MHD and Hybrid Simulation Study of FRC Plasmas

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   i. FRC translation and two FRC collision process
   ii. Comparison with ST merging
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Research Background

Plasma Ion Dynamics

1995-1998
Collisionless pitch-angle scattering at X-point

1998-2006
Adiabaticity breaking of plasma ions in core region
NB injected particle motion

2004-
Origin of spontaneous toroidal spin-up

MHD

2004-2006
Fusion propulsion

2013-
Translation and two FRCs collision

C-2 experiment calculated by LamyRidge code

Electron fluid equation + Ion particle model

Full-particle simulation
Basic equations

Resistive Hall MHD equations

Eq. continuity
\[ \frac{\partial \rho}{\partial t} = - \nabla \cdot (\rho \mathbf{u}) \]

Eq. Motion
\[ \frac{\partial \mathbf{u}}{\partial t} = - (\mathbf{u} \cdot \nabla) \mathbf{u} + \frac{1}{\rho} \left[ \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p - \nabla \cdot \mathbf{I} \mathbf{I} \right] \]

Energy Eq.
\[ \frac{\partial p}{\partial t} = - (\mathbf{u} \cdot \nabla) p - \gamma p \nabla \cdot \mathbf{u} + (\gamma - 1) \left[ \eta j^2 - \mathbf{I} : \nabla \mathbf{u} \right] \]

Faraday’s law
\[ \frac{\partial \mathbf{B}}{\partial t} = - \nabla \times \left( \eta j - (\mathbf{u} \times \mathbf{B}) + \frac{1}{en_e} [(j \times \mathbf{B}) - \nabla p_e] \right) \quad p_e = 0.5 p \]

Hall term
- Cylindrical coordinate system
- G-S equilibrium as initial condition
- Axisymmetric system
- Resistivity dependent on the current density
- 4-th order Runge-Kutta method
Translation by magnetic pressure gradient

**Experiment**

NUCTE-III/T and magnetic field profile

**Present simulation**

Time variation of external field structure

MHD eq. (e.g.) Faraday’s law

\[
\frac{\partial \mathbf{B}_{\text{plasma}}}{\partial t} = \nabla \times \left( \mathbf{u} \times \mathbf{B}_{\text{total}} - \frac{\eta}{\mu_0} \nabla \times \mathbf{B}_{\text{total}} \right)
\]

- Calculation of time evolution of field by plasma
- Inputting external field variation

\[\mathbf{B}_{\text{total}} = \mathbf{B}_{\text{plasma}} + \mathbf{B}_{\text{external}}\]
Collision simulation (1)

w/o magnetic assist: sequential external field control

Two FRCs collision, 0-72[µs].

No magnetic reconnection event is observed.

Small translation velocity

- Max. 20km/s

External magnetic assist is needed

Normalized by Alfven velocity

Volume-averaged translation velocity
Collision simulation (2)

w/ magnetic assist: sequential external field control

Collision simulation with magnetic assist, 0-72[μs].

Magnetic reconnection occurs in the separatrix field line.

Larger translation velocity is observed
- About two times larger

Our result is different from the experiment

Volume-averaged translation velocity
Comparison of field structure

- Only slight difference between two models is observed
- No complete core merging can be reproduced
High resolution resistive MHD simulation

Current sheet width
\[ \sim 4\text{-}5 \text{ axial meshes } \Delta z \]

Fine magnetic structure in the reconnection region can not be calculated by MHD model.

1/10 mesh size region is prepared in high-resolution MHD model.
High resolution simulation results

Current density profile in the reconnection region

2D current density profile. (Top) Conventional model (bottom) high-resolution model

Width of the current sheet

- Up to 5 meshes in conventional model
- Up to 12 meshes in high-resolution model
**Force in reconnection region**

\[ \mathbf{j} \times \mathbf{B} - \nabla p \]  : Dominant force components in our MHD model

- Attracting force acts on two FRC plasmas
- Repelling force, however, is found near the separatrix surface

(Top) 2D Force profile, (Bottom) Axial force profile \((r = R)\)  \(R\): major radius

Difficult in FRC merging by resistive MHD model
ST plasma merging

Required subject of fusion research = Core Fueling


Feasibility study

3D MHD simulation code -MIPS\cite{Y. Todo et al., PFR 5, S2062 (2010)}-

Assist coils (to accelerate the secondary plasma)

Initial poloidal flux and pressure profiles of ST plasmas


Translation and merging process

1. Collision occurs at 260 $t_A$, and then the reconnection process starts.
2. During 600-630 $t_A$, the magnetic axes of two plasmas approach rapidly, and the secondary plasma is absorbed into the main plasma.
Core fueling and current drive

-Merging fueling and current drive will work-

2D plot of the plasma pressure on a poloidal cross-section

Particle

Current

2016/8/25
Can we produce FRC by giving the pressure gradient?

