Hybrid FRC equilibria with fully-kinetic ions and fluid electrons

Loren Steinhauer
Tri Alpha Energy

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Objective: reconstruct FRC equilibrium based on measured data

FRC reconstruction is like "reconstituted orange juice"

- Most of the vitamins and minerals are retained
- You can't go to Florida every time you want an orange juice

Two approaches

(1) Front-loaded, Ohm's-law driven

(2) Evolving sequence of equilibrium

Apply to model C-2U shot #43628

Challenge: how to reconstruct FRC equilibria, working off limited diagnostics?

Measurables

Routine (each time instant):

> Excluded flux radius profile: R_ϕ vs z

> Multi-chord interferometry: $\int n_e dl$ vs y

Occasionally: T_e, T_i, superthermal ions, etc.

Need

Efficient interpretative tool, "equilibrium reconstructor"

> Just enough "physics" to be realistic

> Instant hands-off, numerically-stable

What we *especially* want to know about the "insides" of an FRC

Shopping list

- FRC dimensions: R_s, Z_s (half length) ← <u>not</u> actually measured (R_ϕ , $Z_{2/3}$)
- Poloidal flux ϕ_p
- Scrape-off layer thickness L_n
- Fraction of the current carried by superthermal ions
- Fractions of ion populations: *core*-confined and periphery (mirror confined)
- Stability indices: tearing, interchange, tilt

Ambitious? Yes, but much needed

Equilibrium reconstruction methods

Existing methods

- Analytic formulas: R_s , Z_s, ϕ_p, etc.
- Grad-Shafranov snapshots (static, single-fluid)
- Enhanced-GS: fluid "bulk" ions plus superthermal Monte- Carlo ions

Emerging methods

- Fast, flexible, time-tracking Grad-Shafranov: *mature*
- Hybrid equilibrium "HyEq" : *functional but "developing"*

Hybrid equilibrium model ingredients

FRC realities

- Large orbit ions; even *bulk* ions T_i = 300-800eV
  especially superthermal ions W_i = 10-15keV
- Edge plasma controls:
  > Strong applied mirrors > Divertor biasing

Two balancing acts

- Number of adjustable parameters in model
  > Too many: too complex
  > Too few: too little flexibility
- Numerical burden
  > Monte-Carlo fast ions, numerically intensive
  > Distribution ions, also intensive *Unless…*

Unless: ion distribution with analytic moments

- Only two kinetic constants of motion in axisymmetric system
  - > thermal (Hamiltonian) and
  - > momentum (canonical angular momentum)
- Separable “thermal” & “momentum” parts
- Kinetic confinement criterion
  - > Separates core-confined and mirror-confined populations

Result

- Analytic moments (density and current density)
- Small number of adjustable parameters

Computational architecture of equilibrium reconstruction tool

GS solver $\Delta^* \psi = -\mu_0 r^2 p'(\psi)$

- Model inputs: three parameters in $p(\psi)$:
- Iterate to target data: R_ϕ and $Z_{2/3}$

HyEq solver $\Delta^* \psi = -\mu_0 r j_\theta$

- Model Inputs: T_e, T_i, *plus two parameters in distributions*
- Iterate to target data: R_ϕ and $Z_{2/3}$ and $\langle n_e \rangle$
- T_i(bulk), T_e directly from data

- Post process to find a multitude of key values: R_s, Z_s, ϕ_p and many others
- Confidence check: reproduce untargeted data?
- Repeat: series of time “snaps” over plasma lifetime

Reconstruction of C-2U #43628
Preview:

GS tool

- Time sequence of equilibria from 0.5ms to 5.5ms
- How key parameters vary in time
- Snapshots of profiles (poloidal, radial) at three times
- Confidence checks

HyEq tool

- Snapshots of profiles at three times
- Unique properties of FRCs with a significant super-thermal "beam-ion" component

First: GS tool reconstruction...

