EXTRACTING ELECTRON ENERGY DISTRIBUTIONS FROM PFRC X-RAY SPECTRA:
PREPARING FOR HIGH-POWER, HIGH-FIELD OPERATION OF THE ROTATING MAGNETIC FIELD

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Overview

• The PFRC
  - Seed Plasma
  - RMF-induced plasma

• Novel spectral inversion method
  • Bremsstrahlung
  • The math
  • Calibration using gas-target x-ray tube

• Seed plasma results and discussion
• RMF plasma results and discussion
The PFRC

The PFRC is a long-pulse, collisionless, low s-parameter experiment to form FRCs using odd-parity rotating magnetic fields.

Starting with a capacitively-coupled seed plasma (~1-500W), odd-parity rotating magnetic field (RMF) antennae drive current and heat electrons at 10-20kW. An FRC is formed within 200μs. RMF operation continues for 5-250ms.

Upgrades coming in the next week/month will increase power 10X
Seed Plasma

- When on, produced by 5 to 500W of capacitively-coupled RF power at the far-left of the machine, 27MHz frequency
- A few $10^{10}/\text{cm}^3$, $T_e$~5eV, yet see x-rays out to 7keV!
- Why would a non-Maxwellian seed plasma be interesting?
- How do hot electrons affect RMF coupling/penetration?
- Can we use it as a diagnostic?
- How are hot electrons formed and heated, beam and thermal?

The seed plasma in the Far End Cell strikes a floating plate and causes it to glow red-hot.

Paper pending by Jandovitz et. al.

Extracting Electron Energy Distributions from PFRC X-ray Spectra
RMF Plasma

- Produced by up to 20kW (200kW) of RMF power from odd-parity antennae
- 8MHz frequency
- Density a few $10^{12}$/cm$^3$
- Temperature may be as high as 300eV
RMF Plasma

A punctuated betatron orbit

Orbit in the co-rotating frame illustrating trapping

Time-history of energy of a particle in the PFRC-2 from a single-particle motion code

Why would a non-Maxwellian RMF plasma be interesting?

- Our heating mechanism has an inductive $\vec{E}$ that traps and accelerates particles along the magnetic null.
- Single-particle codes predict our heating mechanism to produce non-thermal distribution, have a hard cutoff
- Need turbulence to heat past the cutoff and equilibrate to thermal: Lower Hybrid Drift mode.

PFRC-1 device saw a Maxwellian tail with density $\sim 10^{12}$ and temperature $\sim 200$eV
RMF: What do we expect?

- More on the RMF heating mechanism: Single-particle motion
- Hamiltonian simulation results

Maximum energy is constrained: 
- PFRC-2 Simulation
- $B_0 = 90 \, \text{G}$, $B_{RMF} = 14 \, \text{G}$
- $f_{RMF} = 7.6 \, \text{MHz}$
- $r_s = 7 \, \text{cm}$, $\kappa = 4$
- $r_0 = 0.5 \, \text{cm}$
- $r_0 = 6 \, \text{cm}$

Average energy is of the same order (~0.3):
- Time-averaged energy
- $r_0 = 0.5 \, \text{cm}$
- $r_0 = 6 \, \text{cm}$

Example distribution of energies: Non Maxwellian, truncated
- PFRC-2 simulation
- $B_0 = 90 \, \text{G}$
- $B_{RMF} = 14 \, \text{G}$
- $f_{RMF} = 7.6 \, \text{MHz}$
- $r_s = 7 \, \text{cm}$
- $\kappa = 4$
X-Ray Detectors

We have Amptek Si-PIN diode detectors. They detect x-rays, determine their energy with some accuracy, and count the number within each energy channel. Have low-energy limit on detected x-rays: 600eV-1keV

http://amptek.com/

Example spectrum (Fe-55)
2016/05/16
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Bremsstrahlung

- Electrons in the vicinity of nuclei accelerate and emit EM radiation. The differential cross section to emit an x-ray between $\nu$ and $\nu + d\nu$ is given by the equation

$$I(\nu) = r_e^3 \frac{m_e c^2 32 \pi^2}{h^3 \sqrt{3}} \int d\beta \frac{f(\beta) G(\beta, \nu)}{\beta \nu}$$

- Or in energy units

$$I(E_x) = K \int dE_x \frac{f(E_e) G(E_x, E_e)}{E_x \sqrt{E_e}}$$

Where $E_x$ is the energy of the x-ray, $E_e$ is the energy of the electron, $G$ is the “Gaunt factor.”

- When discretized, this is a matrix multiplication:

$$\vec{I}_{x,i} = \sum_j M_{i,j} \vec{I}_{e,j} = (M \vec{I}_e)_i$$

Where $M_{i,j} = KG(E_{x,i}, E_{e,j}) / E_{x,i} \sqrt{E_{e,j}}$

- Spectral lines are not considered
Spectral Inversion

\[ f(E_e) \rightarrow \text{Physics} \rightarrow I(E_x) \rightarrow \text{Analysis} \rightarrow f_r(E_e) \]

- Matrix Inversion. \( M^{-1}I_x = M^{-1}MI_e = I_e \). This method is unstable to numerical noise. It yields unphysical results.

- Instead let us maximize the log-likelihood that our model (\( f(E_e) \)) produces our data (\( I(E_x) \)).
  - Maximizing the log-likelihood comes from Bayes’s Theorem; it’s the formally consistent way to do statistical inference.

- Minimize the log-likelihood with respect to our model variables, in this case \( \{I_e\} \) values. Find the \( \overrightarrow{I_e} \) that minimizes this quantity.

