Two-fluid magnetic relaxation in RFPs*

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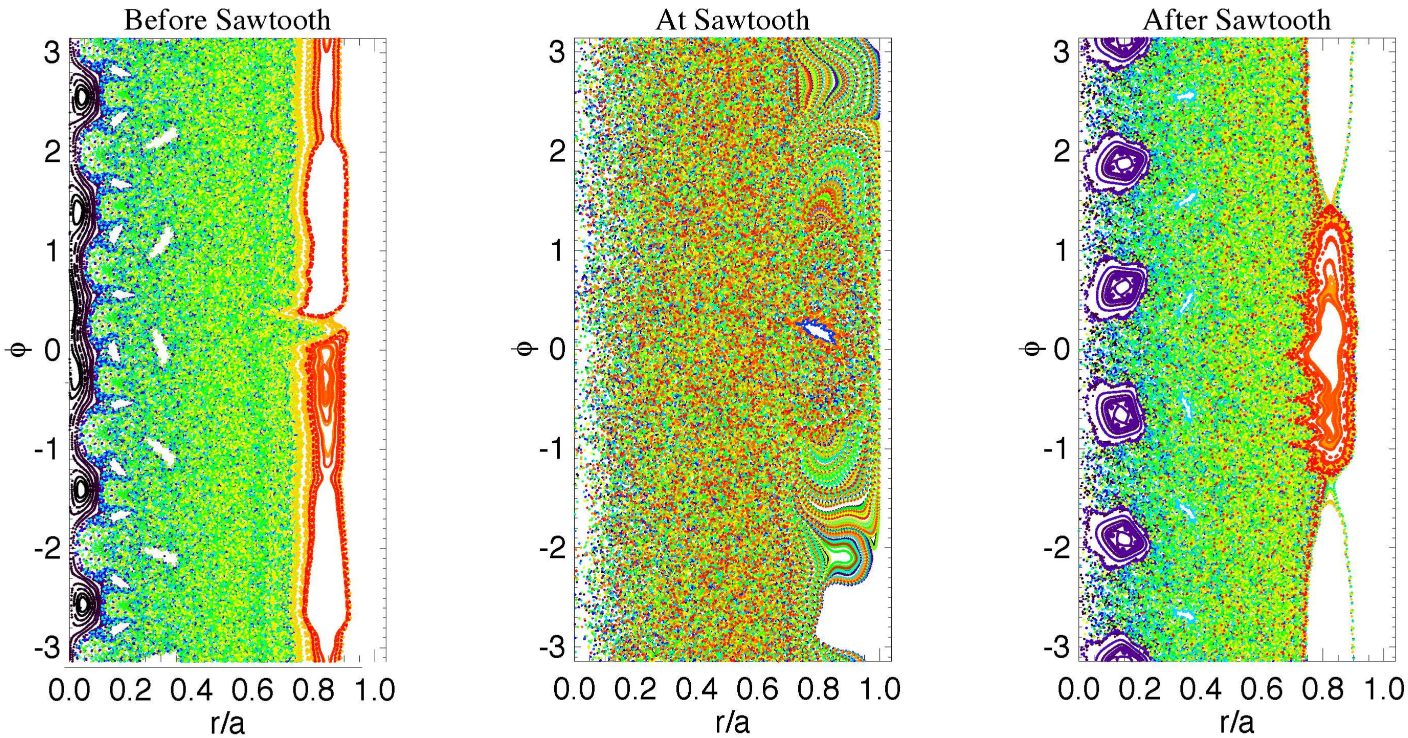
Magnetic relaxation in the reversed-field pinch (RFP) is a demanding test case for comparisons with extended MHD

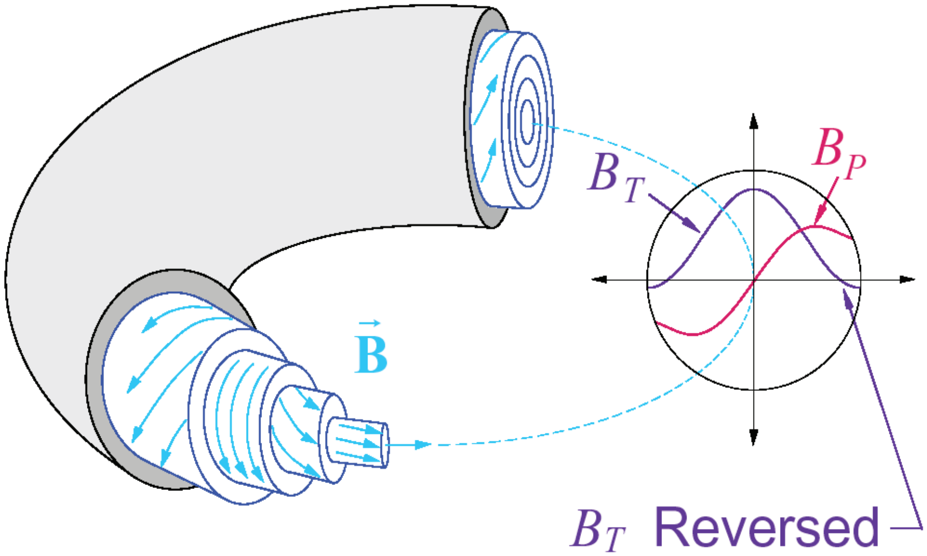


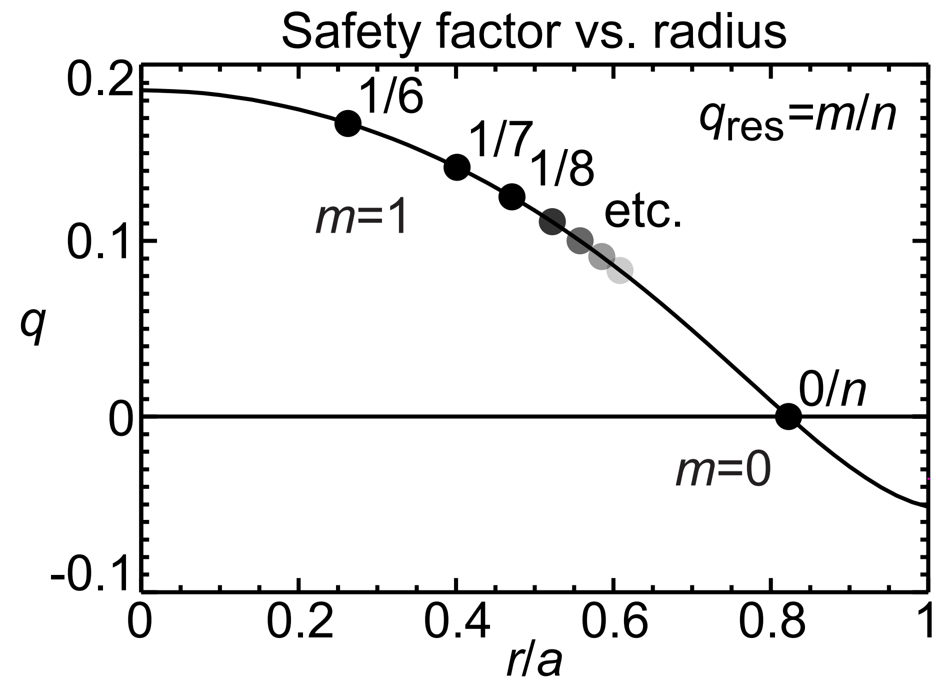
- Sequence of tearing modes nonlinearly coupling core and edge
- Reminiscent of tokamak disruption processes
- Visco-resistive MHD can be extended with two fluids, finite beta, kinetic effects, impurity evolution, radiation, ...

