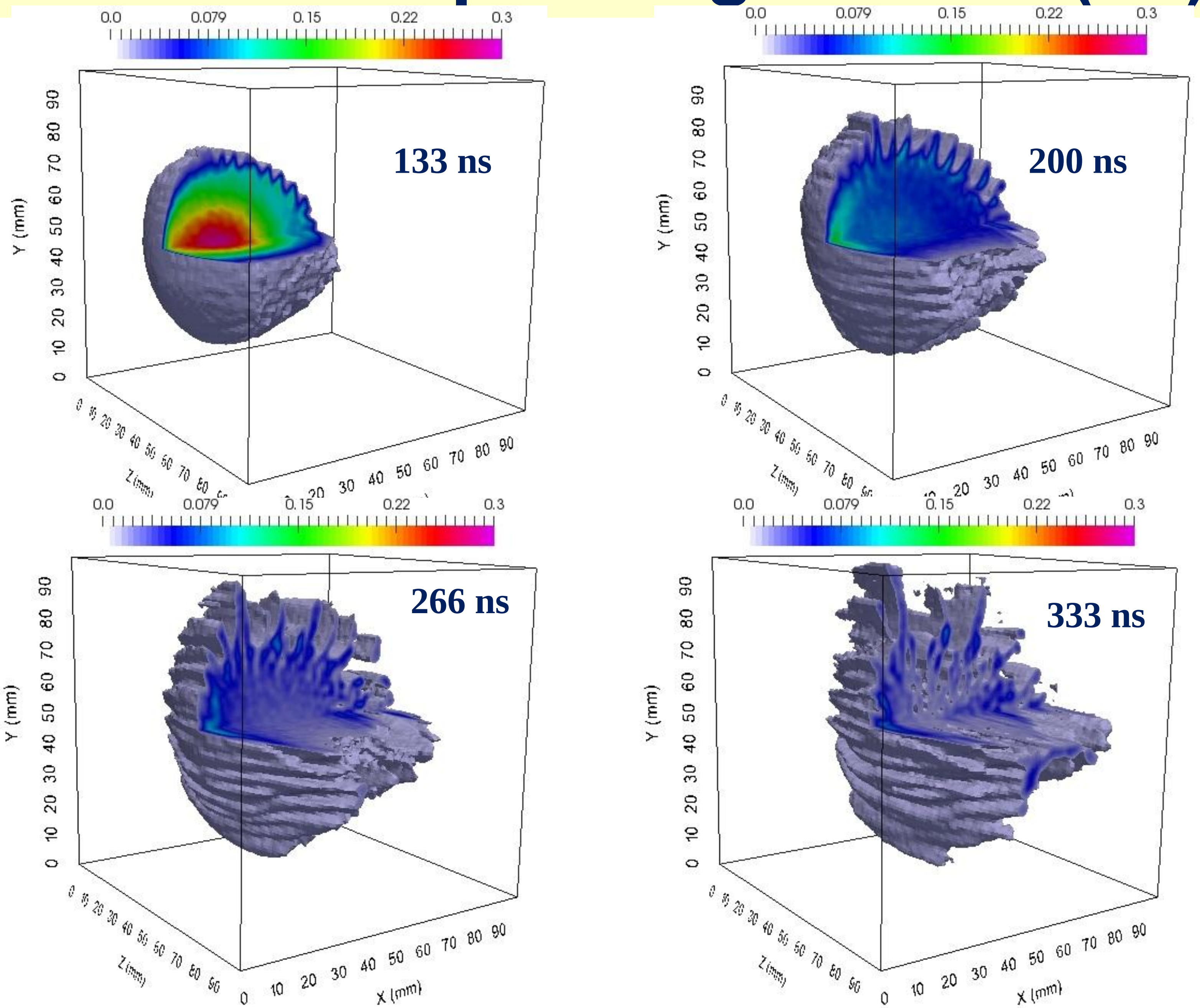
LAPD: Expanding Plasma (Bz)



LAPD: Expanding Plasma (ne)