- An example of this is Tikhonov Regularization [2][3], in which the quantity \( \|MI_e - I_x\|^2 - \lambda\|I_e\|^2 \) is minimized, the log-likelihood for Gaussian measurements. \( \|Q\| \) is the Euclidean norm of quantity Q.
Poisson Regularization

X-ray counts in each channel is a Poisson distributed random variable. Poisson Distribution:

\[ P(a | b) = \frac{b^a e^{-b}}{a!} \]

Thus we minimize the log-likelihood:

\[ \ln[P(b|a)P(a)] \]

minimize \( \sum Ml_e - Ix \ln(Ml_e) + [\ln \sum I_e] + etc \)

That quantity is a function of \( N \) independent variables, \( \{l_e\} \). We use a quasi-Newton method to optimize, but there are many usable algorithms to optimize a high-dimensional problem.
Choice of $M_{i,j}$

We can go even farther. In our matrix we can include the effects of the transmission efficiency and finite resolution.

$$I_x = M I_e \rightarrow W \rightarrow W M I_e \rightarrow N \rightarrow f_r(E_e)$$
Calibration

• We can even directly measure $M_{i,j}$ using gas-target Bremsstrahlung in an x-ray tube.

• We also use radioactive sources
Anatomy of an Inverted Spectrum

- In the x-ray tube, neon gas fill, 3600eV beam energy
- Using the Elwert approximation to the Gaunt factor: Spectral lines not included, produce artificial spikes in spectrum. This is not the case of calibrated analysis.
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Extracting Electron Energy Distributions from PFRC X-ray Spectra

Neon K-α
Aluminum K-α
Silicon K-α

Non-physical features due to spectral lines
2600eV secondaries off of accel grid (at 1kV)

3600eV primary electrons
Spectrum Inversion to find spectral lines

- Spectral lines are not considered by the previous slide’s analysis; produce incorrect electron distributions.
- Re-transforming these electron distributions back into x-ray distributions yield discrepancies.
- No plasma could have produced the measured x-ray spectrum via only Bremsstrahlung. Spectral lines are *required* to produce those peaks.
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Seed Plasma Results: High power H$_2$

- Center Cell, **400W** RF power, H$_2$ gas

Beam integrates to a density of $1 \cdot 10^8$/cm$^3$ of bulk density $\sim 10^{10}$/cm$^3$
Seed Plasma Results: High power H$_2$

• 1300eV beam: In Source End Cell, plasma terminates on carbon cup (left)
  • This carbon cup floated at -1900V with oscillations of 375V$_{pkpk}$
  • Beam has 500eV FWHM
  • Plasma termination paddle at other end floats at -600V
• 2150eV beam: Oscillations must heat our beam. Resonances exist.
Seed Plasma Results: Low power $\text{H}_2$

- Center Cell, **100W** RF power, $\text{H}_2$ gas
- 140 ± 30eV thermal-like

![X-ray spectrum graph](image1)

**Carbon cup floating potential:** -1000V
**Far end paddle:** ~0V

**Beam at 1300eV**

![Inverted electron spectrum graph](image2)

**Beam integrates to a density of** $8 \cdot 10^6 / \text{cm}^3$ **of bulk density** $\sim 10^{10} / \text{cm}^3$
Seed Plasma Results: Argon

• Source End Cell, **350W** RF power, Ar gas

![Graph showing corrected x-ray spectrum](image)
Seed Plasma Results: Argon, high pressure

- Source End Cell, **350W RF power, Ar gas**

These first two show hard cutoffs in energy
Seed Plasma Results: Argon, low pressure

- Source End Cell, **350W** RF power, Ar gas

These three have spectral lines visible

Non-physical hollow and peak before and after spectral line

Beam and sub-beam, as with H₂

Inverted x-ray spectrum

2016/08/05

Inverted x-ray spectrum on a log scale

2016/08/05
Seed Plasma Discussion

- The diagnostic can determine features like beams and cutoffs and measure their amplitudes.
- The diagnostic can be used to identify spectral lines.
- Fast electrons were never considered in RMF calculation and simulation:
  - How do they affect RMF coupling? Penetration?
  - Previous simulation has started with a thermal distribution.
RMF Plasma Results

- Center Cell, 13.5kW RMF power, 300W seed power

- Unexpectedly low count rate.
- Beam at 1500eV
- $T_e = 250 \pm 50\text{eV}$
- Assuming beam, $n_e = 3 \cdot 10^8/\text{cm}^3$
Reminder: What did we expect?

- Single-particle motion Hamiltonian simulation results example distribution

![PFRC-2 simulation graph](image)
RMF Plasma Results: Seed Comparison

- Center Cell, 13.5kW RMF power, 300W seed power

**During RMF Pulse**

- Corrected x-ray spectrum
- Inverted electron spectrum

**Outside RMF Pulse (seed only)**

- Corrected x-ray spectrum
- Inverted electron spectrum

Seed count rate ~30X lower
RMF Plasma Results: Low seed

- Center Cell, 19.5kW RMF power, 30W seed power

- Beam at 1150eV
- $T_e = 130 \pm 60$eV
- Assuming thermal, $n_e = 3 \cdot 10^9$/cm$^3$
- Assuming beam, $n_e = 3 \cdot 10^7$/cm$^3$
RMF Plasma Discussion

- Even if hot electrons are thermal during RMF, only account for 1-3% of electrons

- Clearly interesting physics is happening. Possibilities:
  - Micro turbulence insufficient to equilibrate to thermal
  - RMF heating minority population (seed beam?) preferentially

- This technique will settle these questions
  - Calibrated data will take into account spectral lines, unaccounted-for detector effects
  - New detector can see down to 400eV
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Citations