Outline

- RFPs, DEBS, and NIMROD
- Single-fluid comparisons
- Two-fluid comparisons

Multiple tearing-mode resonances allow complex nonlinear MHD in standard RFP operation





- High beta and perhaps Ohmic ignition are potential advantages for fusion with RFPs
- Transient improved confinement is tokamak-like

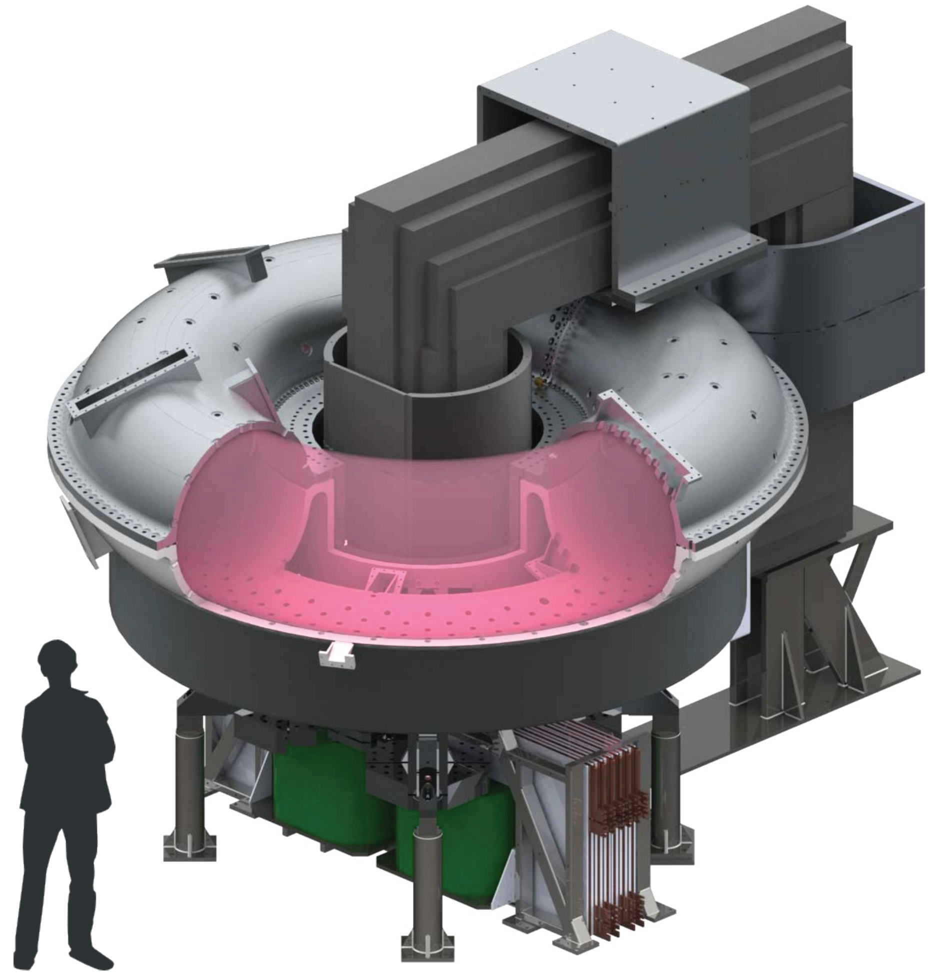
Madison Symmetric Torus (MST)

- $R_0/a = (1.5 \text{ m})/(0.52 \text{ m}) \approx 3$

- $I_\text{p} \lesssim 600$ kA

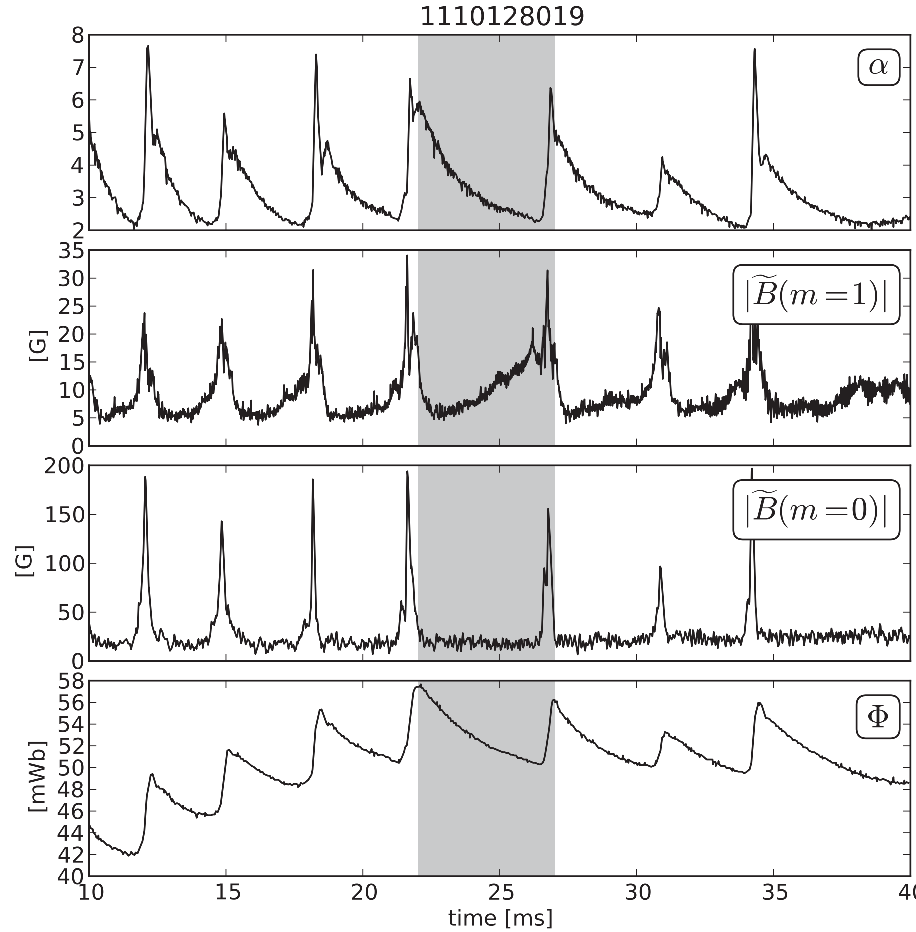
- $n_\text{e} \sim 10^{19}/\text{m}^3$

- $T_\text{e,i} \lesssim 2$ kV



Magnetic relaxation as a sawtooth cycle in MST

- Ohmic drives $\lambda \propto J_{\|}/B$ more peaked: flatness parameter α decreases
- Core-resonant $m = 1$ modes become unstable
- Edge-resonant $m = 0$ stable but nonlinearly driven by $m = 1$ at sawtooth crash
- Crash EMF generates core toroidal flux Φ, flattens λ



- Key mechanism: fluctuation-induced 'dynamo' EMF in mean-field parallel Ohm's law, $\langle \mathbf{E} \rangle_{\|} \simeq \underbrace{-\left\langle \widetilde{\mathbf{V}} \times \widetilde{\mathbf{B}} \right\rangle_{\|}}_{\text{MHD dynamo}} \underbrace{+\left\langle \widetilde{\mathbf{J}} \times \widetilde{\mathbf{B}} \right\rangle_{\|} /(en)}_{\text{Hall dynamo}} + \langle \eta \mathbf{J} \rangle_{\|}$

3D MHD codes DEBS and NIMROD have different capabilities

DEBS: single-fluid visco-resistive MHD in cylindrical geometry

- $\partial \mathbf{A}/\partial t = S\mathbf{V} \times \mathbf{B} - \eta \mathbf{J}$
  $\rho \partial \mathbf{V}/\partial t = -S\rho \mathbf{V} \cdot \nabla \mathbf{V} + S\mathbf{J} \times \mathbf{B} + \nu \nabla^2 \mathbf{V}$,
  where Lundquist number $S \equiv \tau_{\mathrm{res}}/\tau_{\mathrm{A}}$
- Dynamic viscosity for larger time steps for same nominal ν
- Schnack *et al.*, J. Comput. Phys. **70**, 330 (1987)