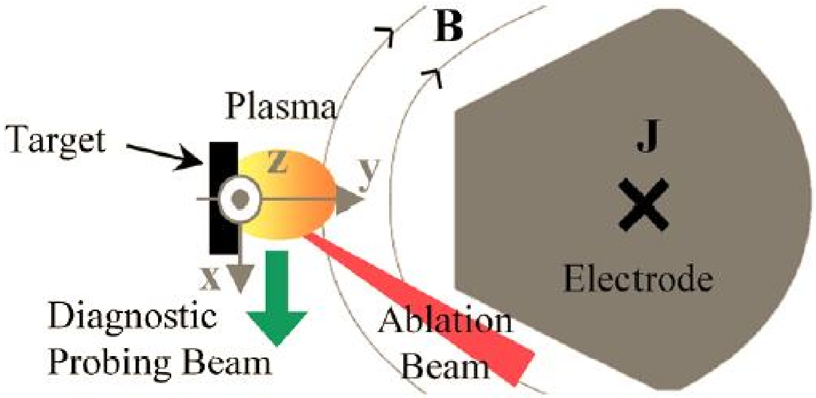
LAPD: Expanding Plasma (ne)



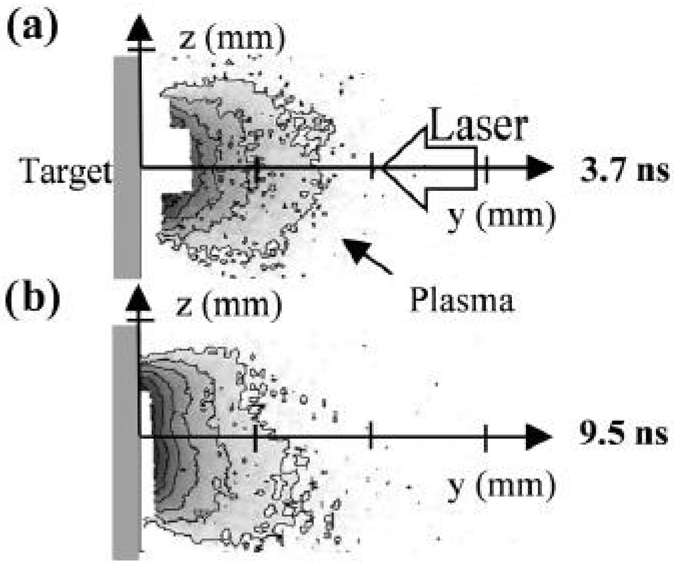
UNR Experiment Simulation (2016)

UNR Experiment

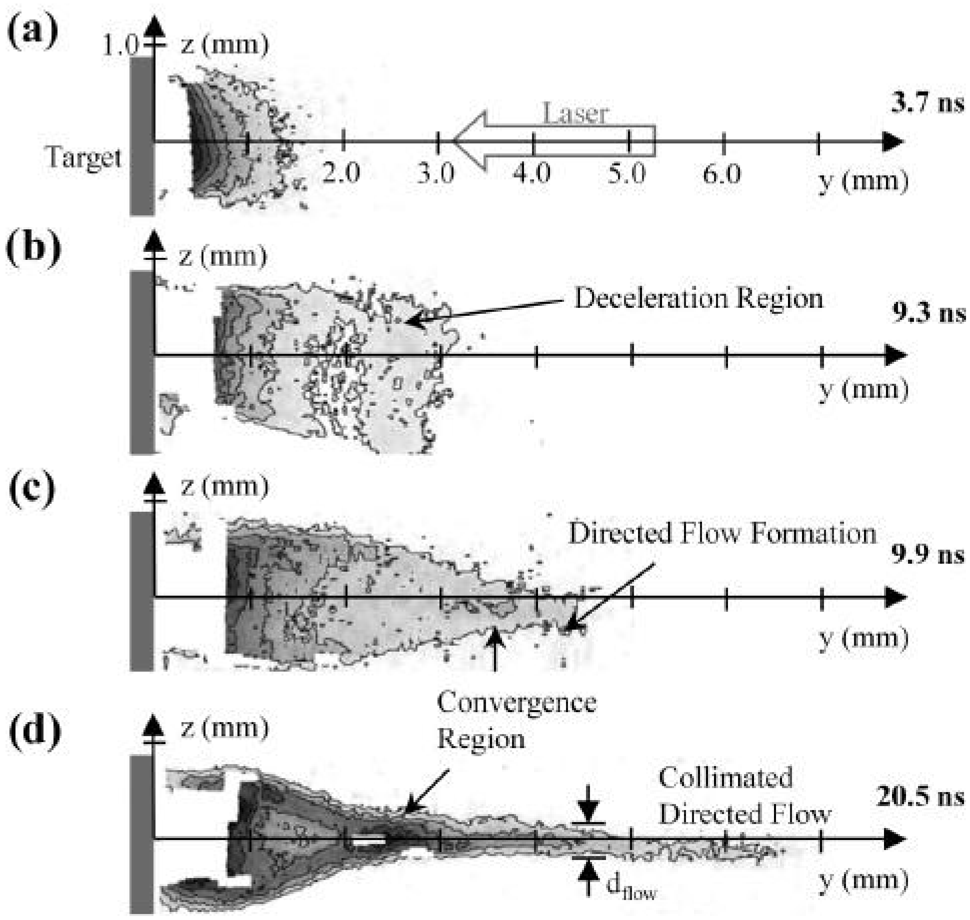
C. Plechaty, R. Presura, and A.A. Esaulov, PRL 111, 185002 (2013)