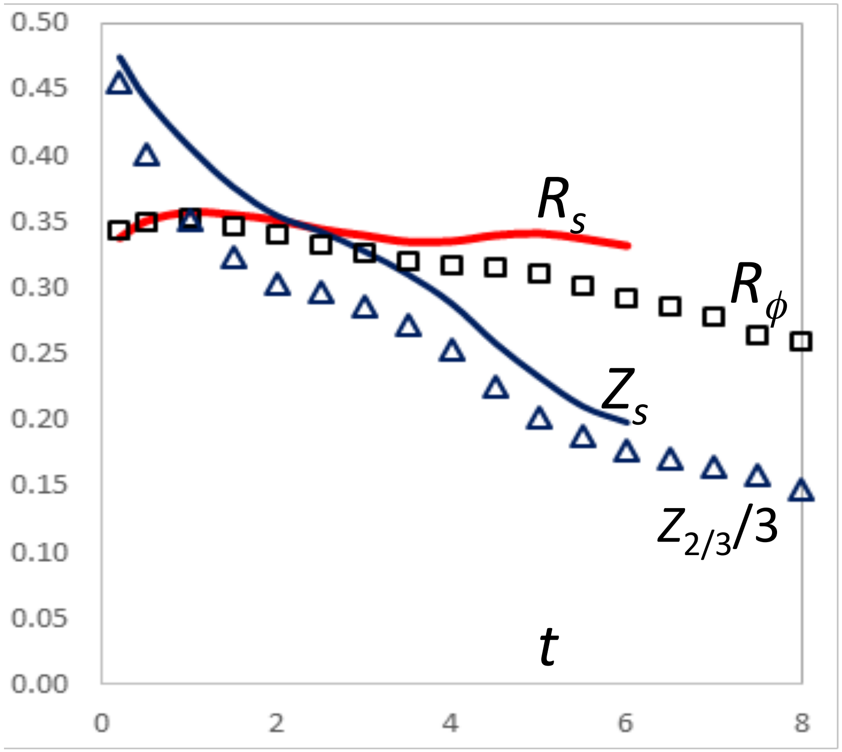
Compare FRC dimensions: R_s and R_ϕ, Z_s and $Z_{2/3}$

GS tool

Time histories →

Symbols = *measured*

Lines = *reconstructed*



Radii: R_s exceeds R_ϕ more and more:

- "Two dimensionality"
- Elongation not large

Half-lengths: Z_s and $Z_{2/3}$ consistent pattern;

- Getting shorter

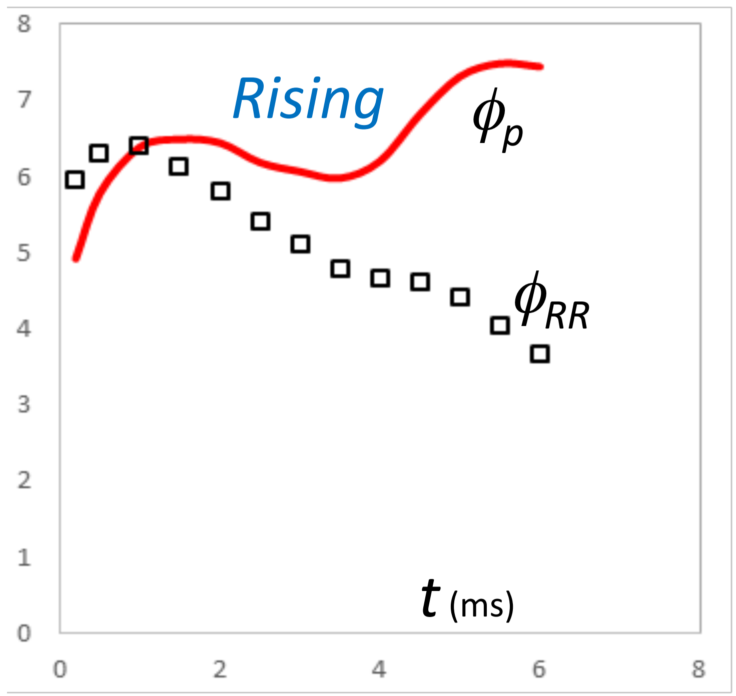
Compare poloidal flux:

actual ϕ_p and "formula" ϕ_{RR}

GS tool

"Rigid-rotor" formula
ϕ_{RR}(Wb) =
B_e(T)R_ϕ^3(m)/R_w(m)

Time histories (mWb) →



Two-dimensionality
(elongation not large):