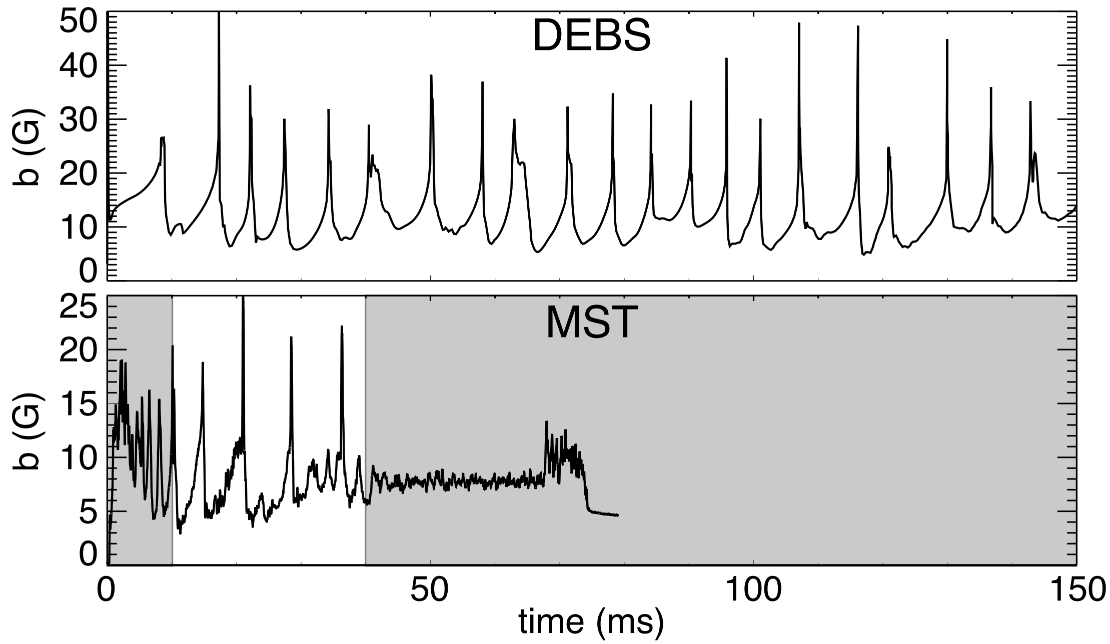
NIMROD: extended MHD in cylindrical or toroidal geometry

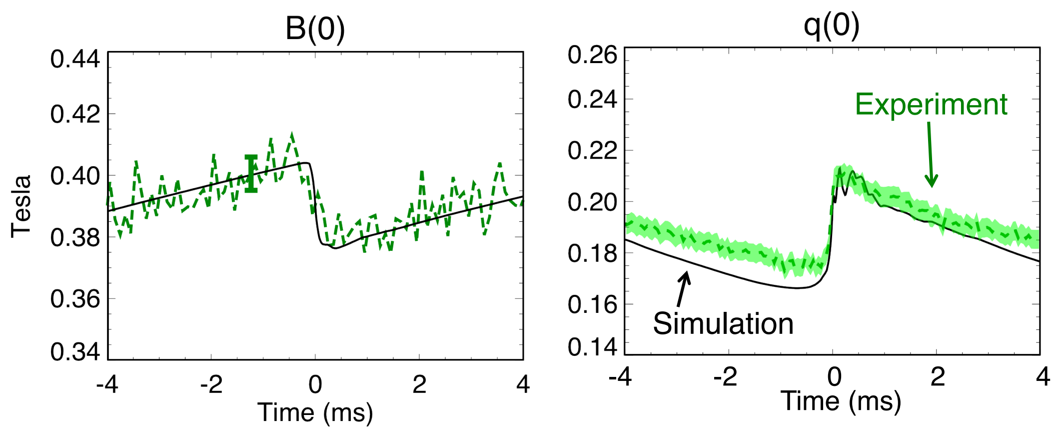
- $\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \frac{\mathbf{J} \times \mathbf{B}}{en} - \frac{\nabla p_{\mathrm{e}}}{en} + \eta \mathbf{J} + \frac{m_{\mathrm{e}}}{e^2 n} \frac{\partial \mathbf{J}}{\partial t}$
  $\rho \frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \cdot \boldsymbol{\Pi}_{\mathrm{gv}} - \nabla \cdot \rho \nu \mathbf{W}$
- Hall term $\mathbf{J} \times \mathbf{B}/(en)$, ion gyroviscous stress $\boldsymbol{\Pi}_{\mathrm{gv}}$, ...
- Sovinec and King, J. Comput. Phys. **229**, 5803 (2010)

DEBS case for single-fluid MHD comparison to MST experiments

- $\beta = 0$ ($\beta_{\text{exp}} \sim 0.1$)
- $S = 4\text{E}6$ ($\approx S_{\text{exp}} \sim I_{\text{p}} T_{\text{e}}^{3/2}$, with $I_{\text{p}} \approx 400$ kA)
- Magnetic Prandtl number $P_{\text{m}} \equiv \tau_{\text{res}}/\tau_{\text{visc}}\,(\propto \nu/\eta) \approx 100$ ($0.1 \lesssim P_{\text{m,exp}} \lesssim 1$ using estimated perpendicular Braginskii coefficient)
- Dynamic viscosity enabled
- J. Reusch *et al.*, PRL **107**, 155002 (2011)

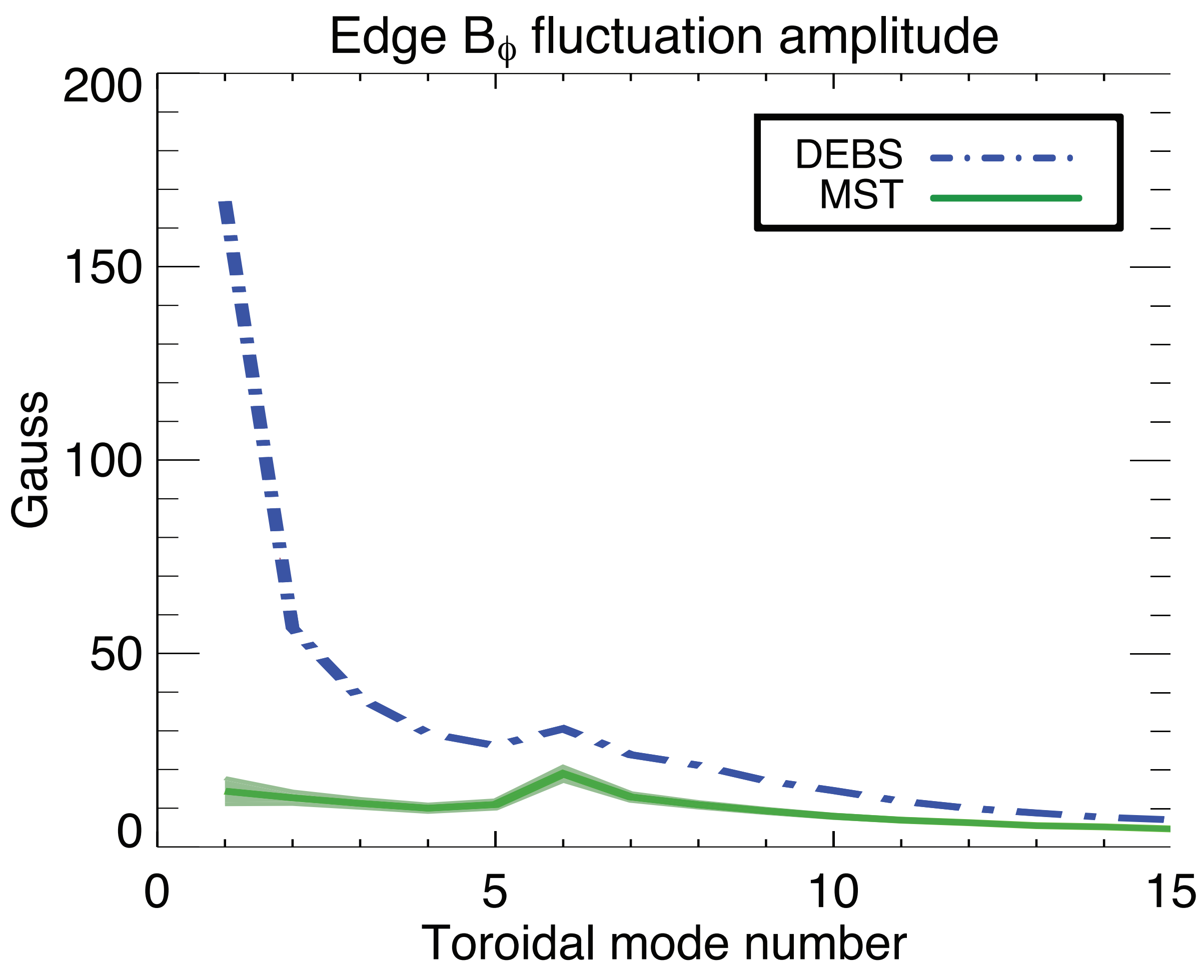
Sawtooth cycle, equilibrium evolution show good agreement