Formation of FRC: Field Reversed Theta Pinch (FRTP) method

\[ \vec{j} \times \vec{B} = \nabla p \]

- Diamagnetic current
- Pressure gradient

Is it possible to generate an FRC by the following scenario?

The pressure gradient
- \( \rightarrow \) the diamagnetic current
- \( \rightarrow \) a field-reversed configuration
3D hybrid simulation

We clarify the possibility to occur field reversal without field-reversed the pinch method by using 3D hybrid simulation

Goal

Difference of sequence

FRTP method

Field reversal

Inductive toroidal field

Radial compression by \( \mathbf{E} \times \mathbf{B} \)

The pressure gradient
\[ \nabla p = \mathbf{j} \times \mathbf{B} \]

Tested method (Present case)

Particle fueling to produce the pressure gradient

Diamagnetic current
\[ \mathbf{v}_D = -\frac{\nabla p \times \mathbf{B}}{qnB^2} \]

Electric field

Field reversal

Order is reversed
Hybrid simulation model

Equation of motion for α species

\[
m_\alpha \frac{d\mathbf{v}_\alpha}{dt} = q_\alpha (\mathbf{E} + \mathbf{v}_\alpha \times \mathbf{B}) - \sum_\beta m_\alpha \nu_{\alpha\beta} (\mathbf{v}_\alpha)(\mathbf{v}_\alpha - \mathbf{u}_\beta)
\]

Collisional pitch angle scattering can be also considered by the Monte-Carlo method

Equation of motion for massless electron fluid

\[
-\varepsilon n_e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) - \nabla p_e + R_{ei} = 0
\]

\[
R_{ei} = -R_{ie} = \int m_i \nu_{ie} (\mathbf{v}_i - \mathbf{u}_e) f_i (\mathbf{v}_i) d\mathbf{v}_i
\]

Faraday’s law \( \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \)

Ampere’s law \( \mu_0 \mathbf{j} = \nabla \times \mathbf{B} \)

Definition of current density \( \mathbf{j} = \varepsilon n_e (\mathbf{u}_i - \mathbf{u}_e) \)

Thermal energy equation for electron fluid

\[
\frac{\partial p_e}{\partial t} + \gamma p_e (\nabla \cdot \mathbf{u}_e) + \mathbf{u}_e \cdot \nabla p_e = (\gamma - 1)(\mathbf{u}_e - \mathbf{u}_i) \cdot R_{ei}
\]
Tested particle fueling model

<table>
<thead>
<tr>
<th>Ring fueling</th>
<th>Circular fueling</th>
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<tbody>
<tr>
<td><img src="image1" alt="Circle Distribution" /></td>
<td><img src="image2" alt="Circle Distribution" /></td>
</tr>
<tr>
<td><img src="image3" alt="Profile" /></td>
<td><img src="image4" alt="Profile" /></td>
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- Particle loading is done by using pseudo random number
- Normal distribution is used for spatial profile
- Velocity distribution is Maxwellian
Ring fueling results

External field: $B_{ex} = 0.4$ T
Ion temperature: $T_i = 50$eV
Ion fueling time 5 $\mu$s
Calculation time 15 $\mu$s

As soon as fueling stops, ring configuration destroys due to axial diffusion.
Ring fueling results

Opposing current

$\mathbf{j} = \mathbf{B} \times \nabla p$

The peak of pressure

$1\text{step} = 5.0 \times 10^{-8} [s]$
Circular fueling

External field: $B_{ex} = 0.4$ T
Ion temperature: $T_i = 50$ eV
Ion fueling time $5 \mu s$
Calculation time $10 \mu s$

As soon as fueling stops, ring configuration destroys due to axial diffusion and instability.
Magnetic field weakening

1. Magnetic field weakening stops just after the end of fueling.
2. As the ion fueling rate increases, magnetic field weakening enhances. However, it can never produce the field-reversal.

External field: $B_{ex} = 0.4 \text{ T}$

Ion temperature: $T_i = 50 \text{ eV}$

Electron temperature: $T_e = 50 \text{ eV}$

Ion fueling rate: $S = 1.4 \times 10^{26} [1/\text{s}]$
The diamagnetic current is generated before the end of fueling. Although the magnetic field is weakened in the central region, however, the field reversal does not occur.

**Circular fueling results**

1 step $= 5.0 \times 10^{-8} \text{[s]}$

- **Field weakening**: Due to no field-reversal and no magnetic reconnection event, the circular plasma configuration destroys by the rapid axial diffusion.

- **Dispersion by axial diffusion**
Summary

• 2D Resistive MHD / Hall MHD simulation has been carried out to study two FRC collision process.

• 3D MHD simulation has also been done for ST merging. After a ballooning instability, the merging process is observed.

• From our high-resolution MHD simulation, the following effects are essential to reproduce FRC merging
  
  ✓ Cross-field thermal conductivity

  ✓ Anomalous resisitivity effect
    = requirement of full-particle simulation

• We can not succeed in producing an FRC plasma by giving only the pressure gradient.