- $B_{ext} > B_{wall}$
- Higher current density

→ Higher poloidal flux

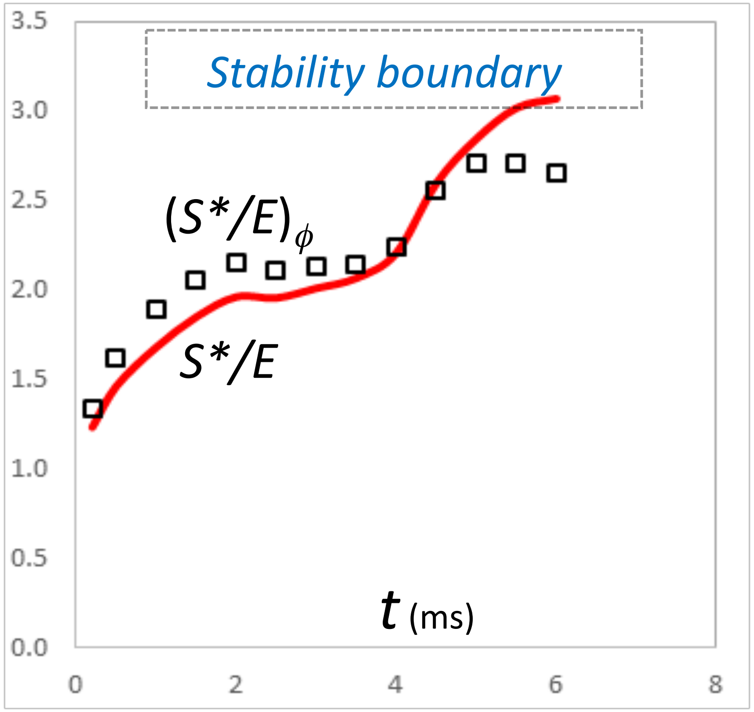
Something <u>drives current</u> to *increase* the flux!

How global stability changes in time

GS tool

Tilt stability: rule of thumb $S_*/E \sim 3 - 3.5$

Time histories →



Stability weakens with time: falling elongation

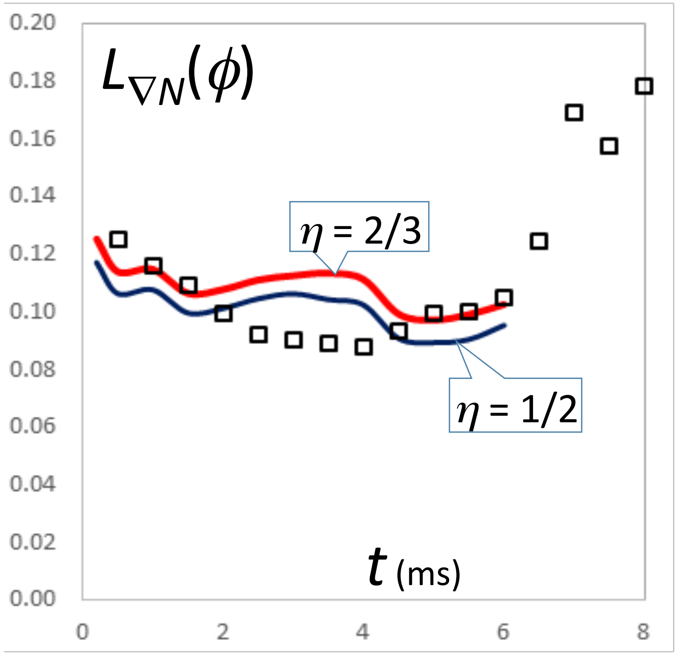
Something happens after about 5.5ms

Confidence check # 1: edge thickness measured and reconstructed

GS tool

Interferometry thickness: gradient length of $\int ndl$ at $y = R_\phi$

Time histories (m) →



Reasonable agreement; not very sensitive to η

Late strong uptrend in observed edge thickness

> Tilt instability?

> Tilt feeding dissipative cascade?

