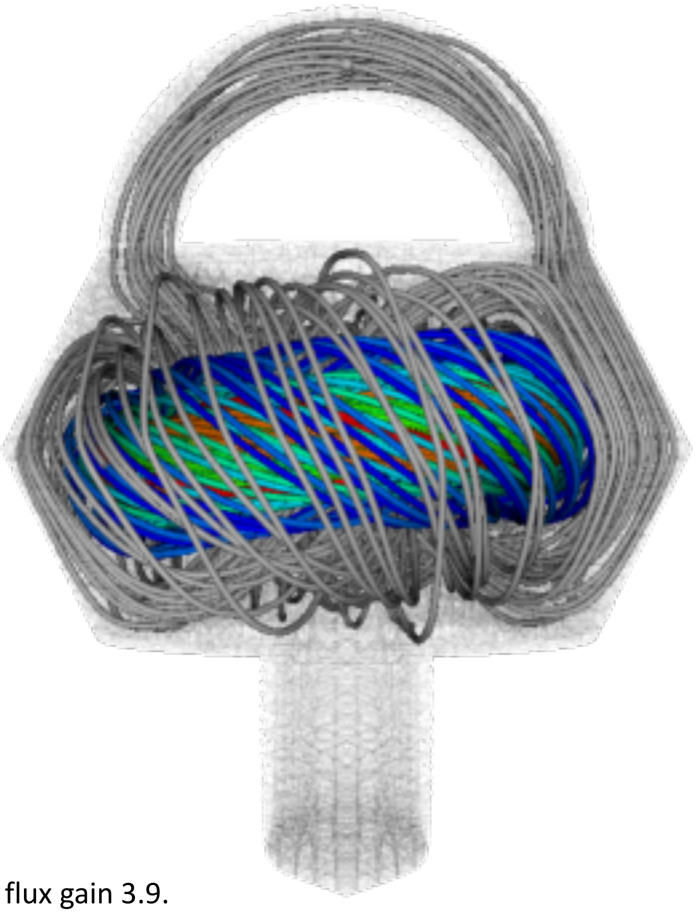
Sustainment of Stable Spheromaks with Imposed-dynamo Current Drive



Composite Taylor state, flux gain 3.9.
Image by Brian Victor

Aaron Hossack, Tom Jarboe, Derek Sutherland, Chris Hansen, Tom Benedett, Kyle Morgan, Chris Everson, James Penna, and Brian Nelson

University of Washington, Seattle, WA

Masayoshi Nagata and Takafumi Hanao

University of Hyogo, Himeji, Hyogo, Japan

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WILLIAM E. BOEING DEPARTMENT OF AERONAUTICS & ASTRONAUTICS
UNIVERSITY of WASHINGTON

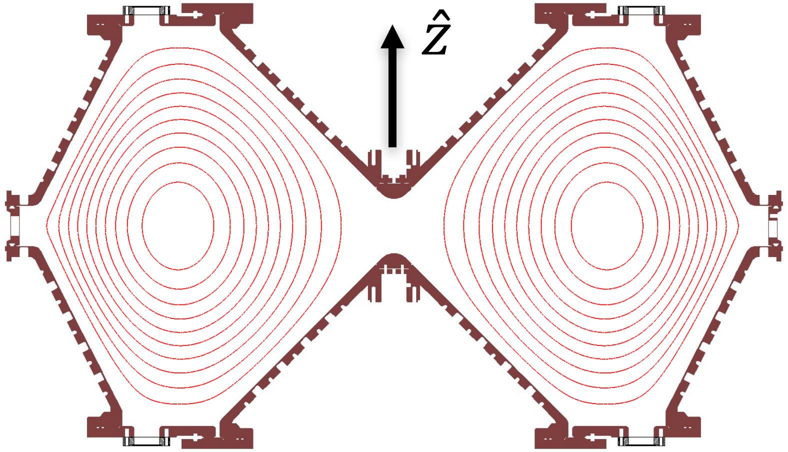
HIT - SI

Outline

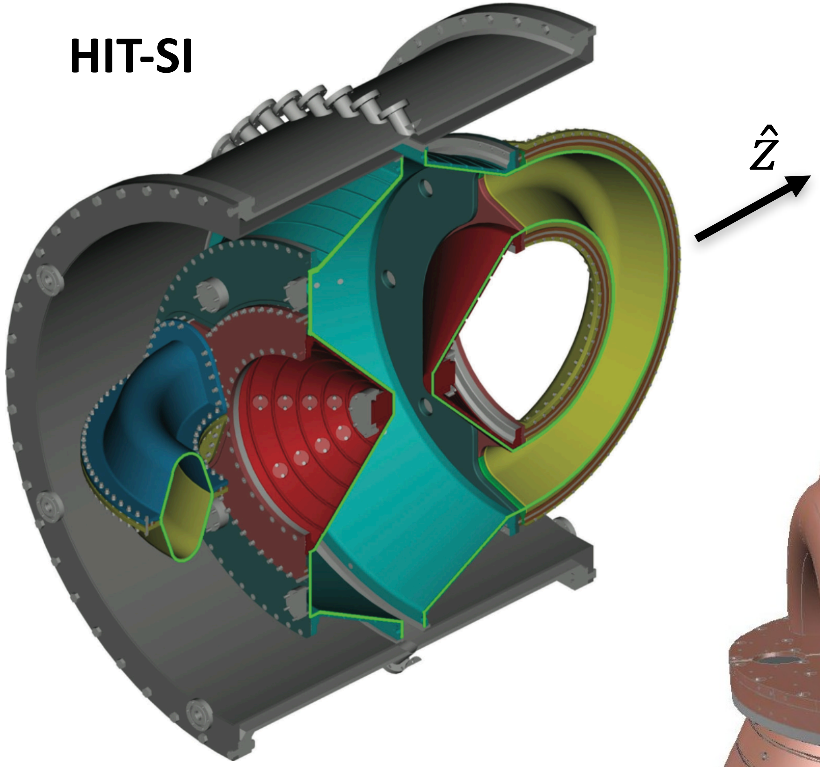
- The HIT-SI and HIT-SI3 experiments
- Imposed-dynamo current drive (IDCD) theory
- Using biorthogonal decomposition to isolate coherent structures
- Observed stability in high frequency, sustained HIT-SI discharges
- Coherent, imposed motion and stability in high-current (90 kA) sustained HIT-SI discharges
- Comparison of spheromak size to closed-flux predictions
  - Taylor minimum energy equilibria
  - IDCD predicted λ profile
- Progress on HIT-SI3

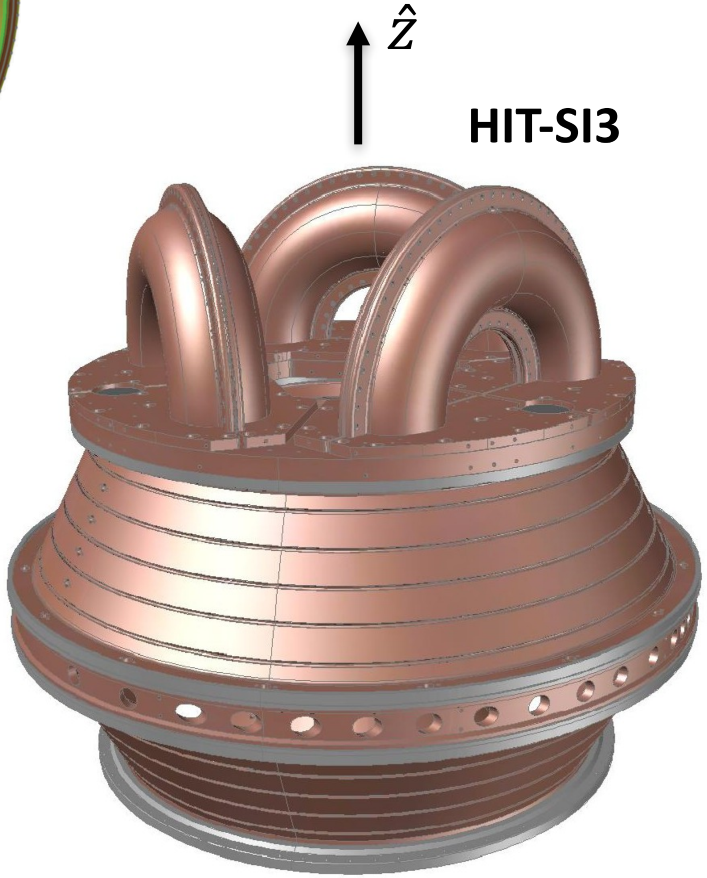
The HIT-SI & HIT-SI3 experiments

- "Bow tie" flux conserver:
  - R = 55 cm, a = 23 cm



- Semi-toroidal injector ducts
  - Two injectors on HIT-SI, opposite sides, rotated 90° toroidally
  - Three injectors on HIT-SI3, same side, equally spaced





CAD renderings by John Rogers

The HIT-SI & HIT-SI3 experiments

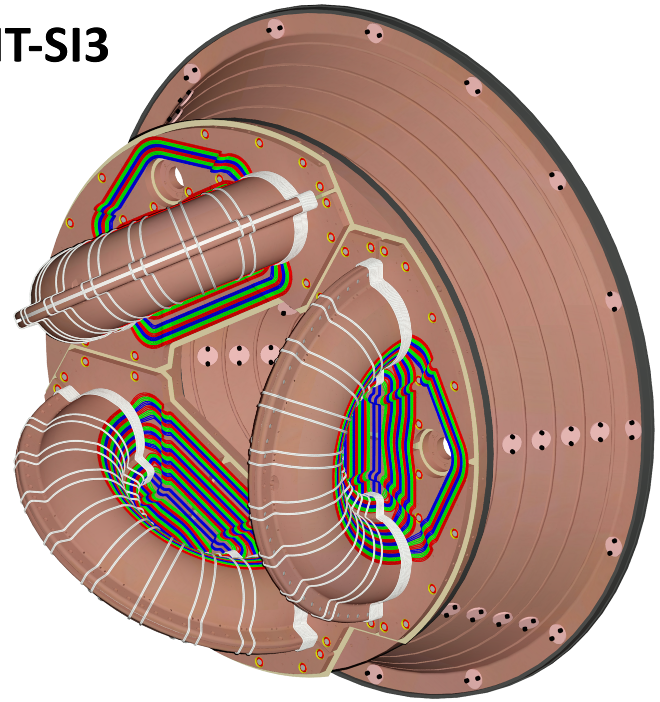
- Voltage coil induces loop voltage, drive injector current
- Flux coil injects magnetic flux
- On each injector, flux and voltage are oscillated in phase, giving positive helicity injection

$$\dot{K}_{inj} = 2 \int \vec{E}_{vac} \cdot \vec{B}_{vac} \, dV$$

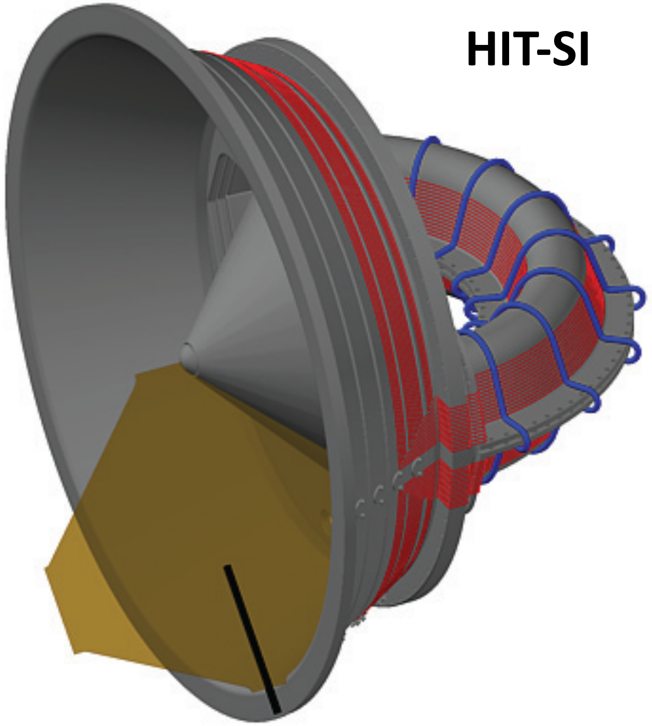
HIT-SI3

Flux

Voltage



HIT-SI



Flux

Voltage

Relative injector phasing for steady helicity injection

- HIT-SI, injectors 90° out of phase, steady helicity injection:

$$\dot{K} = 2V_0\ \psi_0[\sin^2(\omega t) + \cos^2(\omega t)]$$

$$\dot{K} = 2V_0\ \psi_0$$

- HIT-SI3, injectors 60° or 120° out of phase:

$$\dot{K}_{inj} = 2V_0\psi_0[\overbrace{\sin^2(\omega_{inj}t)}^{A_{inj}} + \overbrace{\sin^2(\omega_{inj}t + \phi)}^{B_{inj}} + \overbrace{\sin^2(\omega_{inj}t + 2\phi)}^{C_{inj}}]$$