- Fluctuation-induced EMF behaves similarly to experiment

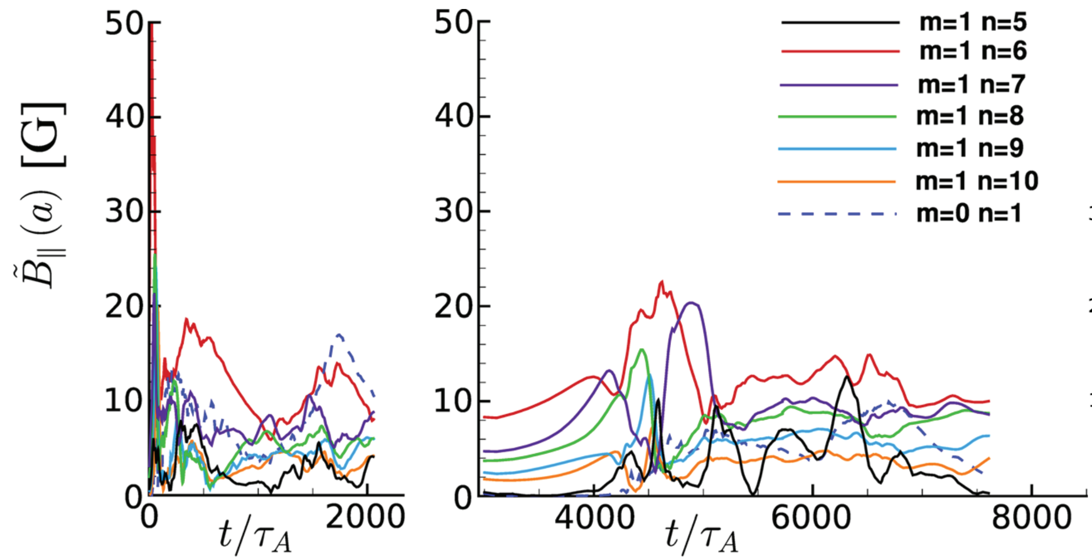
MST magnetic fluctuation amplitudes strongly overpredicted



NIMROD cases for extended MHD simulations of RFP

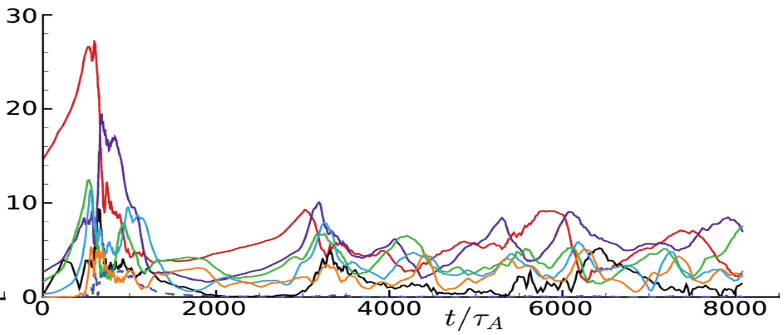
- Cylindrical geometry
- Single fluid or two-fluid with cold or warm ($\beta_0 = 0.1$) ions
- Uniform thermal pressure
- $S \leq 8 \times 10^4$, much smaller than most MST cases
- $P_\mathrm{m} \leq 1$, similar to MST perpendicular value assuming Braginskii
- King, Sovinec, & Mirnov, POP **19**, 055905 (2012)

Two-fluid MHD with ion gyroviscosity has saturated magnetic-fluctuation amplitudes 2x smaller than single-fluid



(a) zero-β single-fluid computation

(b) finite-β single-fluid computation



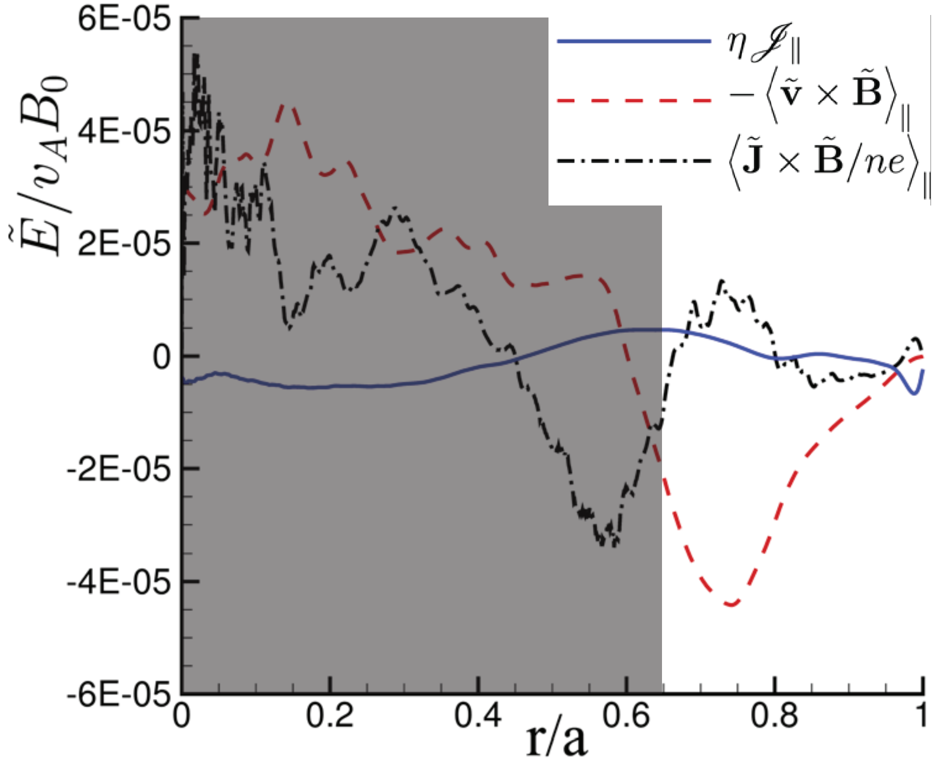
finite-beta two-fluid computation

King, Sovinec, and Mirnov, POP 19, 055905 (2012)