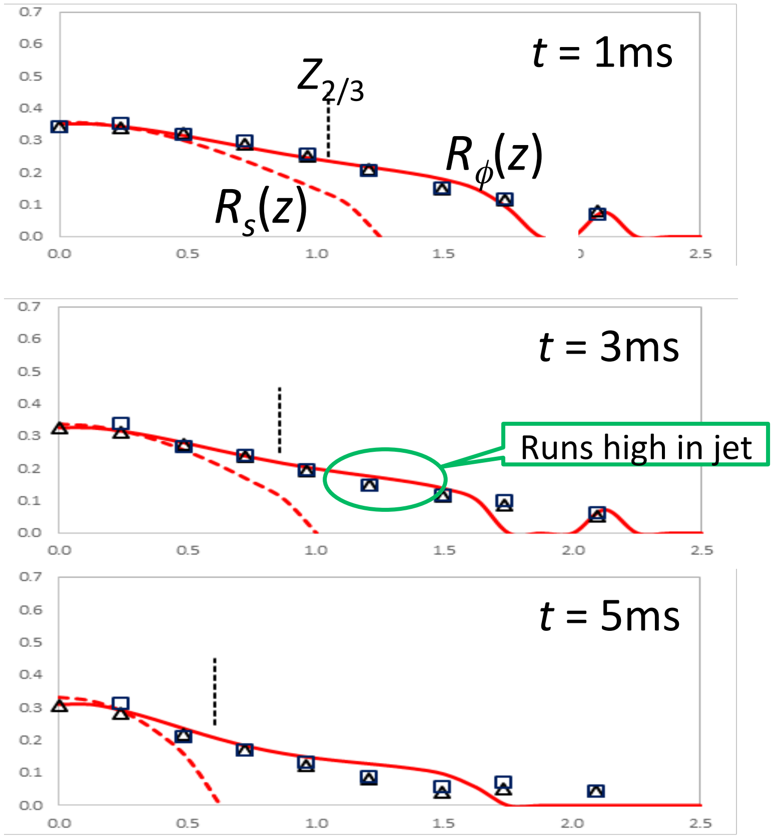
Confidence check # 2: poloidal shape of core measured and reconstructed

GS tool

Snapshots →

Observed:
Symbols measured

Reconstructed:
Red solid, $R_\phi(z)$
Red dashed $R_s(z)$



Reasonable agreement although a bit high in jet

Grad-Shafranov tool performs reasonably well

Reconstruction of C-2U #43628 with HyEq

Preview:

- Dimensions: radius & half length
- Poloidal shape: *r* vs *z*
- Poloidal flux, ϕ_p