Without magnetic field:

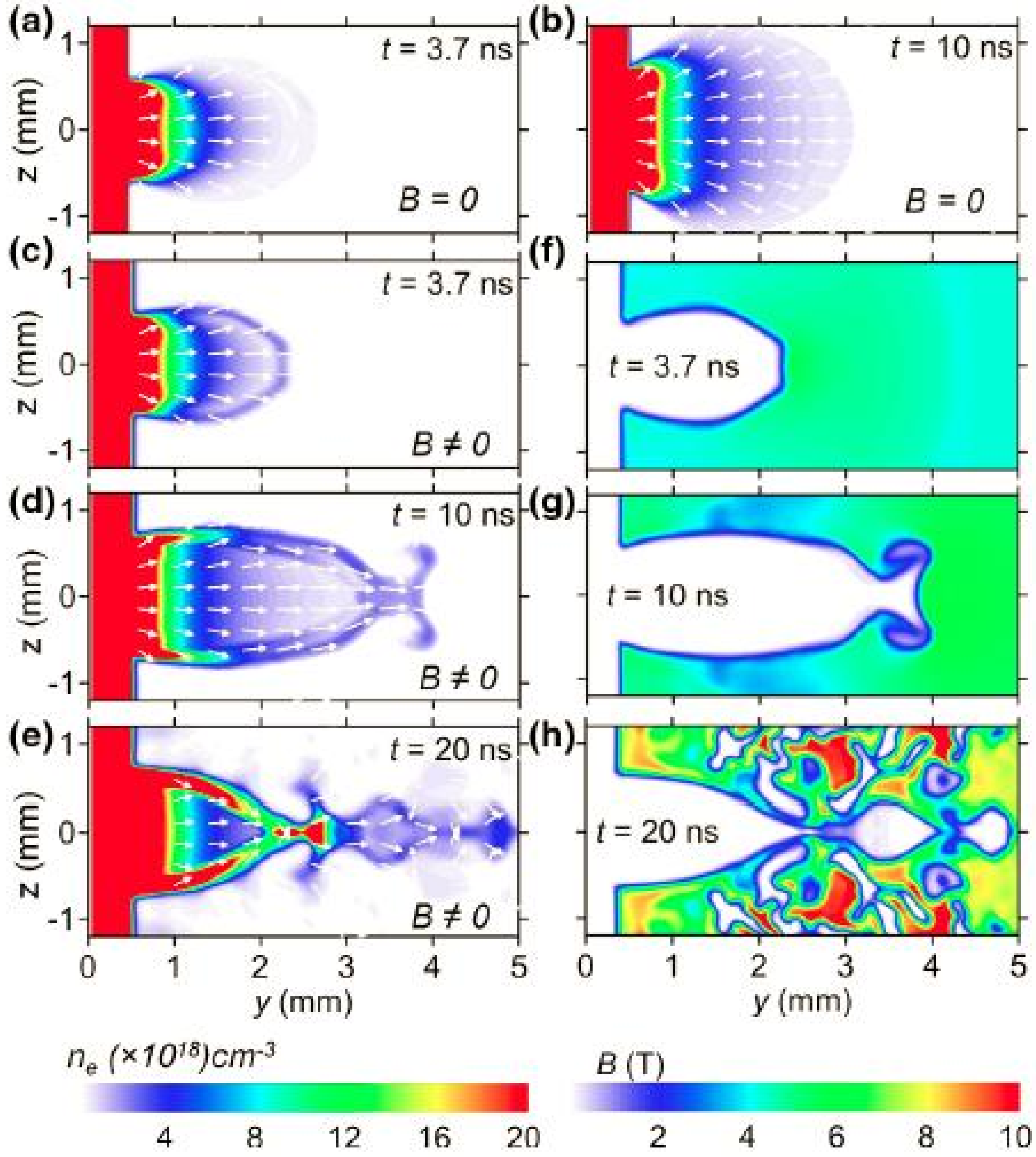


With magnetic field:



