

# MHD and Hybrid Simulation Study of FRC Plasmas

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# Outline

1. Research Background
2. MHD simulation
  - i. FRC translation and two FRC collision process
  - ii. Comparison with ST merging
3. Hybrid simulation
  - i. Feasibility of slow formation by plasma pressure supply
4. Summary



# (my personal) Research Background

## Plasma Ion Dynamics

1995-1998

**Collisionless pitch-angle scattering at X-point**

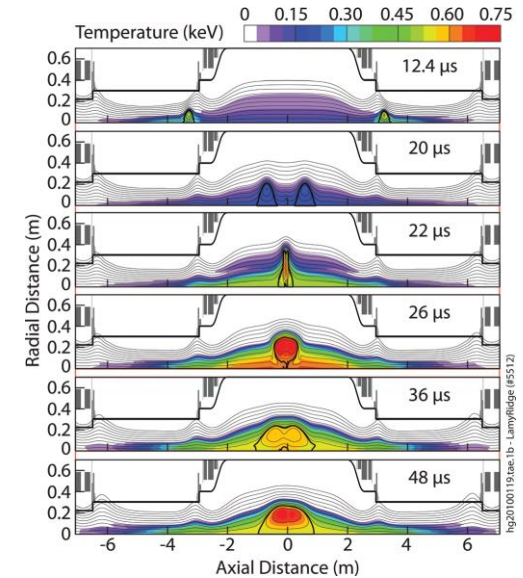
1998-2006

**Adiabaticity breaking of plasma ions in core region**

**NB injected particle motion**

2004-

**Origin of spontaneous toroidal spin-up**



C-2 experiment calculated  
by LamyRidge code

H. Y. Guo et al, Phys.  
Plasmas 18, 056110 (2011).

## MHD

2004-2006

**Fusion propulsion**

2013-

**Translation and two FRCs collision**

Electron fluid equation  
+ Ion particle model



Full-particle simulation



2016/8/25

# Basic equations

## Resistive Hall MHD equations

Eq. continuity  $\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u})$

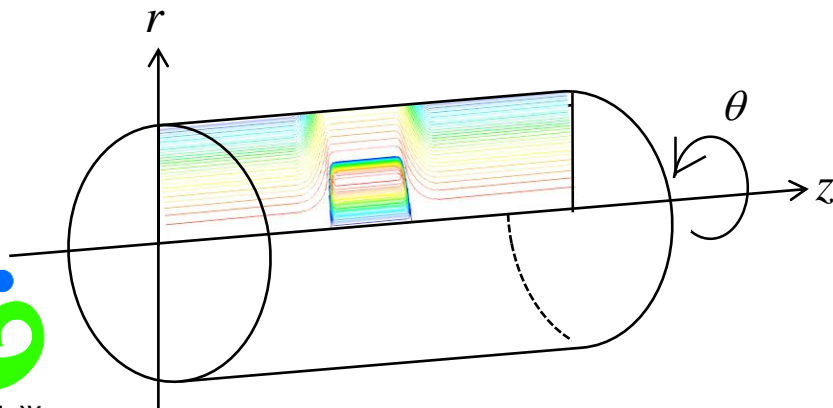
Eq. Motion  $\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla) \mathbf{u} + \frac{1}{\rho} \left[ \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p - \nabla \cdot \tilde{\Pi} \right]$

Energy Eq.  $\frac{\partial p}{\partial t} = -(\mathbf{u} \cdot \nabla) p - \gamma p \nabla \cdot \mathbf{u} + (\gamma - 1) \left[ \eta j^2 - \tilde{\Pi} : \nabla \mathbf{u} \right]$

Faraday's law  $\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left( \eta \mathbf{j} - (\mathbf{u} \times \mathbf{B}) + \frac{1}{en_e} [(\mathbf{j} \times \mathbf{B}) - \nabla p_e] \right)$   $p_e = 0.5p$