$$\phi = 120°, 60°$$

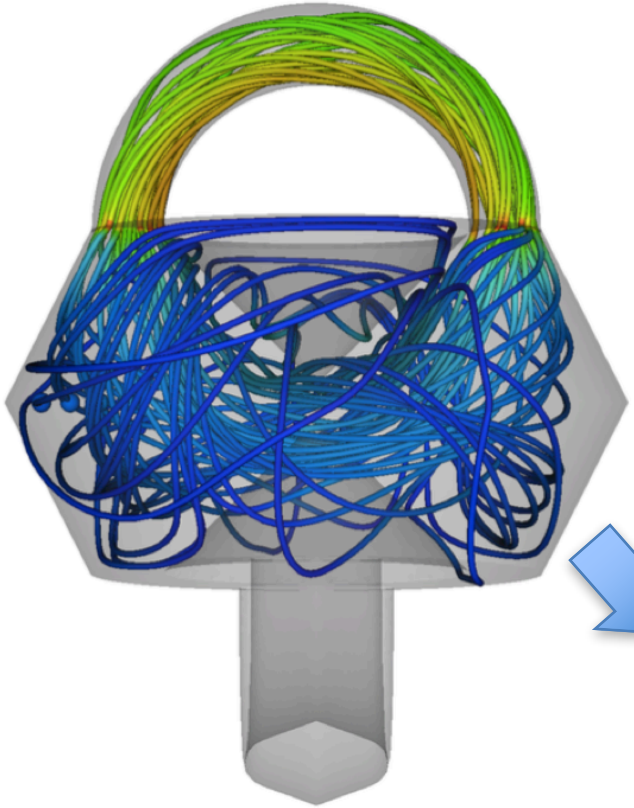
$$\dot{K}_{inj} = 3V_0\psi_0$$

- HIT-SI3, injectors in phase, ***non-steady*** helicity injection:

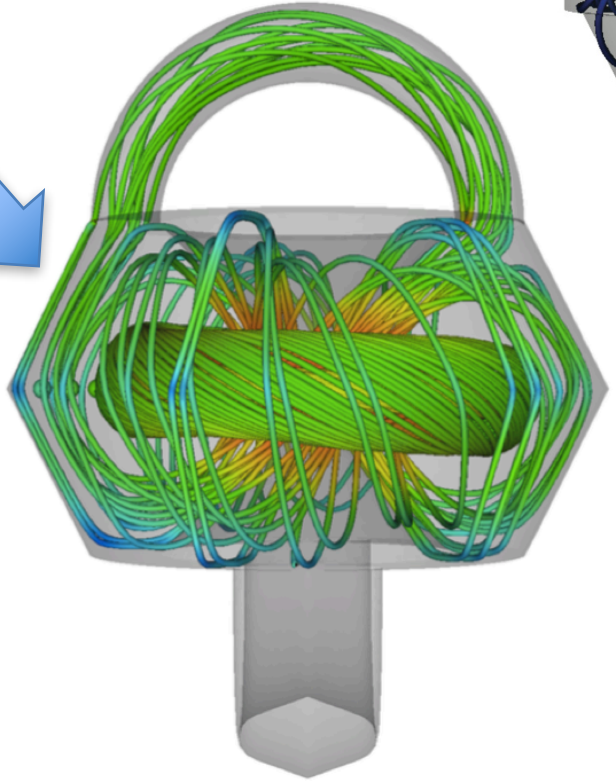
$$\dot{K}_{inj} = 6V_0\psi_0 \sin^2(\omega_{inj}t)$$

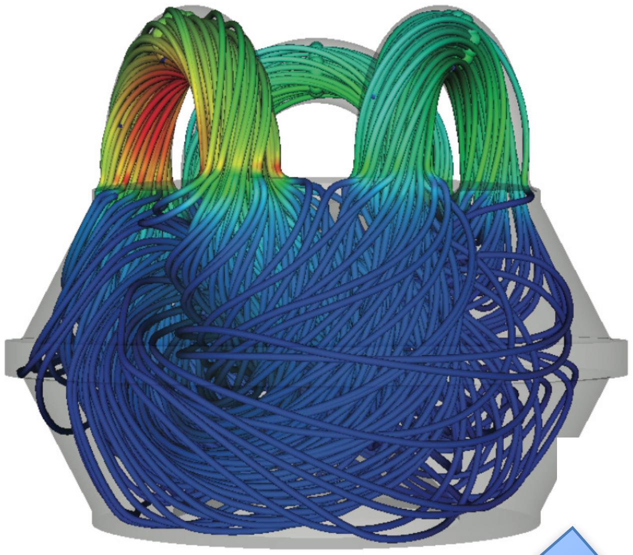
Spheromak formation by relaxation to lower spheromak $\lambda = \mu_0 j/B$

Injector Taylor states only

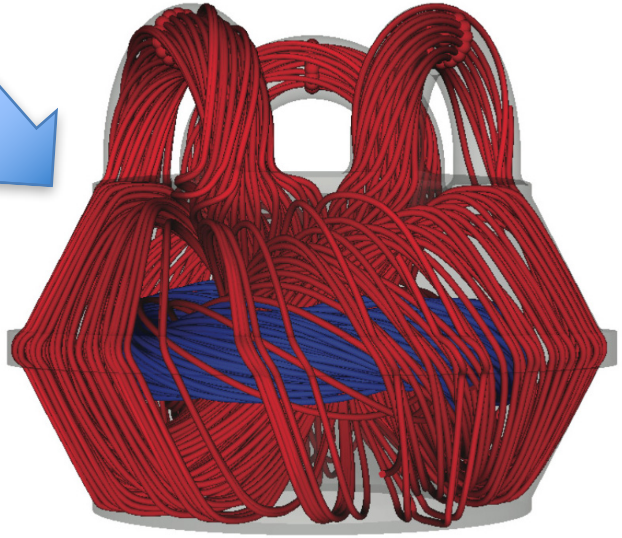


HIT-SI





HIT-SI3



Composite Taylor states
with spheromak

Figures courtesy of Chris Hansen

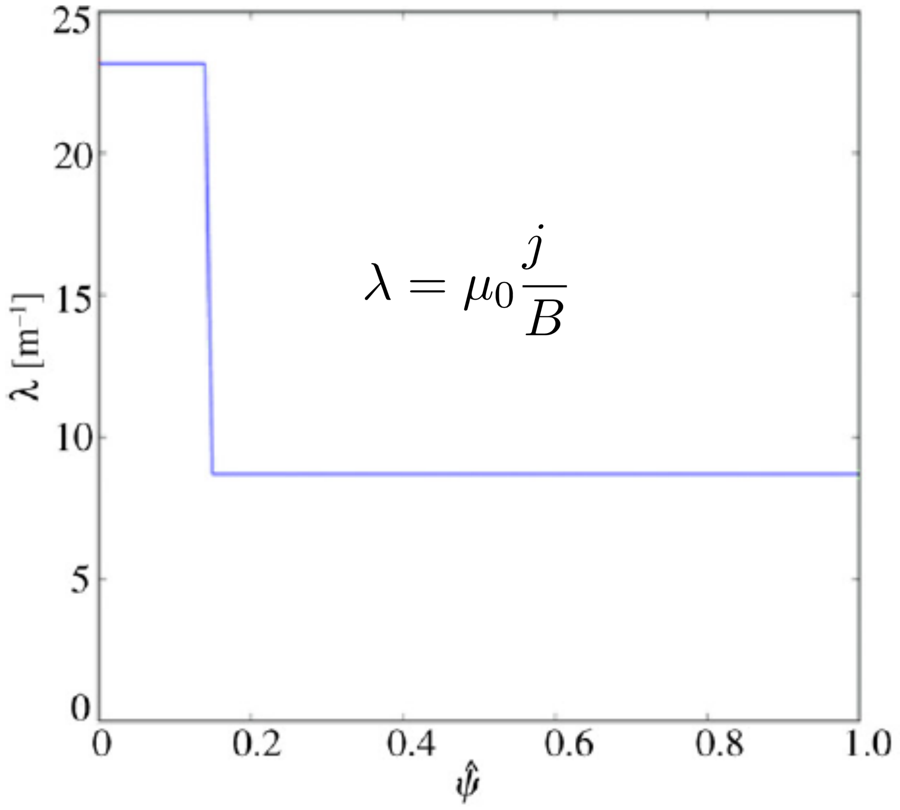
Sustainment with Imposed-dynamo Current Drive (IDCD)

- IDCD requires driving the edge-λ higher than the spheromak λ, while imposing non-axisymmetric, magnetic perturbations.

- The dynamo terms in Hall-MHD Generalized Ohm's Law leads to a dynamo electric field that **drives current parallel to the equilibrium field.**

- For steady-state sustainment, electromagnetic energy in a closed-flux volume must remain constant, i.e.: resistive decay must be balanced:

$$\int \vec{j} \cdot \vec{E}\, dV = 0$$



IDCD 2-step λ model

T.R. Jarboe, *et al.*, *Nucl. Fusion*, **52.8** (2012) 083017

T.R. Jarboe, B.A. Nelson, and D.A. Sutherland, *Phys. Plasmas* **22** (2015) 072503

Derivation of sustainment inside closed volume

Starting with generalized MHD Ohm's law,

$$\vec{E} = -\vec{v}\times\vec{B} + \frac{\vec{j}\times\vec{B}}{ne} + \eta\vec{j}$$

Assume relevant quantities can be separated into equilibrium and perturbative components: $\vec{j} = \overrightarrow{j_o} + \delta\vec{j}$, $\vec{B} = \overrightarrow{B_o} + \delta\vec{B}$, and $\vec{v} = \overrightarrow{v_o} + \delta\vec{v}$

Next, assume the perturbation is frozen into the electron fluid and the electron fluid exclusively carries the equilibrium current

In the ***perturbation*** frame of reference the ion fluid moves at the drift speed, $\overrightarrow{v_o} = \frac{\overrightarrow{j_o}}{ne}$

Derivation of sustainment inside closed volume

Solving for $\vec{j} \cdot \vec{E}$,

$$\vec{j} \cdot \vec{E} = (\overrightarrow{j_o} + \delta\vec{j}) \cdot \left(-(\overrightarrow{v_o} + \delta\vec{v}) \times (\overrightarrow{B_o} + \delta\vec{B}) + \frac{(\overrightarrow{j_o} + \delta\vec{j}) \times (\overrightarrow{B_o} + \delta\vec{B})}{ne} + \eta(\overrightarrow{j_o} + \delta\vec{j}) \right)$$

Performing the dot product and simplifying yields:

$$\boldsymbol{\vec{j} \cdot \vec{E} = \left[(\overrightarrow{j_o} \times \delta\vec{B}) + (\delta\vec{j} \times \overrightarrow{B_o}) \right] \cdot \delta\vec{v} - (\delta\vec{B} \times \delta\vec{j}) \cdot \overrightarrow{v_o} + \eta(\vec{j} \cdot \vec{j}) + O(\delta^3)}$$

Or in integral form as required for closed-flux sustainment,

$$\int \boldsymbol{\vec{j} \cdot \vec{E}}\,\mathbf{dV} \approx \int \boldsymbol{\left\{ \left[(\overrightarrow{j_o} \times \delta\vec{B}) + (\delta\vec{j} \times \overrightarrow{B_o}) \right] \cdot \delta\vec{v} - (\delta\vec{B} \times \delta\vec{j}) \cdot \overrightarrow{v_o} + \eta(\vec{j} \cdot \vec{j}) \right\} d^3x} = 0$$

Key dynamo term is parallel to equilibrium current

Neglecting the velocity perturbation for simplicity recovers the equation from Jarboe *et al.*, Phys. Plasmas (2015):