- Trends toward better agreement with MST experiments

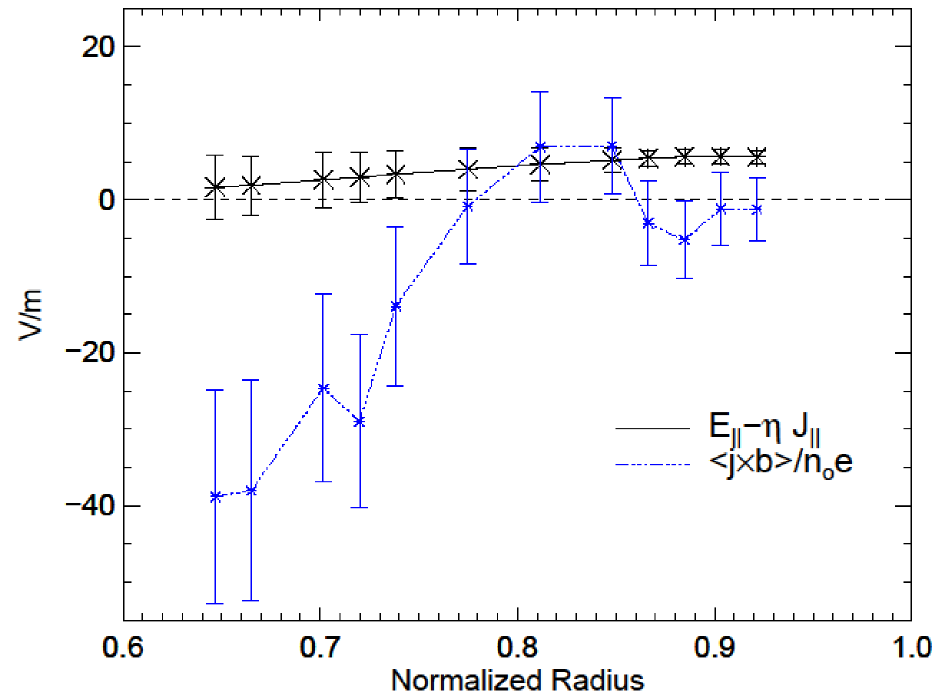
Hall dynamo has complex radial structure in both simulation and experiments with deep-insertion probe

Simulation



King, Sovinec, and Mirnov, POP 19, 055905 (2012)

Hall dynamo probe



- $\langle \mathbf{E} - \eta \mathbf{J} \rangle_{\|} \simeq - \left\langle \widetilde{\mathbf{V}} \times \widetilde{\mathbf{B}} \right\rangle_{\|} \underbrace{+ \frac{1}{en} \left\langle \widetilde{\mathbf{J}} \times \widetilde{\mathbf{B}} \right\rangle_{\|}}_{\text{Hall dynamo}}$

In two-fluid MHD, magnetic and flow relaxation couple through common $\mathbf{J} \times \mathbf{B}$ term in Ohm's law and momentum equation

- Mean-field Ohm's law:

$$\langle \mathbf{E} \rangle_{\parallel} \simeq \underbrace{-\left\langle \widetilde{\mathbf{V}} \times \widetilde{\mathbf{B}} \right\rangle_{\parallel}}_{\text{MHD dynamo}} \underbrace{+\frac{1}{en}\left\langle \tilde{\mathbf{J}} \times \widetilde{\mathbf{B}} \right\rangle_{\parallel}}_{\text{Hall dynamo}} +\eta \langle \mathbf{J} \rangle_{\parallel}$$

  - MHD and Hall dynamos often compete

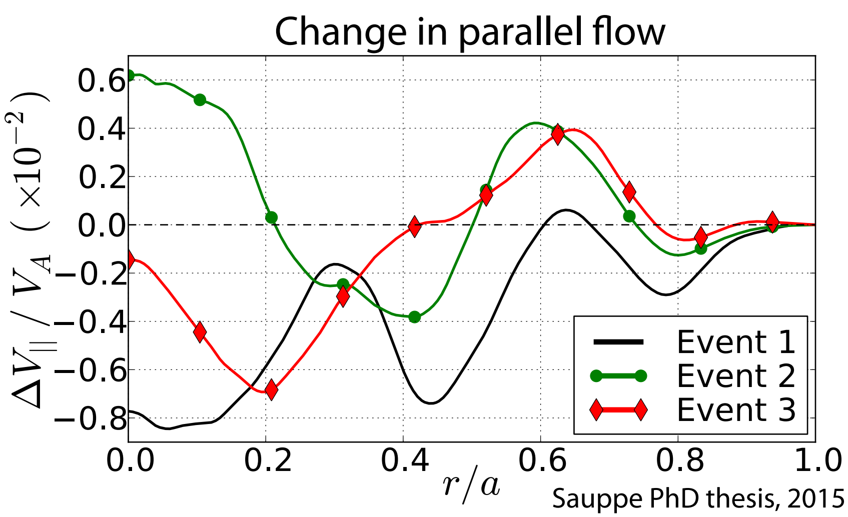
- Mean-field momentum equation:

$$\rho \frac{\partial \langle \mathbf{V} \rangle_{\parallel}}{\partial t} \simeq \underbrace{\left\langle \tilde{\mathbf{J}} \times \widetilde{\mathbf{B}} \right\rangle_{\parallel}}_{\text{Maxwell stress}} \underbrace{-\rho \left\langle \widetilde{\mathbf{V}} \cdot \nabla \widetilde{\mathbf{V}} \right\rangle_{\parallel}}_{\text{Reynolds stress}} +\rho\nu \nabla^2 \langle \mathbf{V} \rangle_{\parallel}$$

  - Reynolds stress tends to oppose the larger Maxwell stress

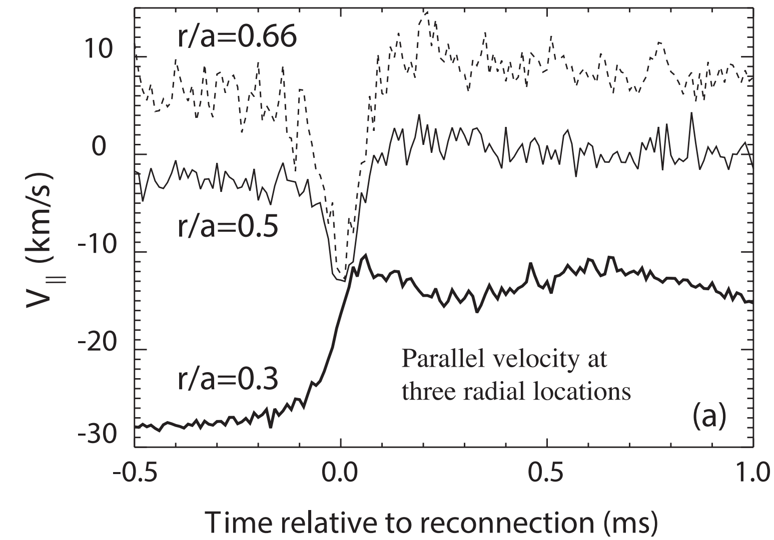
Two-fluid MHD relaxation events significantly change mean flow profiles, as sawteeth do in experiment