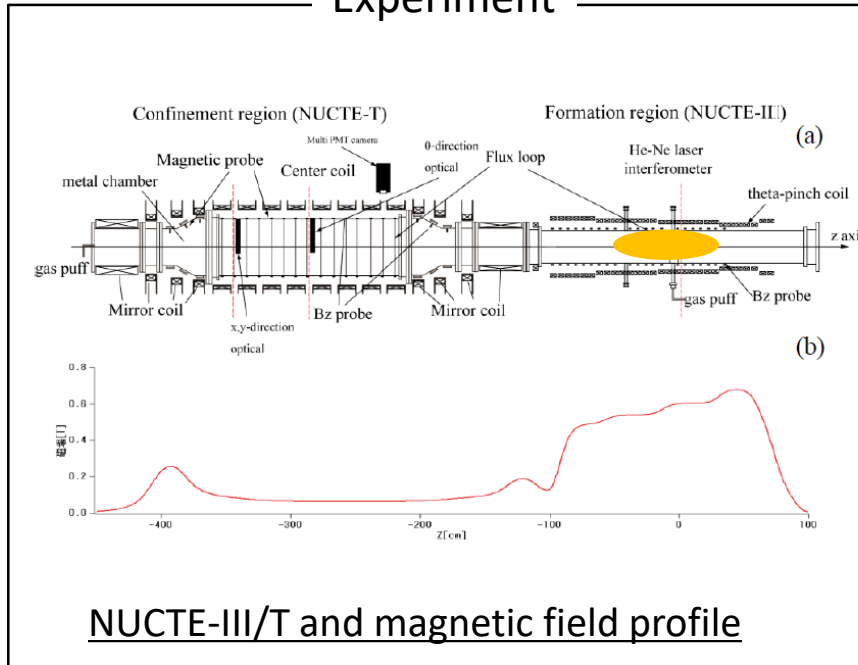
Hall term

- Cylindrical coordinate system
- G-S equilibrium as initial condition
- Axisymmetric system
- Resistivity dependent on the current density
- 4-th order Runge-Kutta method

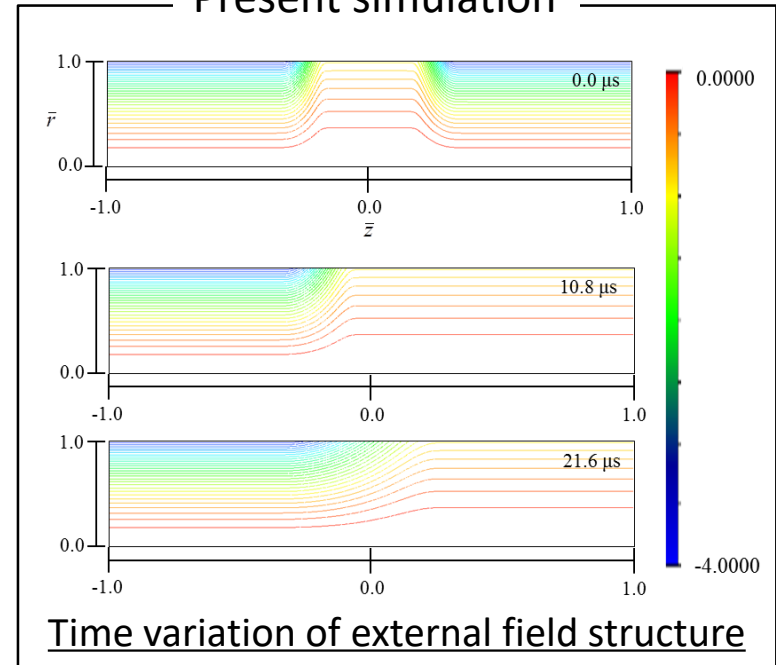


# Translation by magnetic pressure gradient

Experiment



Present simulation



MHD eq.  
(e.g.) Faraday's law

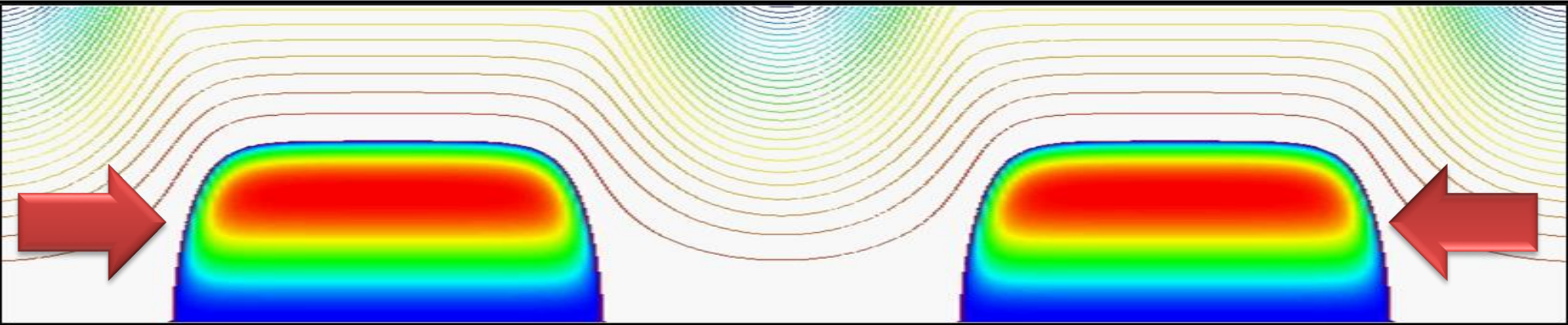
$$\frac{\partial \mathbf{B}_{\text{plasma}}}{\partial t} = \nabla \times \left( \mathbf{u} \times \mathbf{B}_{\text{total}} - \frac{\eta}{\mu_0} \nabla \times \mathbf{B}_{\text{total}} \right)$$