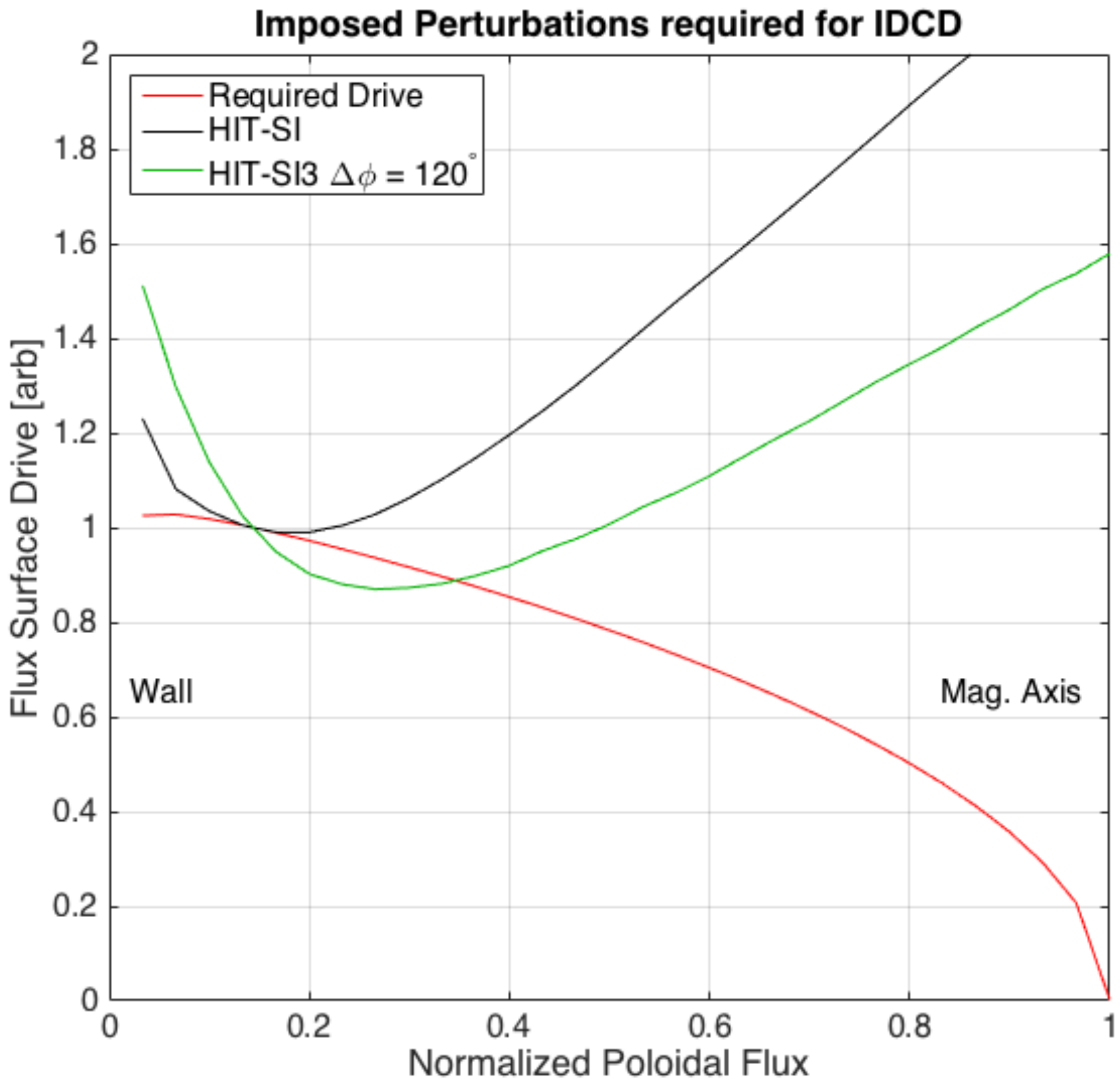
$$\int \vec{j}\cdot\vec{E}\,\mathrm{dV} \approx \int \left\{-\left(\boldsymbol{\delta}\vec{\boldsymbol{B}}\times\boldsymbol{\delta}\vec{\boldsymbol{j}}\right)\cdot\overrightarrow{\boldsymbol{v_o}} + \boldsymbol{\eta}(\vec{\boldsymbol{j}}\cdot\vec{\boldsymbol{j}})\right\}\boldsymbol{d}^3\boldsymbol{x} = 0$$

Note that a component of the dynamo drive is in the direction of equilibrium current,

$$\overrightarrow{v_o} = \frac{\overrightarrow{j_o}}{ne}$$

Thus, it is possible for appropriately phased perturbations to sustain a closed-flux configuration against resistive decay

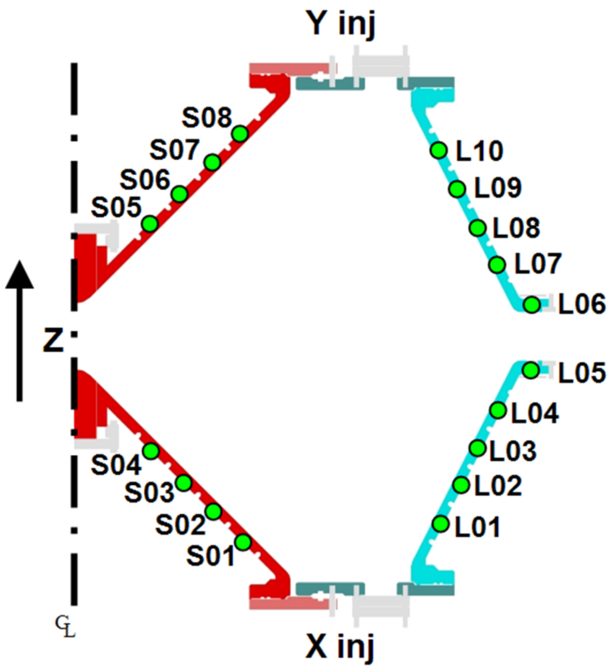
Separate, stable spheromak predicted by IDCD

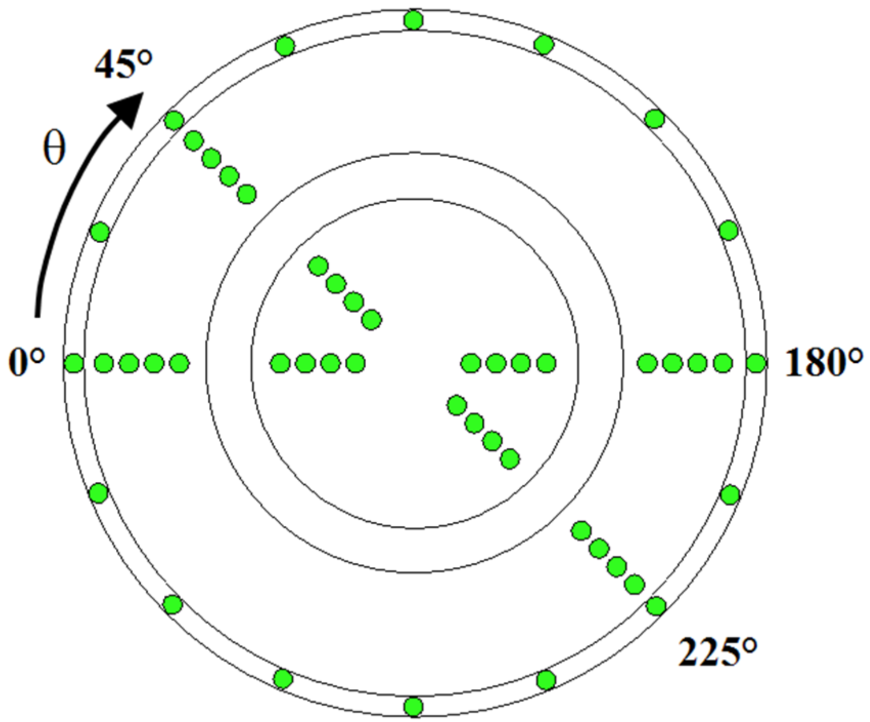


- Perturbations in HIT-SI exceed current-drive requirement
  - Electron flow "locked" inside current separatrix
- IDCD λ-profile in HIT-SI is "step"
- A plasma which is stable to injector perturbations resists deformation
- If the inner spheromak plasma is stable, it will move as a coherent object

Calculations from Chris Hansen's Ph.D. thesis, University of Washington, 2014

Surface magnetic probes measure toroidal current and modes





- Four arrays of 16 probes at toroidal angles 0°, 45°, 180°, and 225° are used to calculate toroidal current
  - Total toroidal current is the average of the four arrays

- Two arrays of 16 probes around midplane measure Fourier modes up to $n = 7$

Biorthogonal decomposition isolates *empirical* modes

- Arrange data in 2D array, space vs. time
- Separate into empirical modes based on spatial and temporal coherence, ordered by "weight" (amplitude)

$$y\left(x_i, t_j\right) = \sum_{k}^{K} A_k \phi_k\left(x_i\right) \psi_k\left(t_j\right)$$

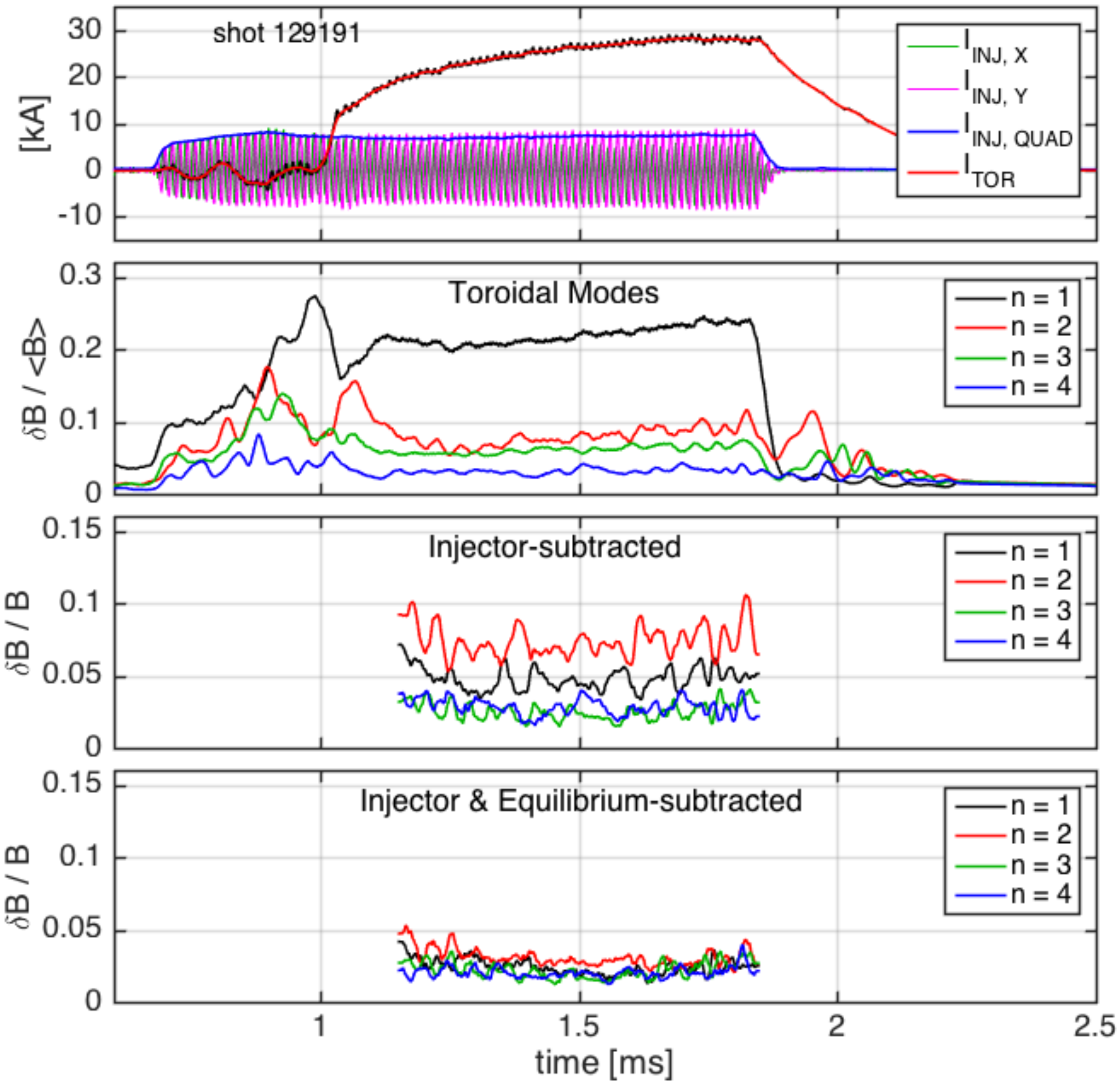
- Append injector currents to surface magnetic probe signal array
  - Force signals correlated with injectors to be grouped together

$$B\left(\left[x_m, CI_{inj,x}, CI_{inj,y}\right], t_n\right) = \sum_{k=1}^{K} A_k \phi_k\left(\left[x_m, CI_{inj,x}, CI_{inj,y}\right]\right) \psi_k\left(t_n\right)$$

- Subtract injector-correlated components (and/or equilibrium-correlated)