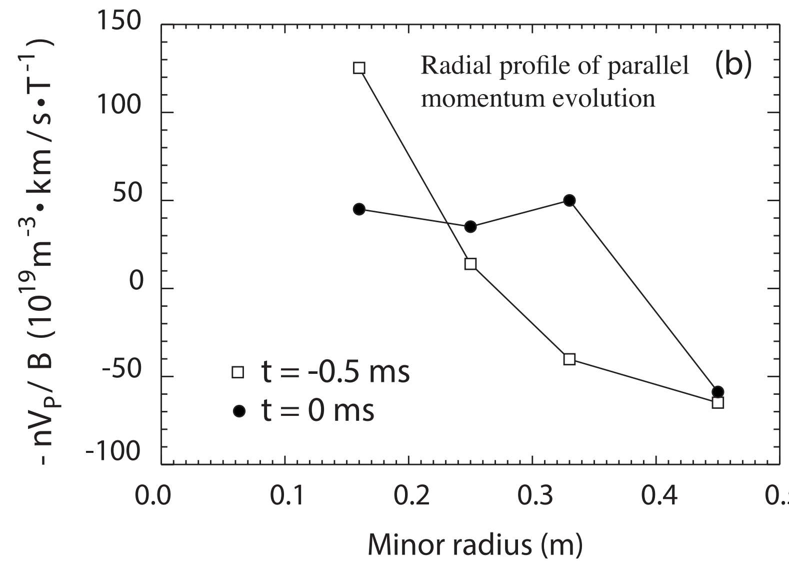
Simulations



- Simulated radial structure is complex
- Only a few simulated events available

Experiments





Kuritsyn *et al.*, Phys. Plasmas **16**, 055903 (2009)

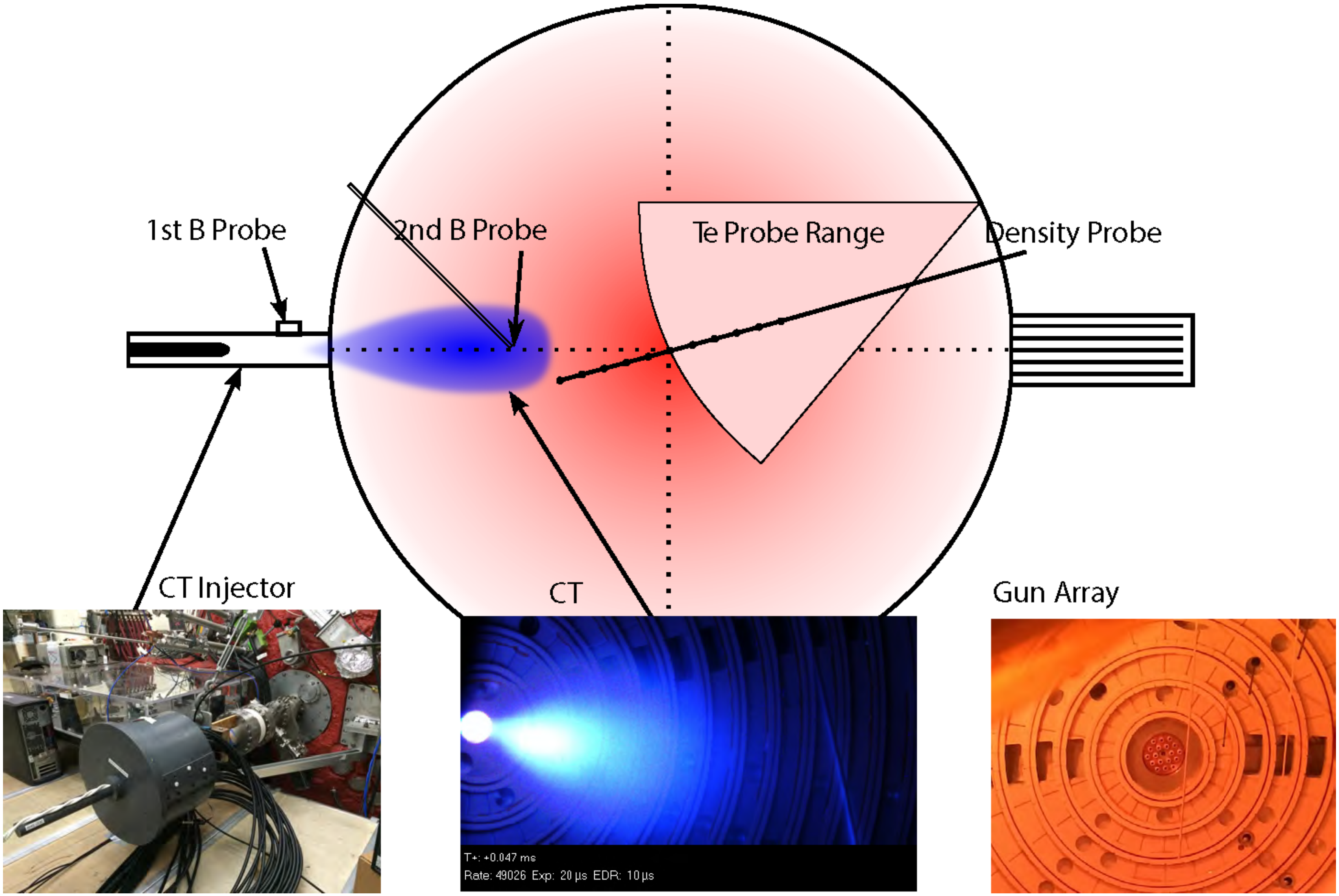
Summary

- Magnetic relaxation in RFP experiments provides demanding comparisons for nonlinear simulations of extended MHD models
- Single-fluid DEBS simulations closely reproduce equilibrium evolution observed in MST but strongly overpredict $\widetilde{B}$
- Single- and two-fluid NIMROD simulations reveal that the Hall dynamo and ion gyroviscosity terms may improve this agreement
- Hall-like term also appears as Maxwell stress in momentum equation alongside Reynolds stress, coupling magnetic relaxation to flow relaxation
- Deep-insertion Hall dynamo probe results on MST show complex radial structure consistent with extended MHD simulations

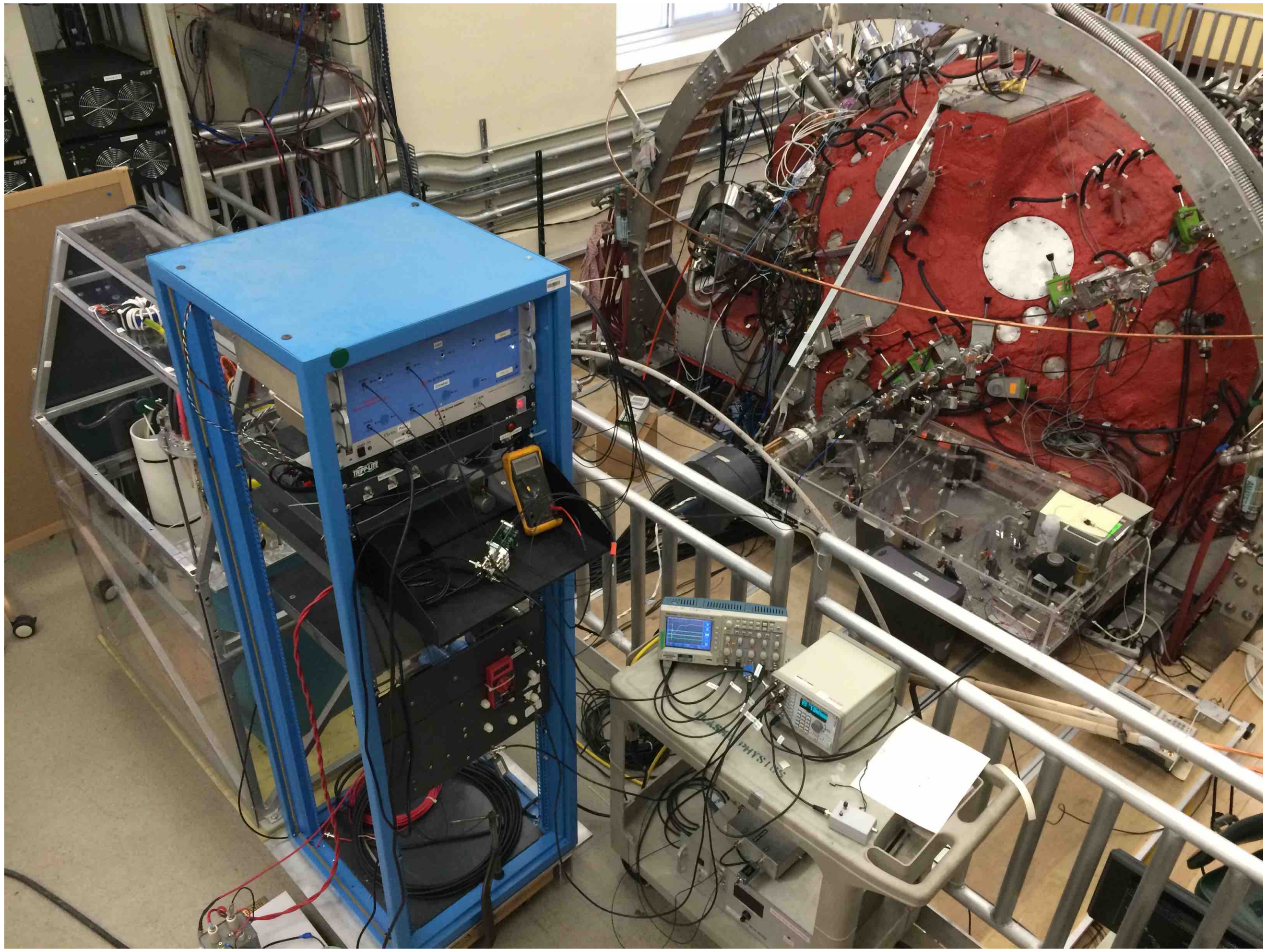
Initial CT injection experiments at WiPAL (Wisconsin Plasma Astrophysics Laboratory)