Compare to GS

- Core- and mirror-confined populations

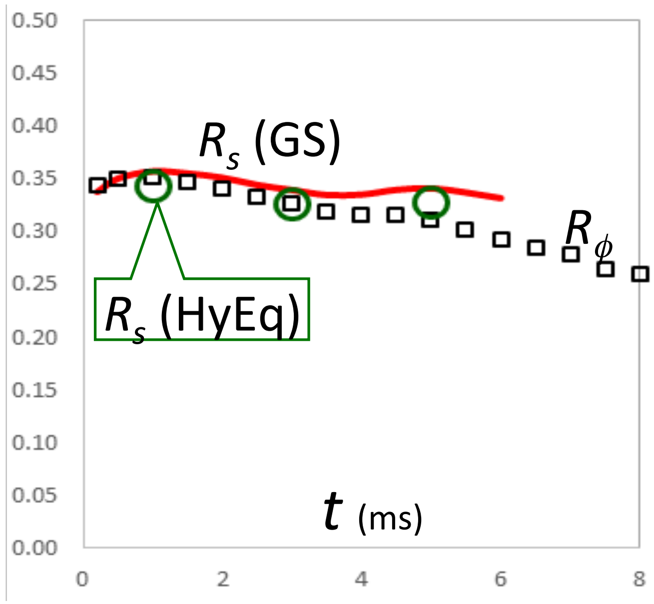
Unique to HyEq

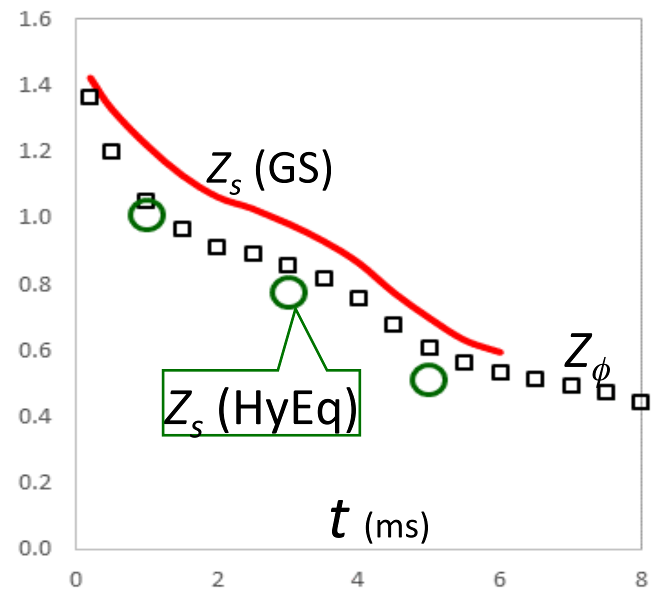
- A key comparison: HyEq &GS

Dimensions: radius & half length

HyEq tool

Distinguish measurables, R_ϕ, $Z_{2/3}$ from reconstructions R_s, Z_s



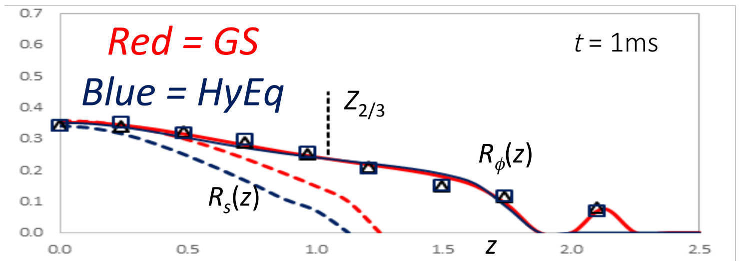


R_s: HyEq similar but ~5% lower than GS

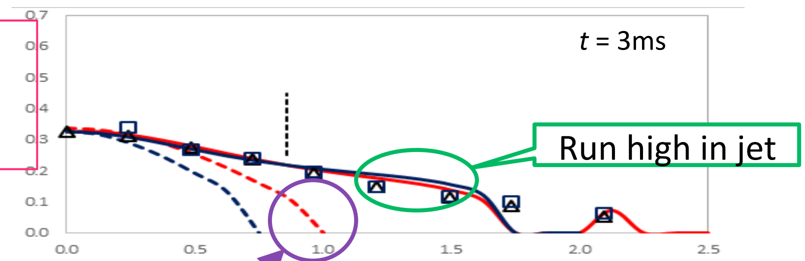
Z_s: similar trends but HyEq ~20% shorter than GS

Poloidal profile: *r* vs *z*

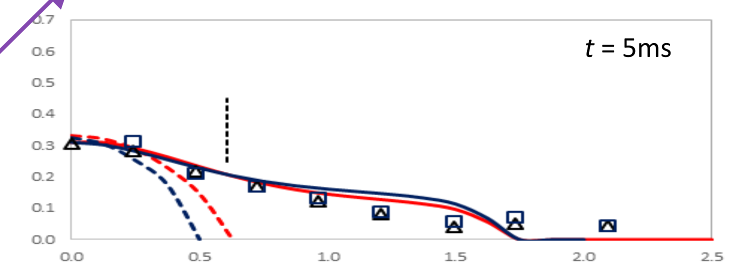
HyEq
tool



Both fit fairly well except
- Run high in jet



Differences:
HyEq: • shorter separatrix
• “fuller” X-point region

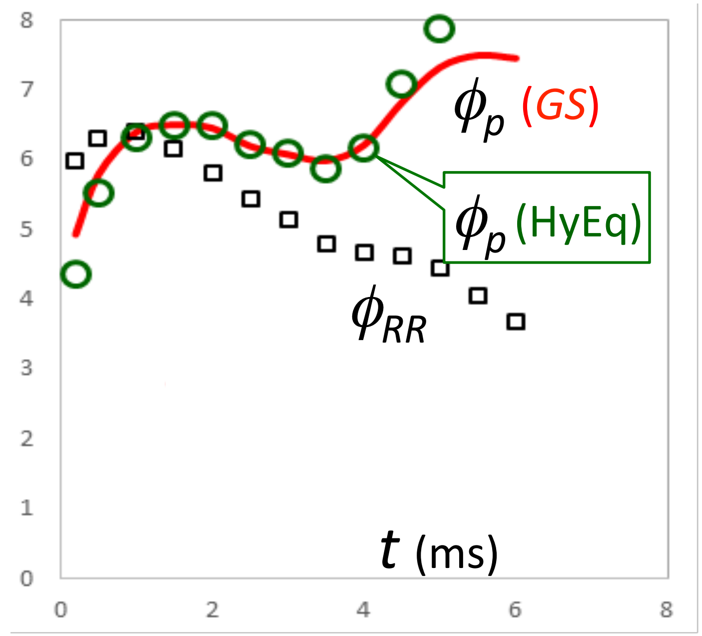


- Shorter ⟶ reaches S_*/E threshold at ~ 3.5 ms
- What inflates X-point region?