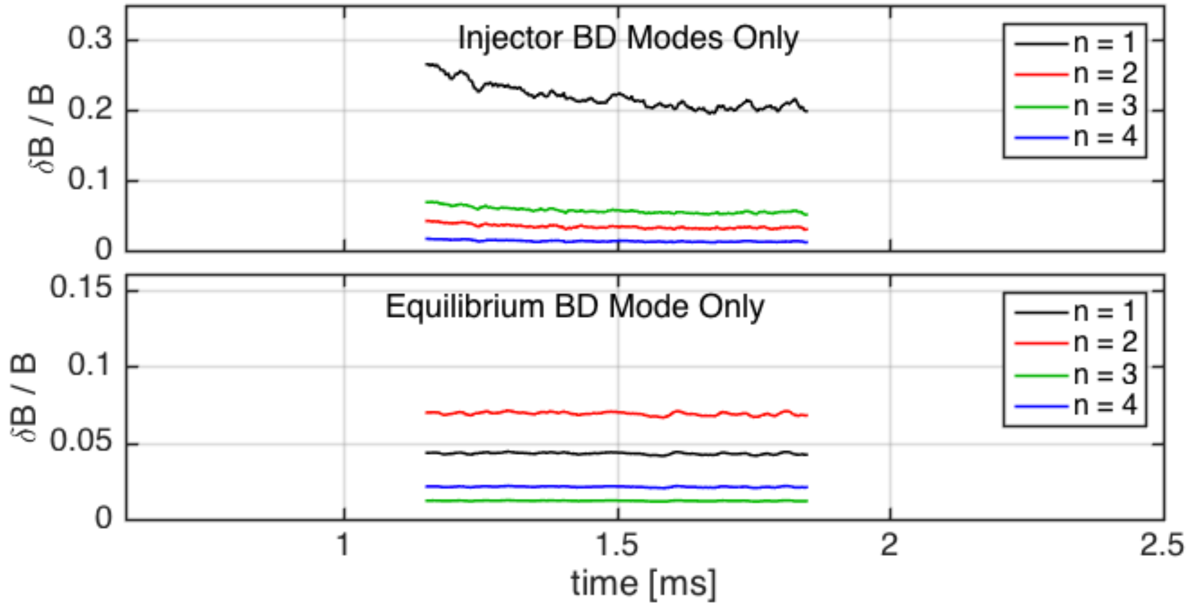
$$B_{sub}\left(x, t\right) = B\left(x, t\right) - A_2 \phi_2\left(x\right) \psi_2\left(t\right) - A_3 \phi_3\left(x\right) \psi_3\left(t\right)$$

Fourier mode structure of injectors, equilibrium, instabilities isolated with BD



- Most n = 1 energy is directly correlated with injector currents
- Equilibrium has n = 2 and n = 1 distortions
- Remaining "plasma-generated" nonaxisymmetric activity is small, $\delta B/B \approx 3\%$

Following method of B.S. Victor, et al., *Physics of Plasmas* **21** (2014) 082504.



Plasma-generated δB cannot sustain toroidal current

$$E_\parallel = -\langle \delta v_e \times \delta B \rangle_\parallel + \eta j_\parallel$$

Electron fluid is frozen to magnetic field,

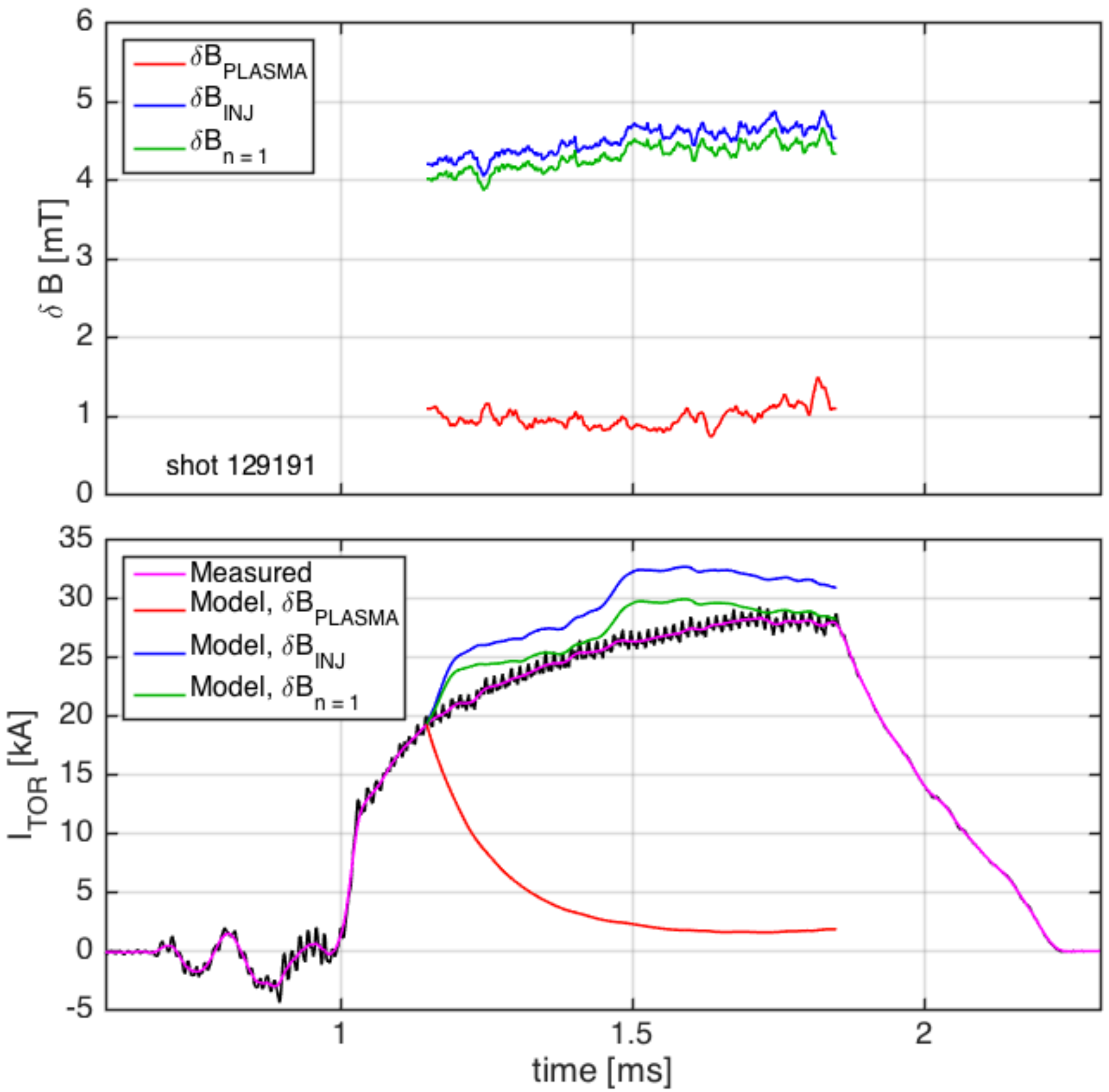
$$-\langle \delta j \times \delta B \rangle_\parallel / ne = \eta j_\parallel - E_\parallel$$

Integrate over volume inside a toroidal flux surface, calculate Maxwell stress on the surface:

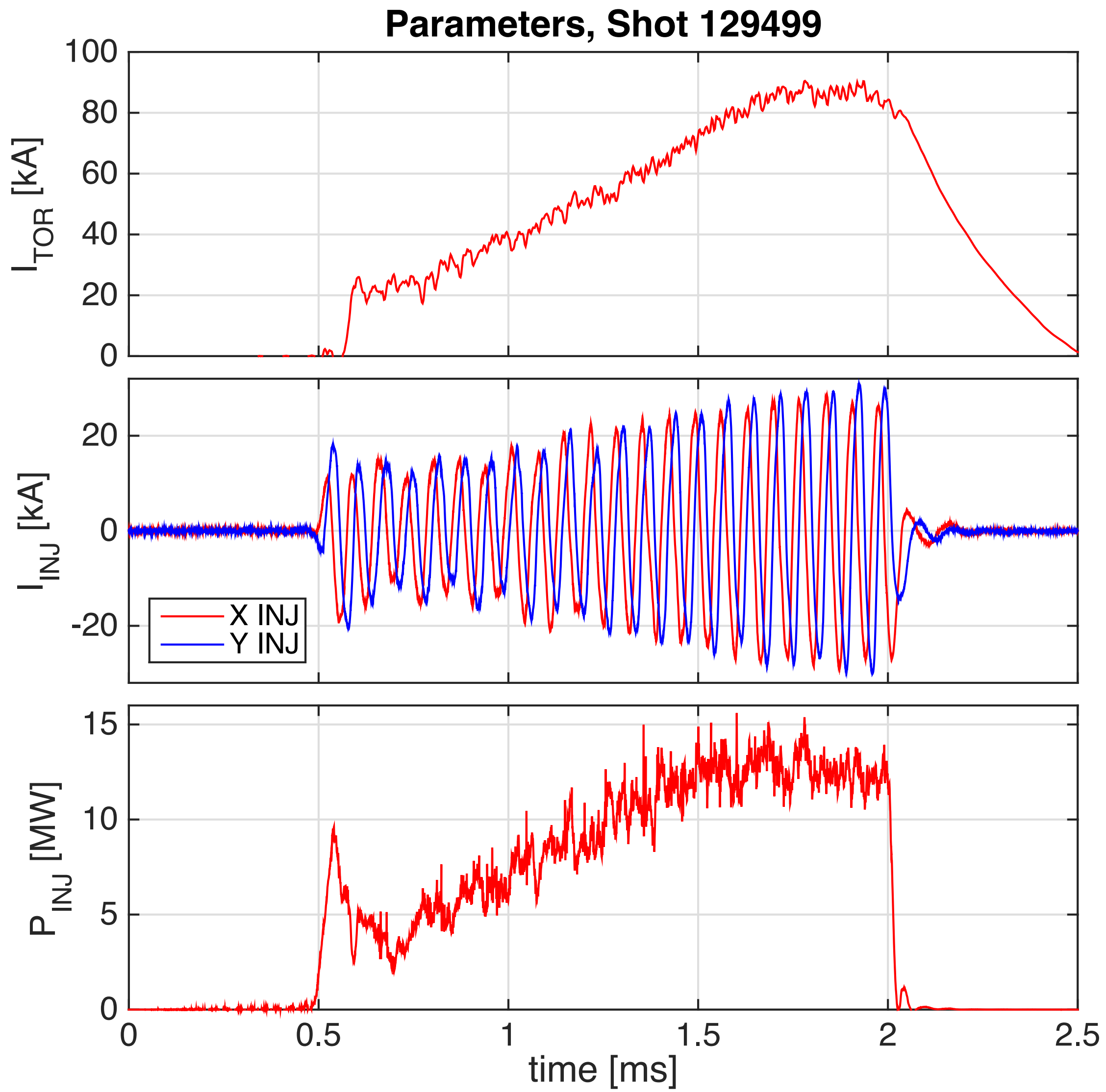
$$-\int \frac{\delta B_\perp \delta B_{tor}}{\mu_0} da = \int ne(\eta j_{tor} - E_{tor}) dV$$

$$\frac{(\delta B_{\perp rms})^2}{2\mu_0} 2\pi R_0 2\pi r \geq (\eta j_{tor} - E_{tor}) ne \pi r^2 2\pi R_0$$

$$\boxed{\dot{I} = \frac{-I_{sph}}{\tau_{L/R}} + \frac{4\pi \delta B_\perp^2}{\mu_0^2 ner}.} \qquad \tau_{L/R} = \frac{2K}{\dot{K}_{inj} - \dot{K}_{total}}$$