UNR Experiment: MHD

C. Plechaty, R. Presura, and A.A. Esaulov, PRL 111, 185002 (2013)



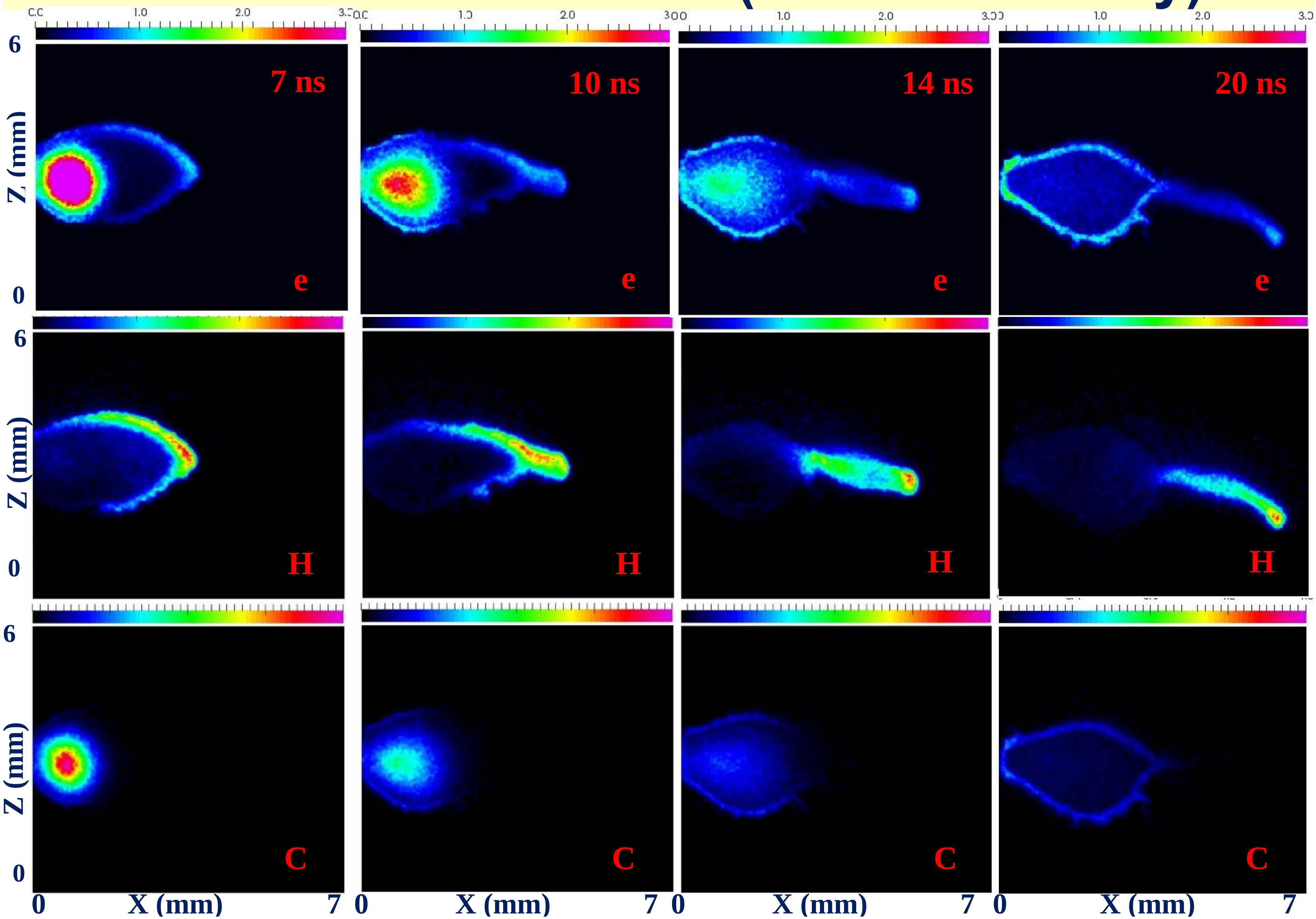
Only the plasma focusing effect is captured by single-fluid MHD.

The collimated flow description requires ion kinetic and Hall effects!