Poloidal flux ϕ_p

HyEq tool

Time histories (mWb) →

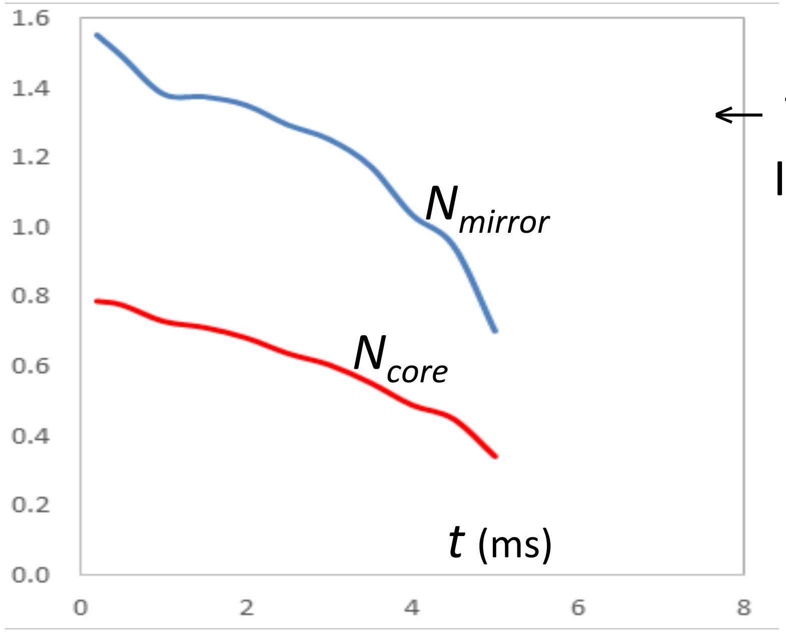


- GS & HyEq very close & well above RR formula

Core- and mirror-confined inventories

HyEq tool

- GS (fluid): ***regions***, e.g. "core" = inside separatrix
- HyEq (kinetic): ***populations***, core-, mirror-confined



← Time histories of Inventories (10^{19})

- Core-inventory *half* the mirror-confined
- Decay time of N_{core} ~ 8.7ms; tail-off begins 3.5 – 4ms
- Mirror population plays an *outsized* role in overall confinement

How close are HyEq and GS equilibria?

Essential property of GS equilibria: surface functions

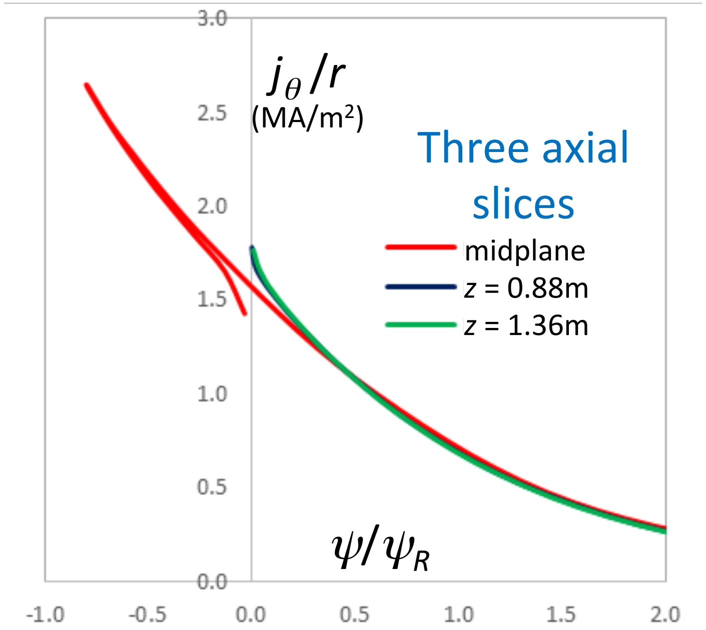
$p = p(\psi)$ and $j_\theta/r = p'(\psi)$

HyEq: How close is j_θ/r to a surface function?

Quite close.

> Inboard – outboard

> Jet consistent with mid plane



Grad-Shafranov tool still useful:

- Less sophisticated;
- Captures many features of hybrid equilibria

Summary: reconstruction tools

- GS and HyEq tools give similar reconstructions of #43628
- Both notably different from standard formulas, especially poloidal flux
- Evidence of current drive for ~4ms
- MHD stability degrades with time; leads to prolonged death rattle rather than abrupt termination
- Periphery ion population *double* the core population; plays an *outsized* role in overall confinement
- Development of both GS and HyEq tools continue