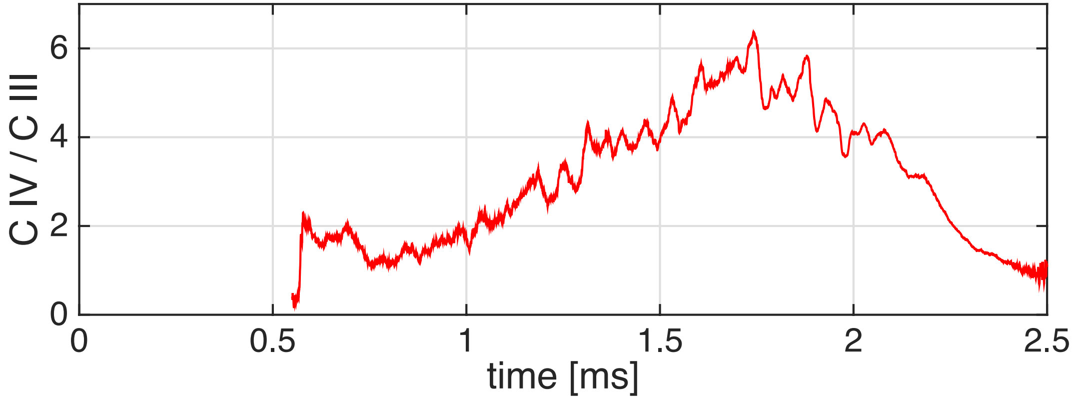


High current, low frequency discharges

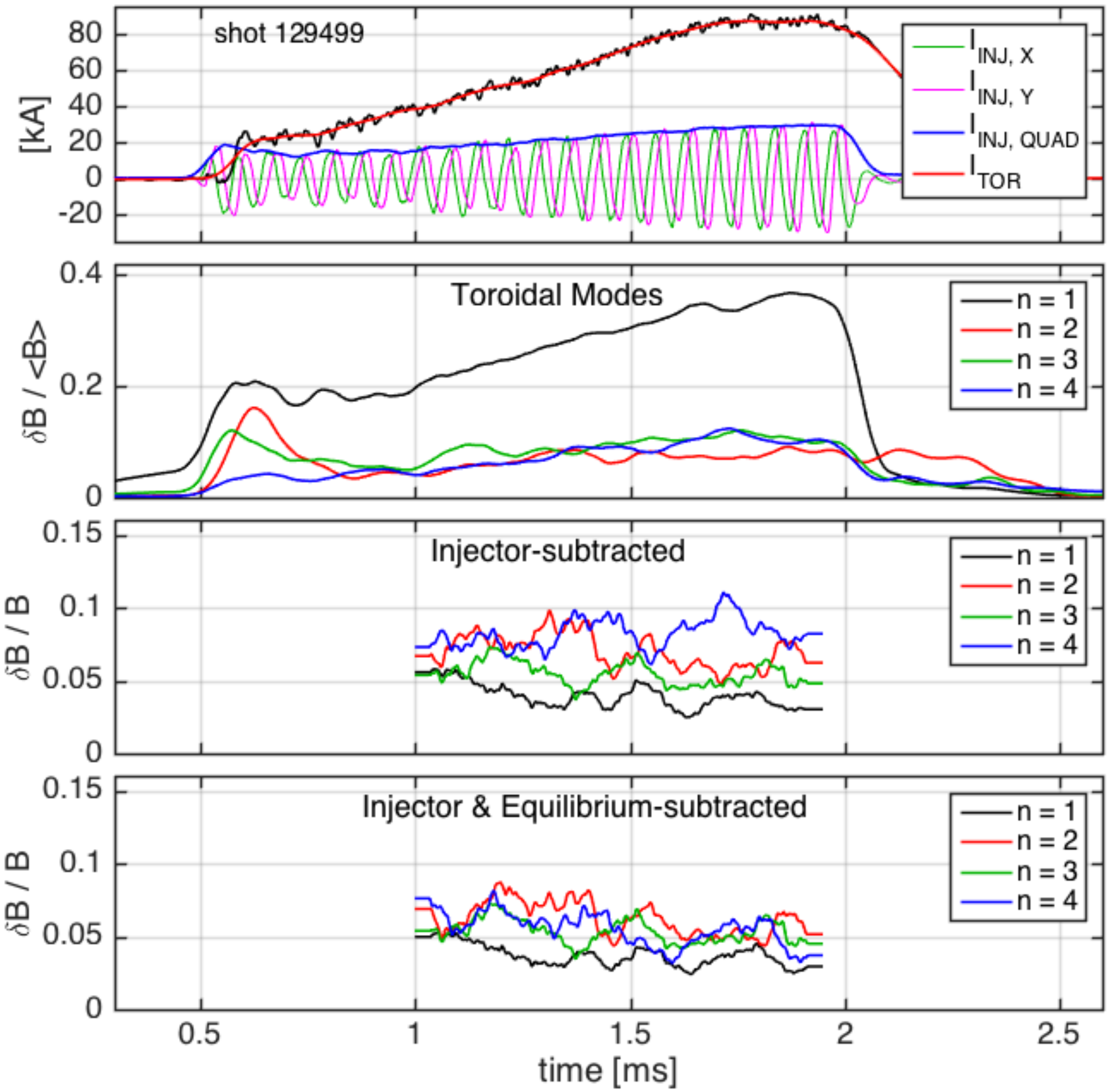


- High power & density regime enabled sub-injector-cycle resolution with ion Doppler spectrometer (IDS)

- f_{INJ} = 14.5 kHz
- $n_e \approx 5 - 7 \times 10^{19}$ m^{-3}
- I_{TOR} / $I_{INJ,\ QUAD} \approx 3$

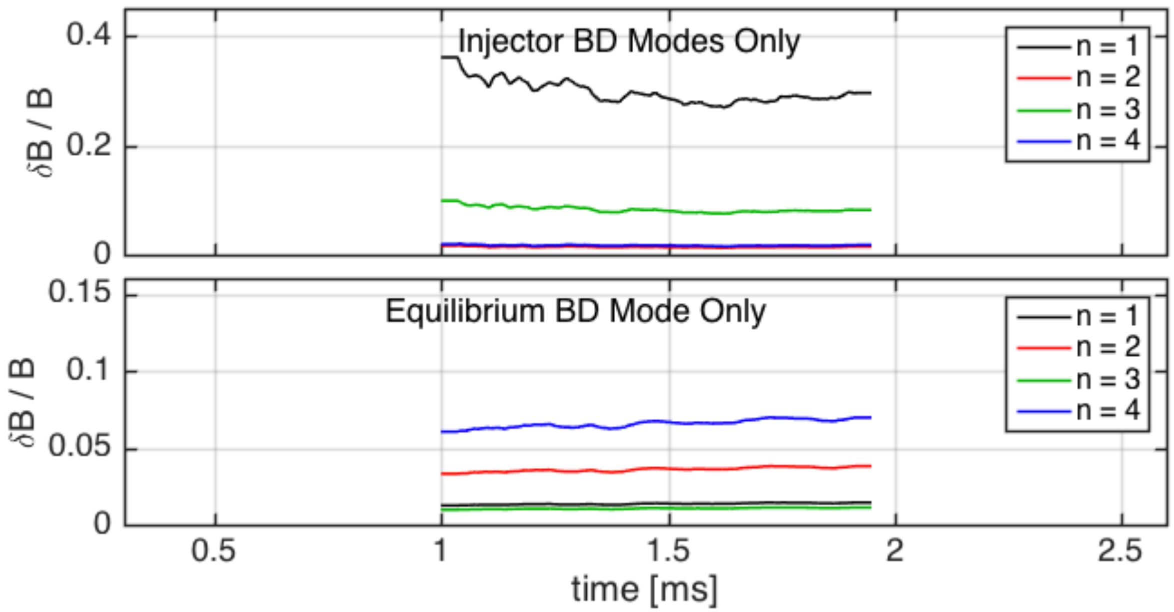


Fourier mode structure of injectors, equilibrium, instabilities isolated with BD



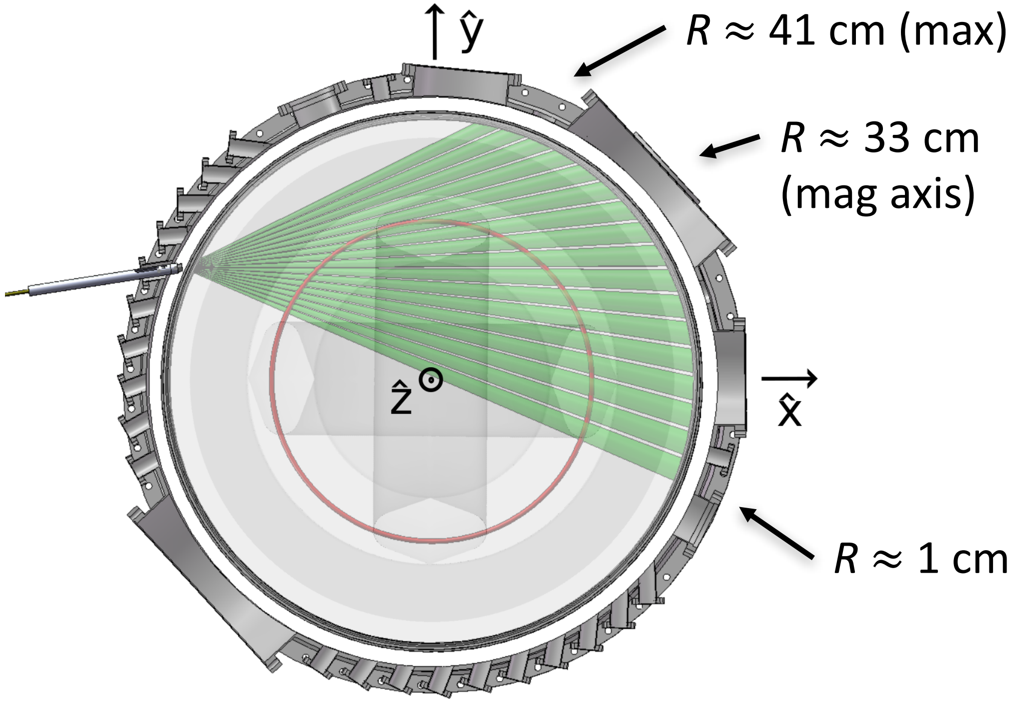
- Most n = 1 energy is directly correlated with injector currents
- Equilibrium has n = 2 and n = 4 distortions
- Remaining “plasma-generated” nonaxisymmetric activity is small, $\delta B / B \approx$ 6%

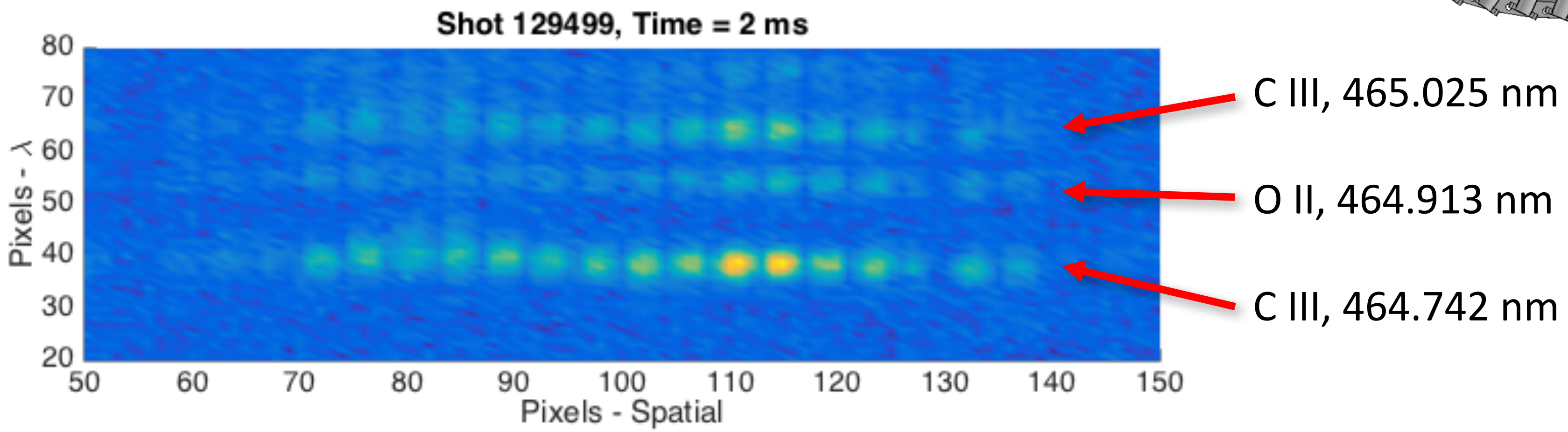
Following method of B.S. Victor, et al., *Physics of Plasmas* **21** (2014) 082504.