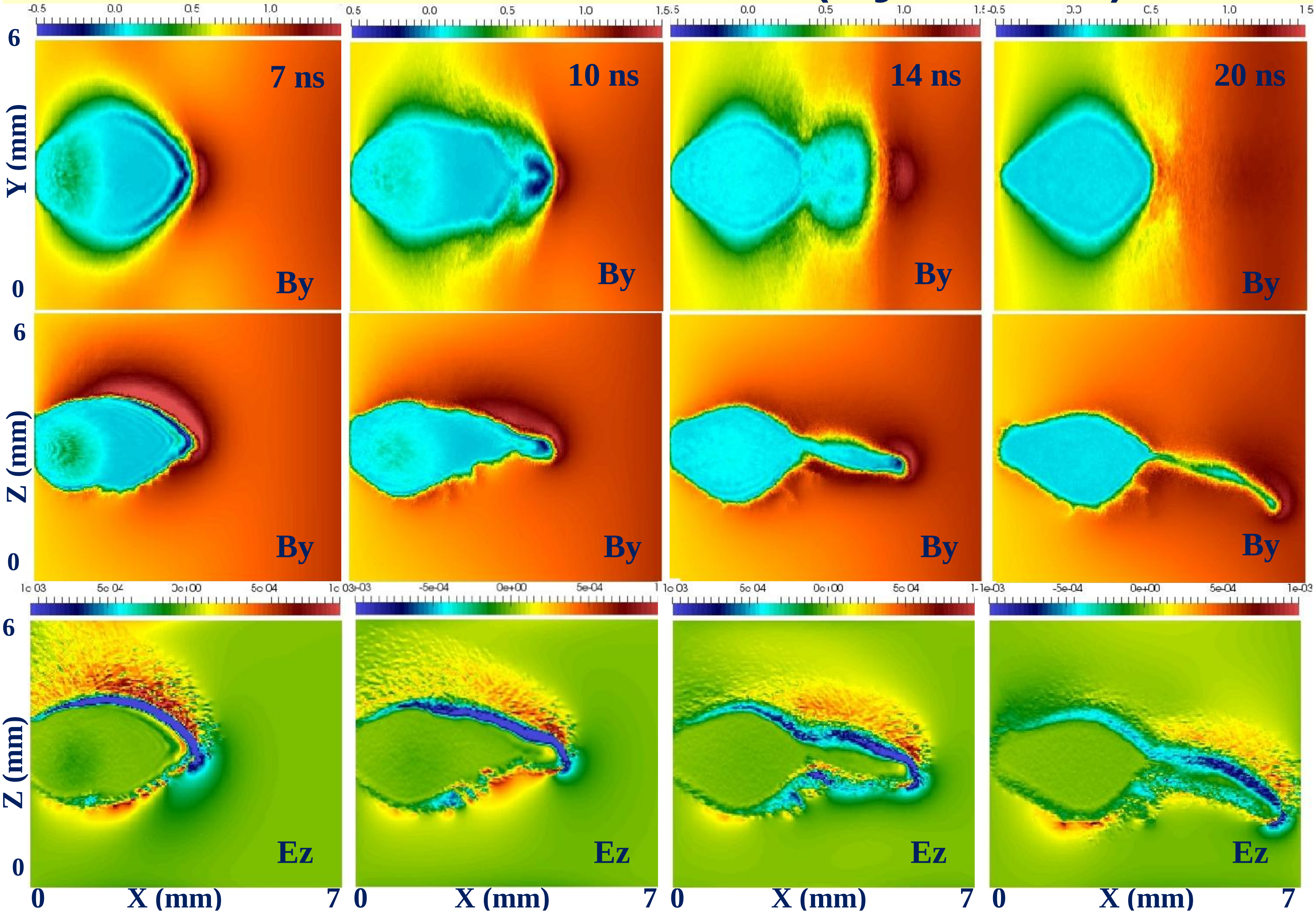
UNR Experiment Simulation (2016)

- Simulation box: 7x6x6 mm, $\lambda_0 = 0.1$mm
- C^{+4} and H^{+} plasma with energy E=0.5KeV
- By ≈ 10T
- $n_0 \approx 5\text{x}10^{24}\ m^{-3}$, $T_e = 5$eV
- Grid Nx x Ny x Nz = 210x180x180 cells
- Ion-ion collisions (frequency > cyclotron frequency)
- $r_{LH} \approx 0.07$ mm, $r_{LC} \approx 0.17$ mm

UNR: Collimated Flow (Plasma Density)



UNR: Collimated Flow (By and Ez)



Summary

What HYPERS can do in 3D:

- FRC formation (θ-pinch, merging), compression
- Global equilibrium studies of inhomogeneous plasmas
- Heating of FRCs with ion (charge-neutralized) beams
- Refueling of FRCs with spheromaks, ion rings, etc.
- Turbulence and reconnection studies
- Propulson, interactions of plasmoids and plasma streams with magnetized and unmagnetized targets

Planned development:

- Fast ions: slowing down/scattering collisions
- Neutral beam/gas: ionization, charge exchange (CEX)
- Plasma-vacuum interface: XHYPERS
- Conforming walls (cut cells)
- Magnetostatic solver