D. Endrizzi, C. Forest, L. Laufman-Wollitzer, *University of Wisconsin*

MPDX (Madison Plasma Dynamo Experiment) tests with CT injector on loan from TAE/U. Nihon group

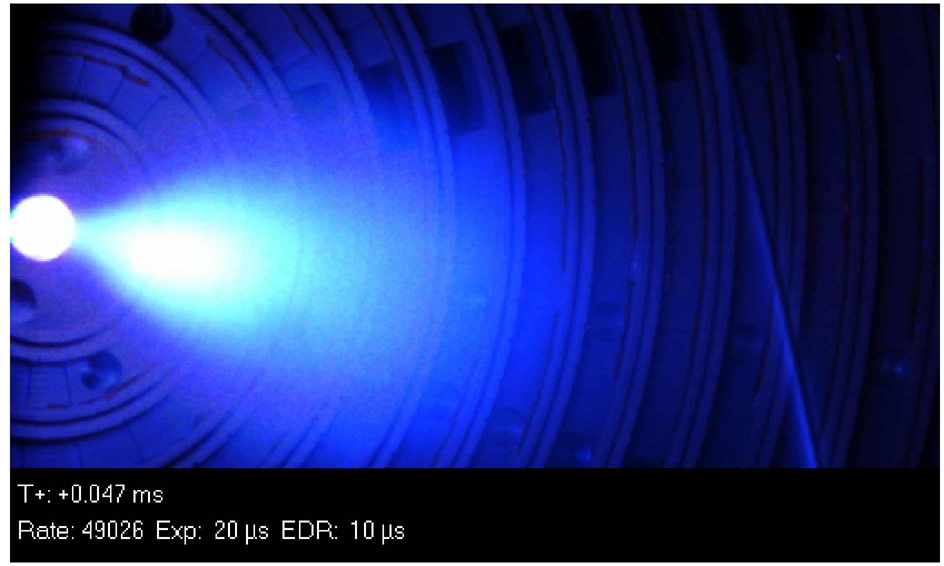


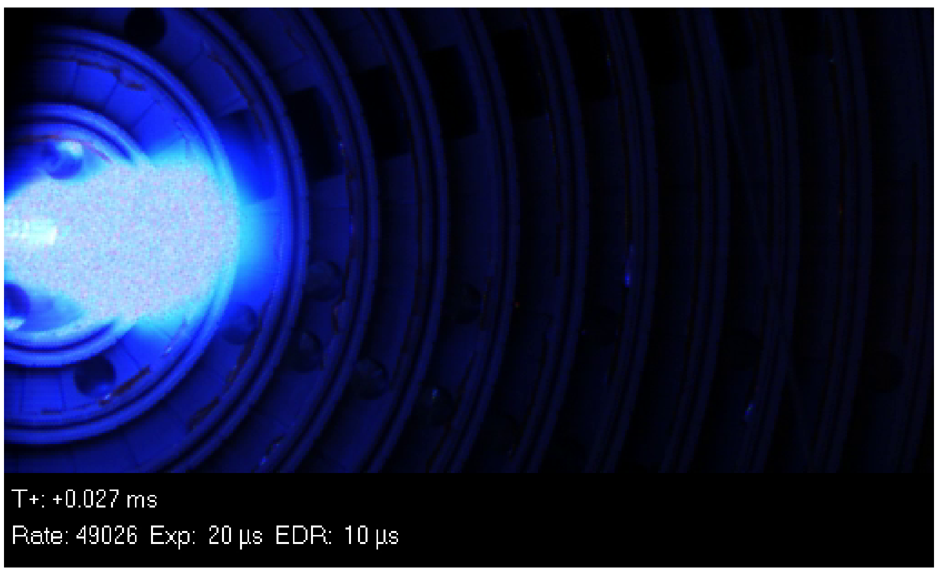
Injector was installed on MPDX with help of H. Gota et al.

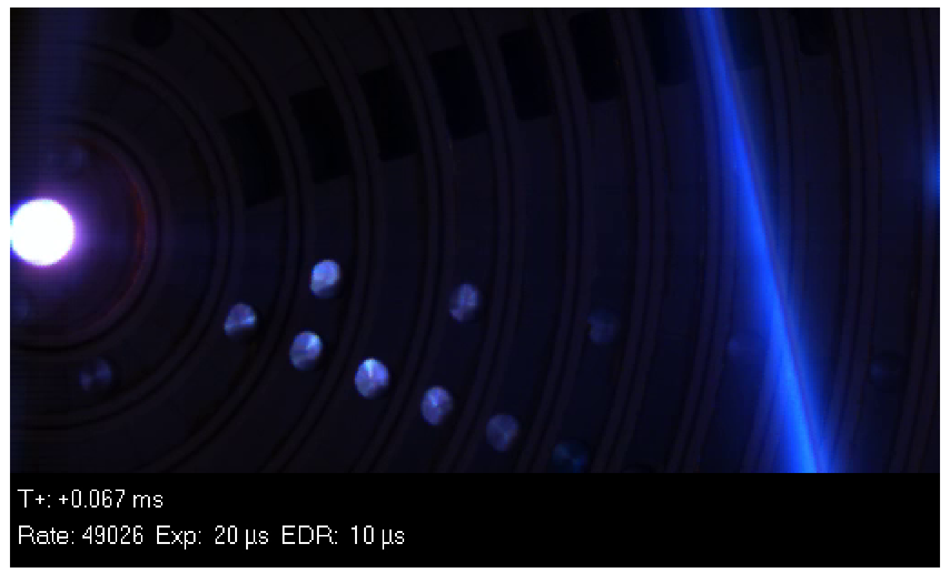


- Matsumoto et al., RSI **87**, 053512 (2016), e.g. for injector details
  - $n_{\mathrm{CT}} \sim 5 \times 10^{21}/\mathrm{m}^3$, $T_{\mathrm{CT}} \sim 40$ eV, $N_{\mathrm{CT}} \sim 10^{19}$