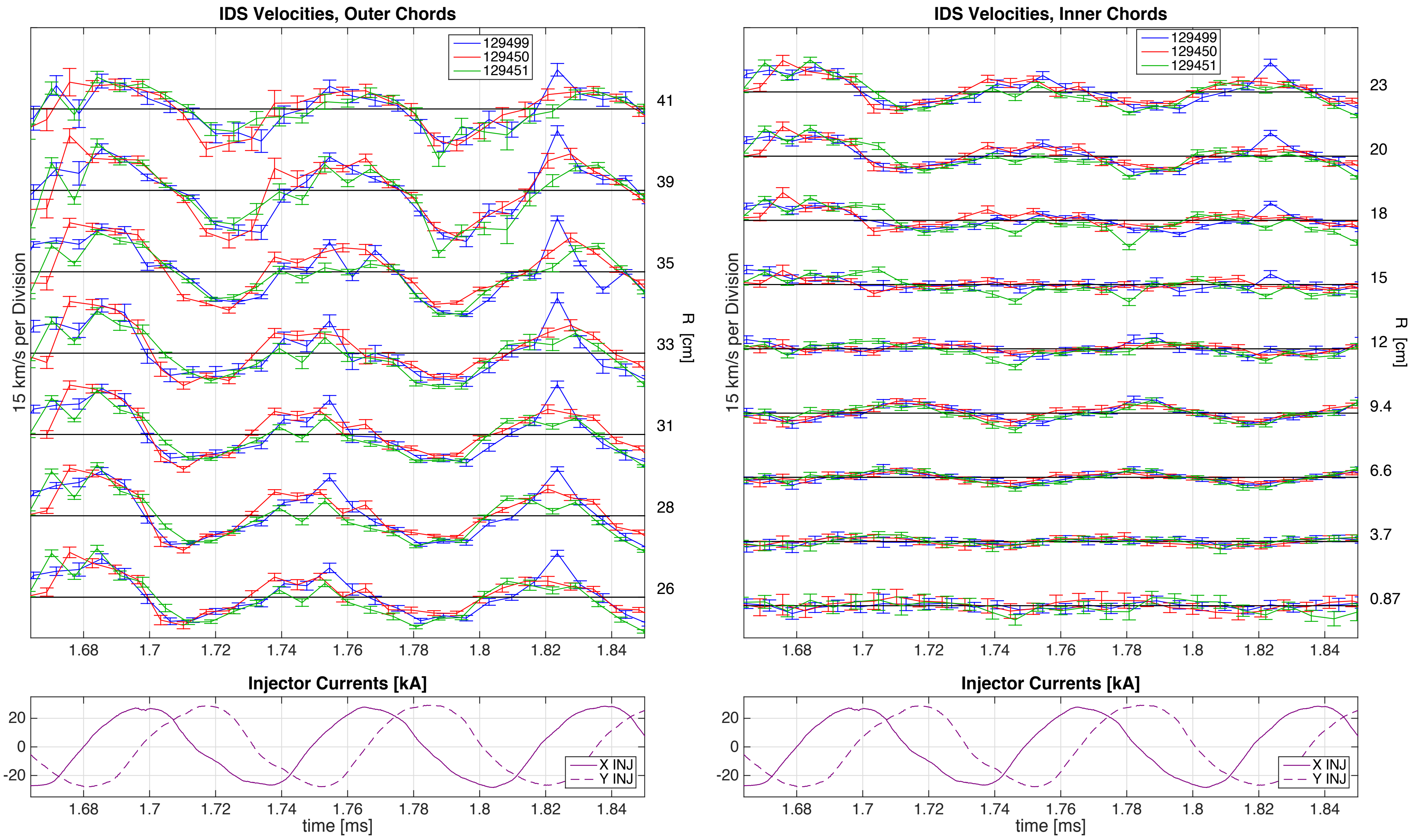
The Ion Doppler Spectrometer (IDS)

- 1 meter, f/8.5 spectrometer
  - On loan from Prof. Nagata
- Linear fiber optic array
- Small, wide-angle lens
- 17 conical chords in toroidal midplane
- Phantom v710 high speed camera
  - 145 kHz = 10, 7 μs exposures per injector cycle





Similar velocities observed in 3 shots

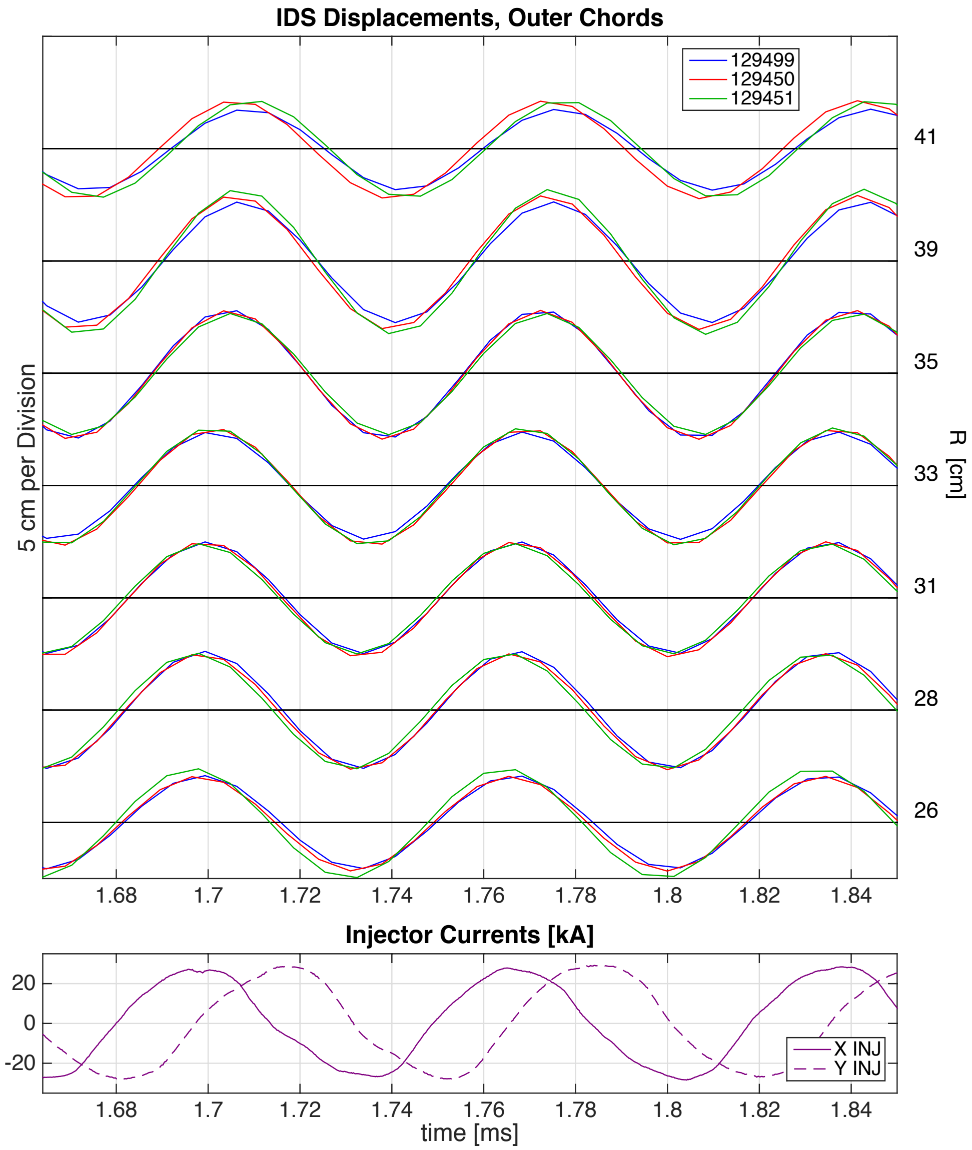


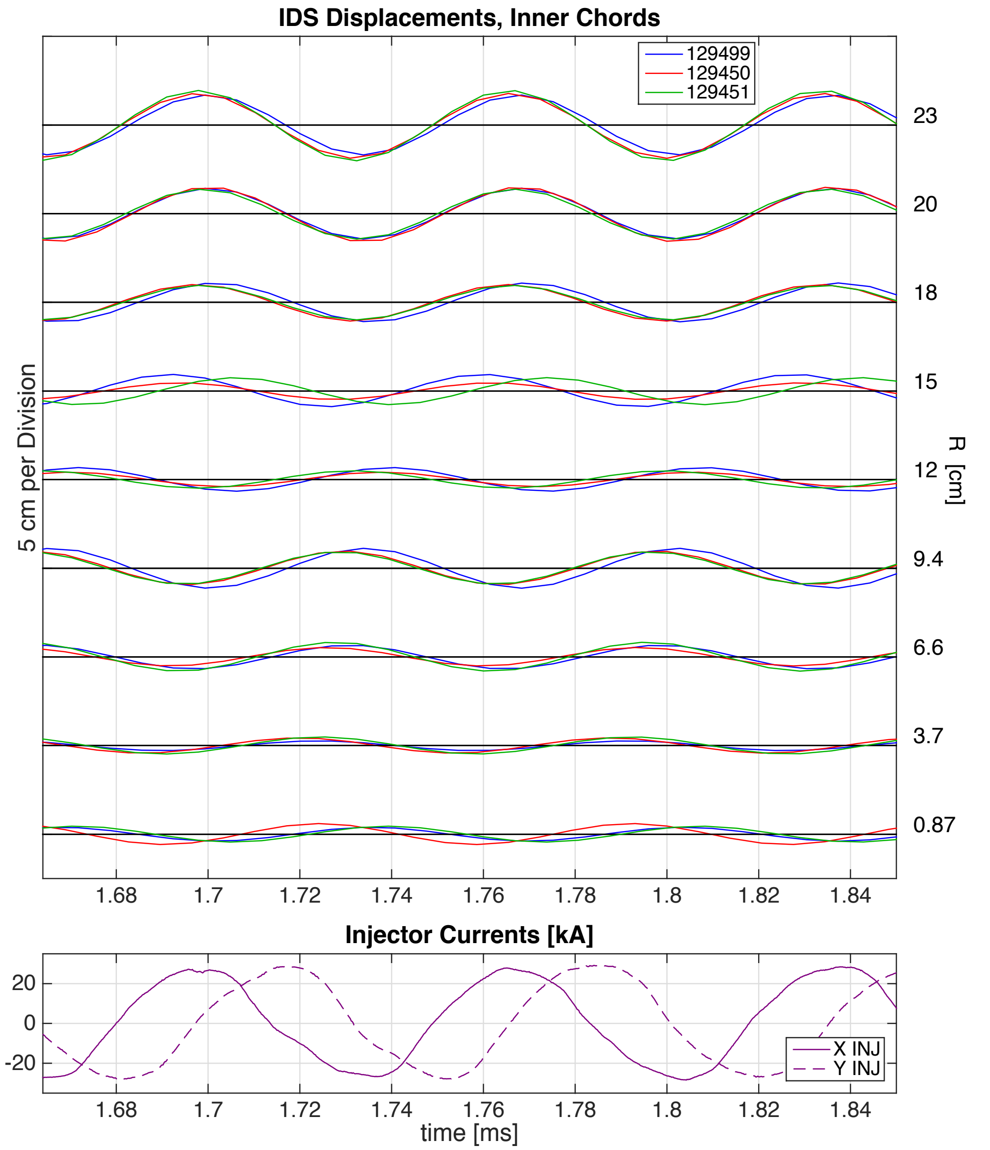
Velocities repeatable for similar shots

Dominated by injector frequency

Aaron Hossack, Ph.D. thesis, University of Washington, 2015

Filter velocity to injector frequency, calculate "displacement" with running integral





Velocities in-phase from beyond magnetic axis (41 cm) to ~18 cm

Coherent motion ±2.5 cm

Phase shift from R≈12-15 cm

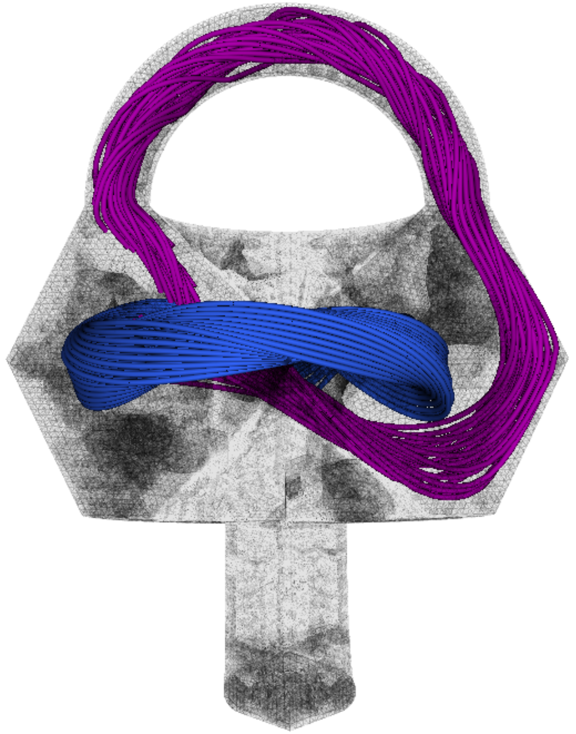
Opposite phase from 9.4 cm inboard – injector plasma displaced by spheromak

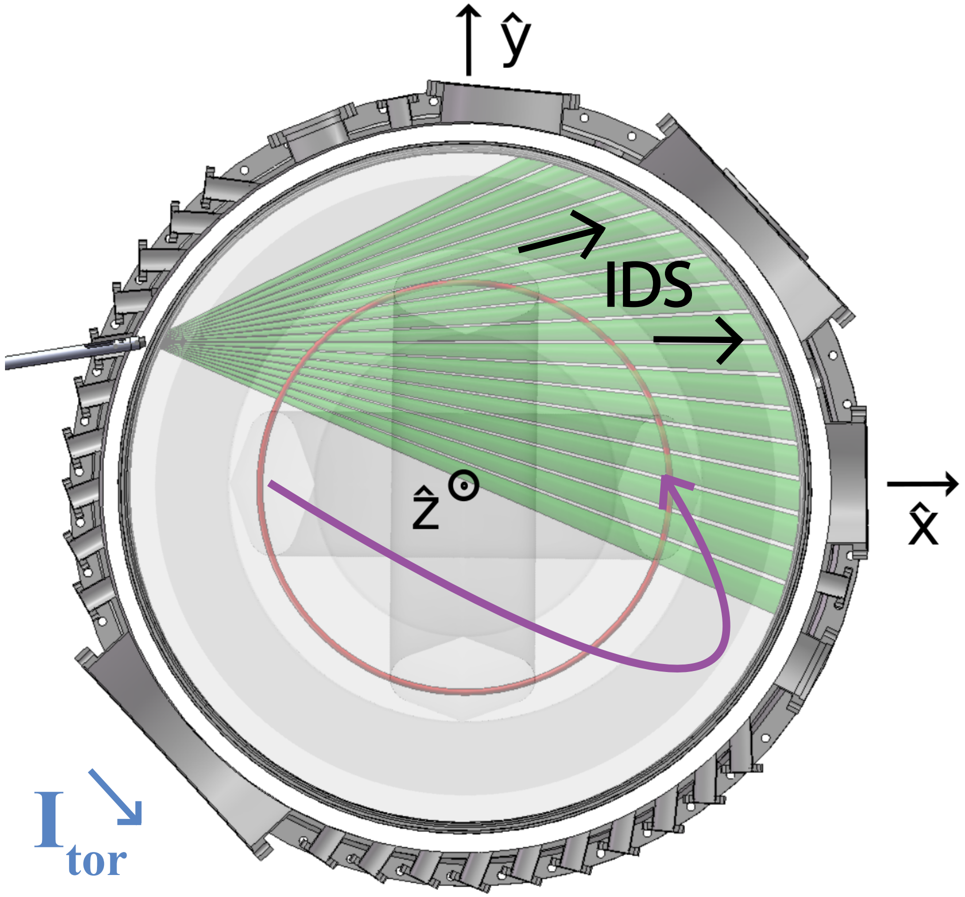
Aaron Hossack, Ph.D. thesis, University of Washington, 2015

Maximum displacement correlated with injector current

- Injector current has been shown to follow toroidal current to one side of geometric axis, hence downward arc in right figure (*Victor et al.*, PRL 2011)

- Injector fields align with spheromak fields – mostly poloidal at edge



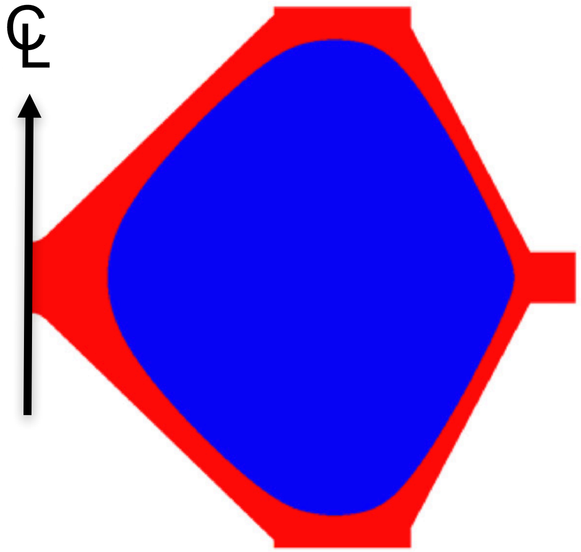


- Two opposing forces:
  - Increased flux around edge pushed spheromak inboard (on right side)
  - Parallel currents attract, pull spheromak outboard
- Data show spheromak displaced to the right at X injector maximum, therefore attractive current force dominates

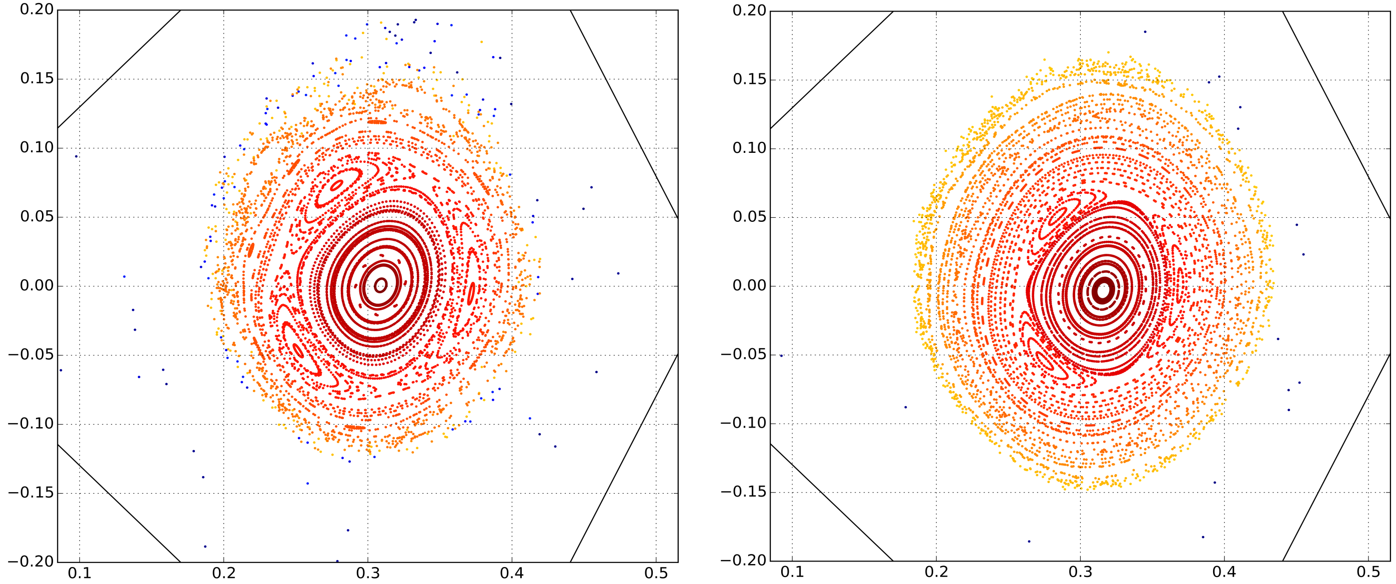
Spheromak size compared with Taylor and IDCD theory

- IDS measured inboard separatrix $R \approx 16 - 18$ cm
- IDCD equilibrium predicts inboard current separatrix at $R \approx 9.3 \pm 0.5$ cm
- Real profile may have "limited-slip" transition region
- Composite Taylor states predict separatrix at $R \approx 20 - 24$ cm
  - Long (not closed) field lines near separatrix

IDCD 2-step Profile



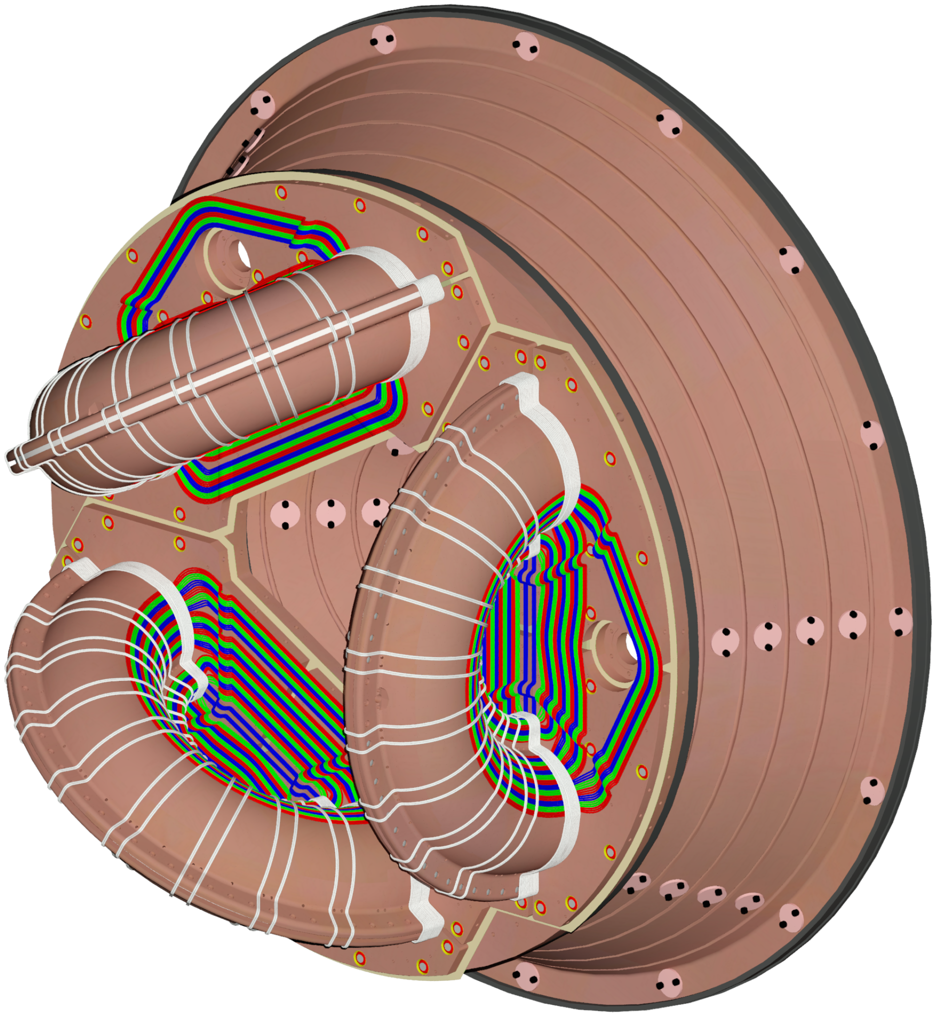
3D Taylor States, Flux Amplification 3 (left) and 4.8 (right)



- Coherent, imposed motion is indicative of stability
  - Larger size than predicted by Taylor theory
  - Smaller size than 2D IDCD theory

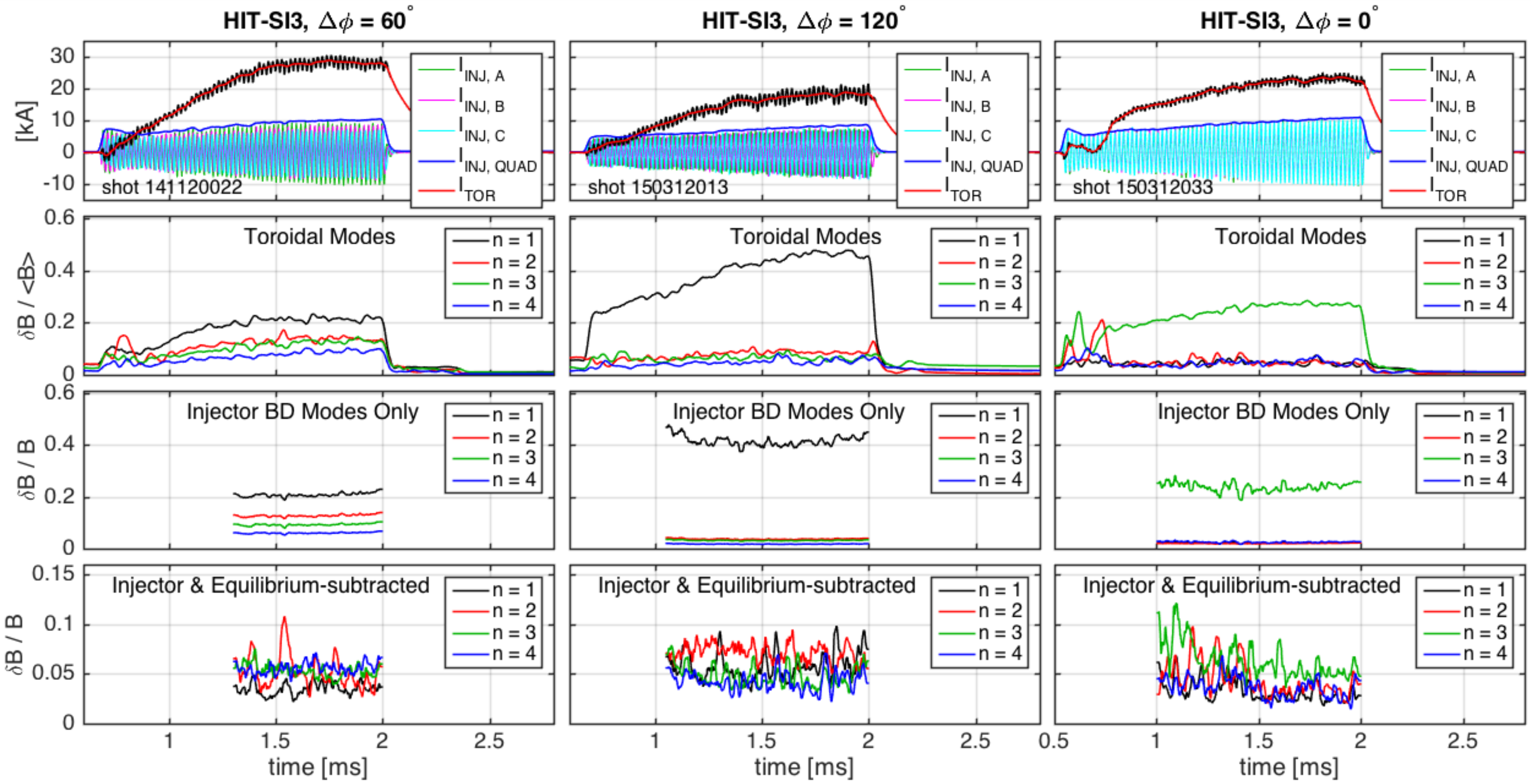
Figures courtesy of Chris Hansen and Thomas Benedett