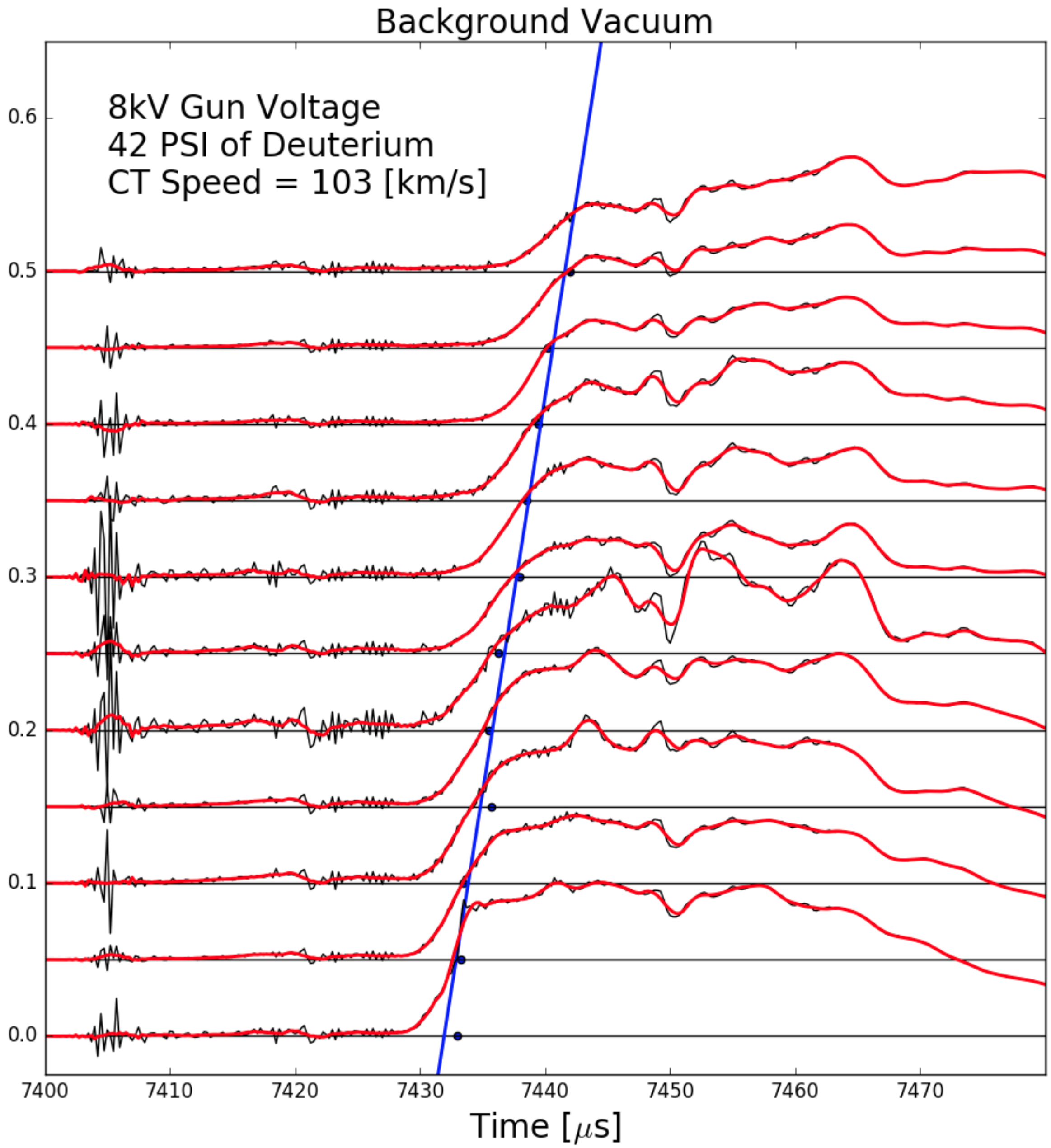
Fast camera footage of injection into vacuum



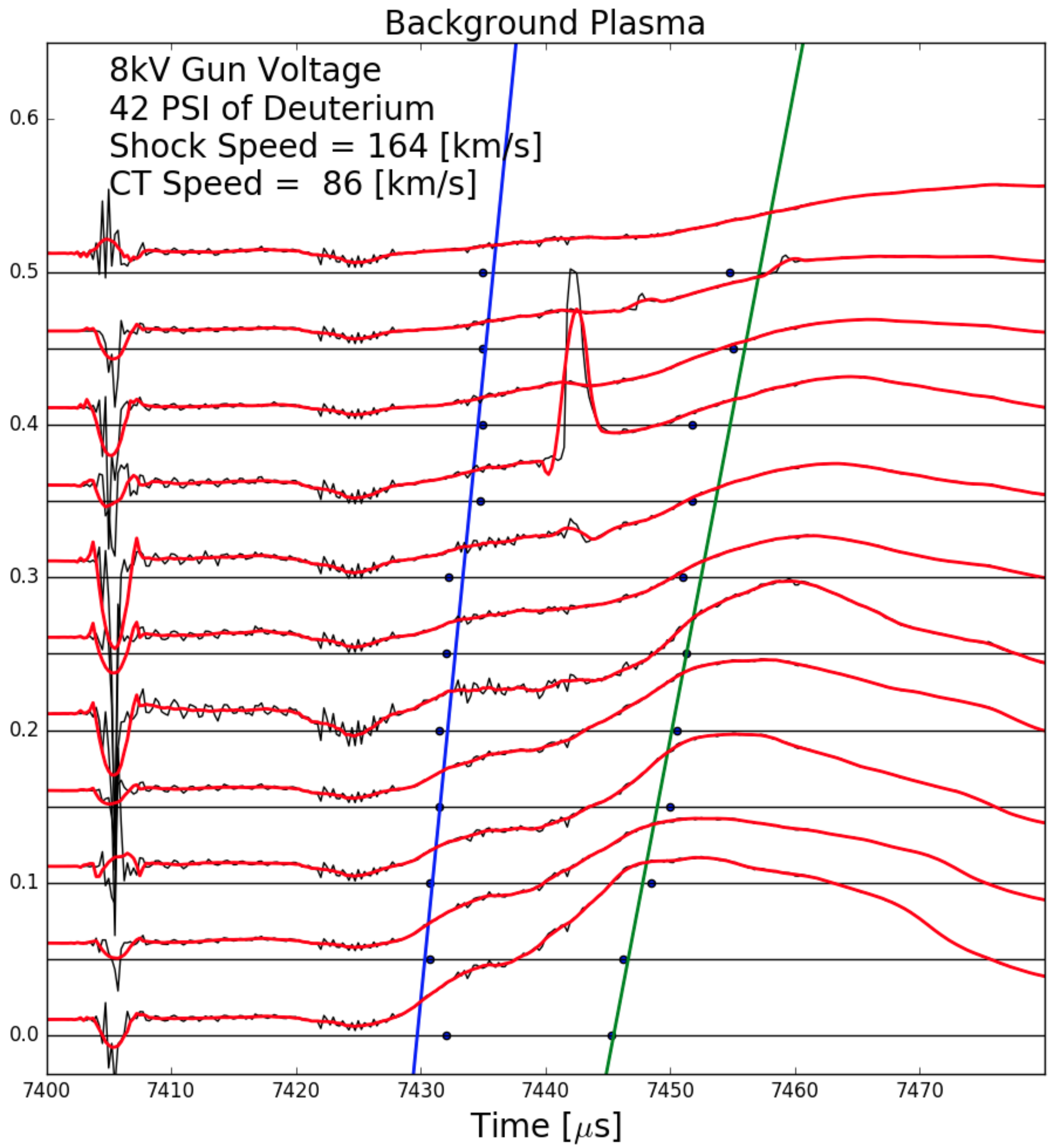




Injected CT speeds measured by Isat array



Injection into target plasmas may show leading shock



- $n_{\text{target}} \sim 10^{17}/\text{m}^3$, $T_{\text{target}} \sim 10$ eV ($c_s \sim 20$ km/s)

Scan of target parallel B shows no clear trend