BD and Fourier mode analysis on HIT-SI3

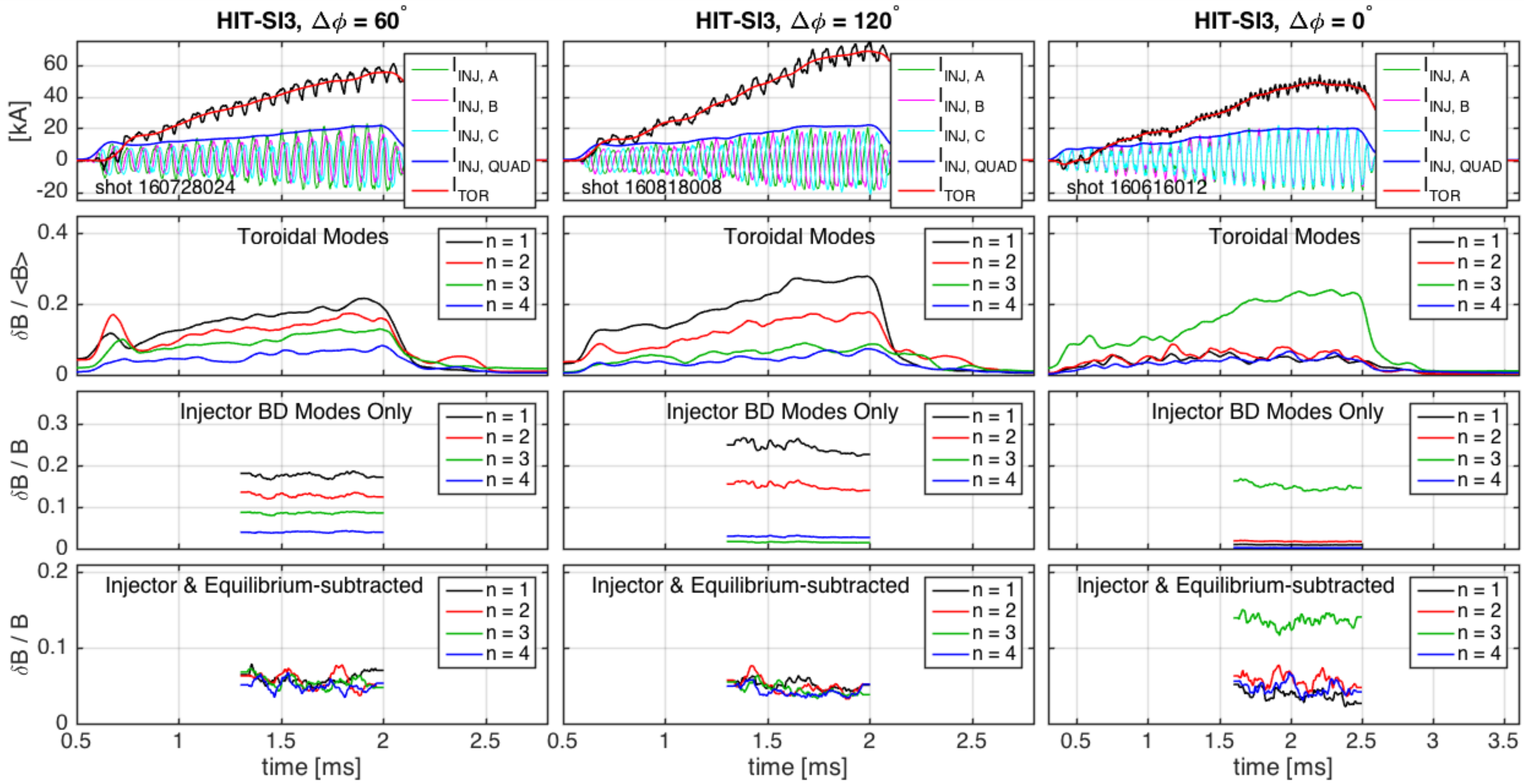


- 60°, 120°, 0° relative phasing between three injectors
- High (47.5 kHz) and low (14.5 kHz) injector frequency

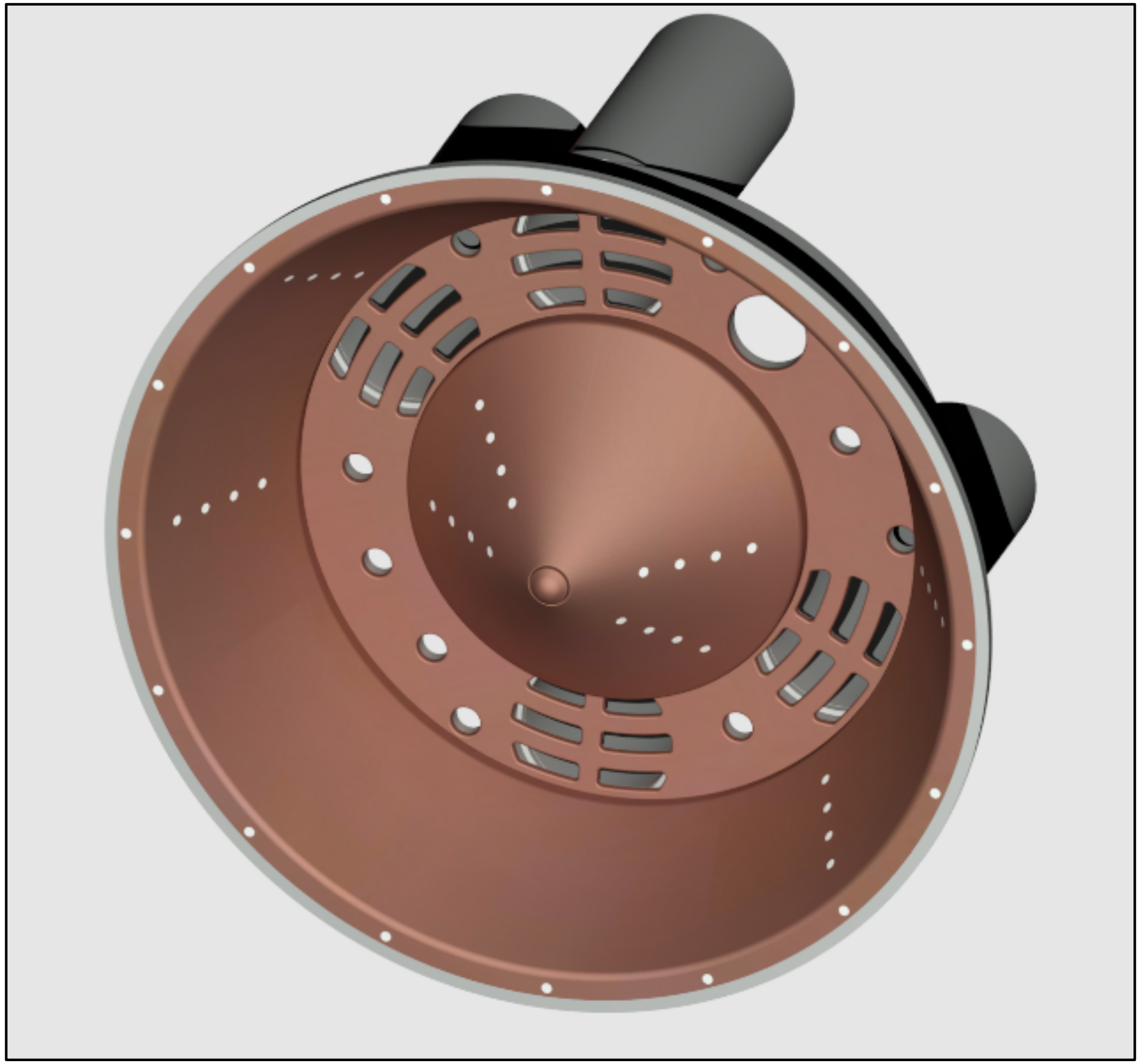
Mode spectra varied in HIT-SI3 by changing injector phasing

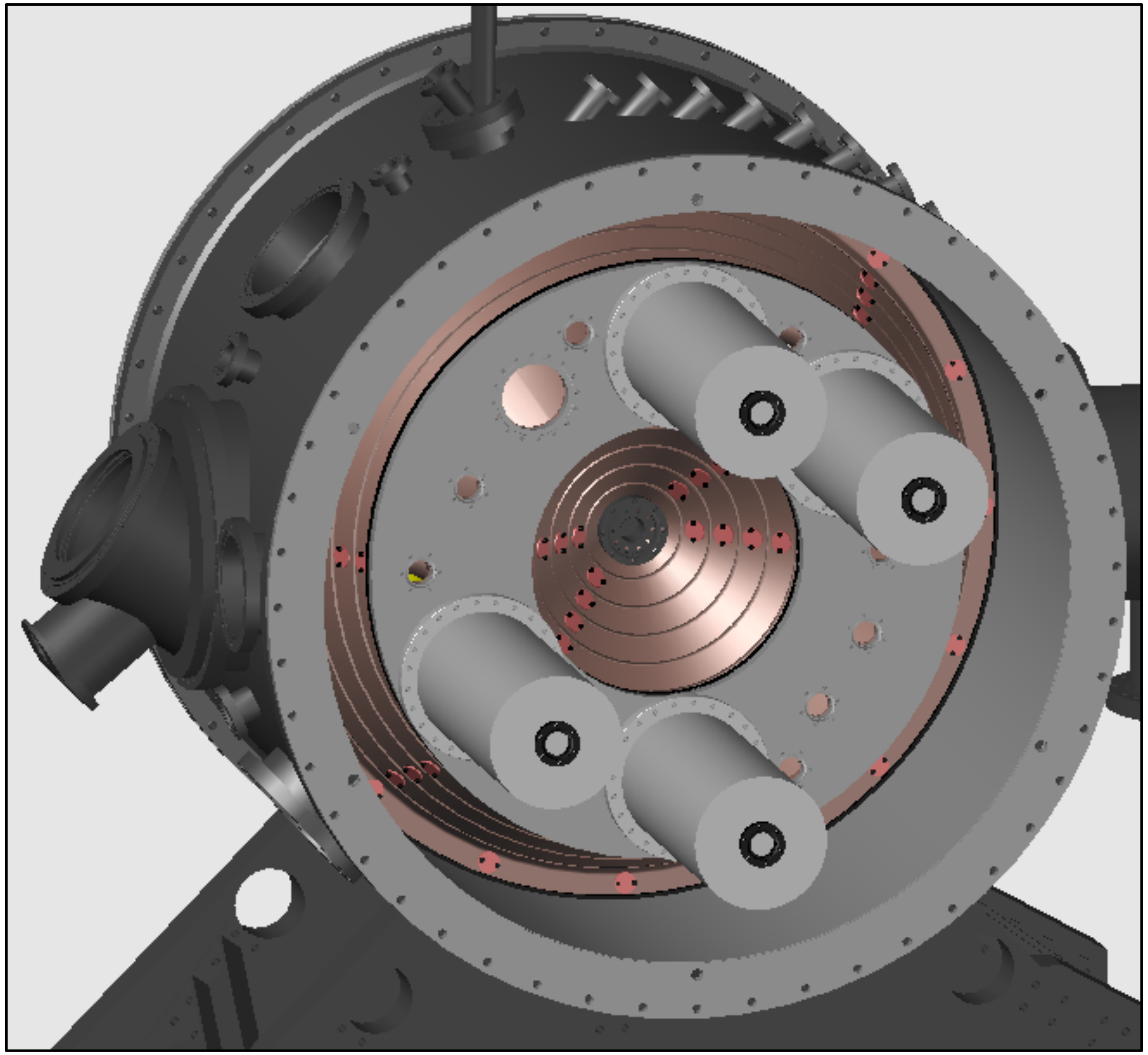


Slight differences at low frequency: injector-correlated *n* = 2 at 120°



Status update: adding pumping plate for density control & Thomson scattering





Summary

- IDCD theory shows that dynamo sustainment of equilibrium current is possible
  - Requires electron velocity gradient and nonaxisymmetric perturbations
- Using biorthogonal decomposition, observed that almost all nonaxisymmetric energy is imposed by injectors
- Plasma-generated instabilities cannot sustain measured current, but imposed perturbations can
- Coherent, imposed plasma motion $\pm$ 2.5 cm observed with IDS
  - Correlated with attractive force of time-varying injector currents
- Size of coherent volume larger than predicted by 3D composite Taylor equilibria
- Consistent with IDCD theory – transition region instead of sharp separatrix
- Coherent motion is indicative of stable equilibrium
- HIT-SI3 is testing plasma response to perturbation spectrum