$$\mathbf{B}_{\text{total}} = \mathbf{B}_{\text{plasma}} + \mathbf{B}_{\text{external}}$$

- Calculation of time evolution of field by plasma
- Inputting external field variation

# Collision simulation (1)

w/o magnetic assist: sequential external field control



Two FRCs collision, 0-72[ $\mu$ s].

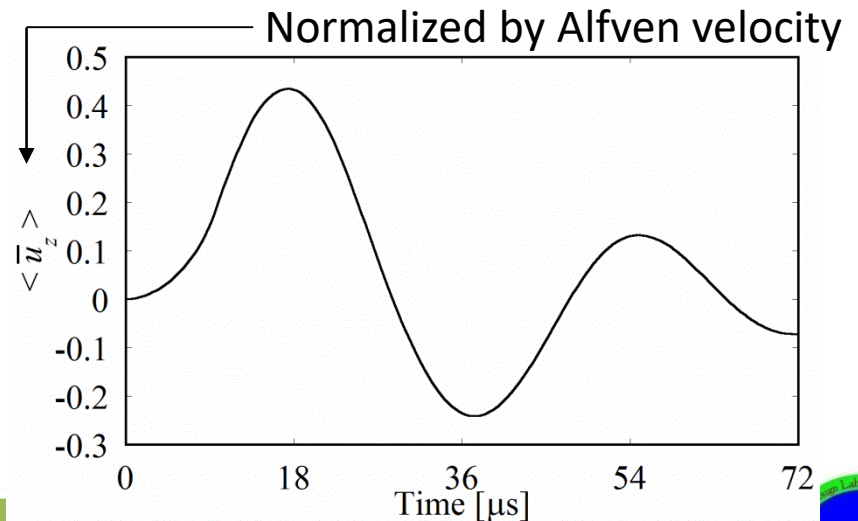
No magnetic reconnection event is observed.

Small translation velocity

- Max. 20km/s



External magnetic assist is needed



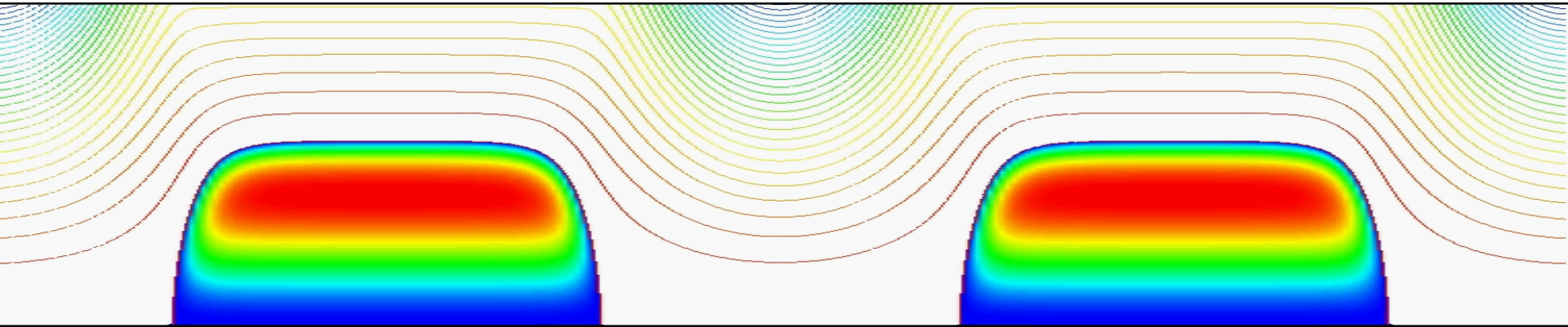
Volume-averaged translation velocity





# Collision simulation (2)

w/ magnetic assist: sequential external field control



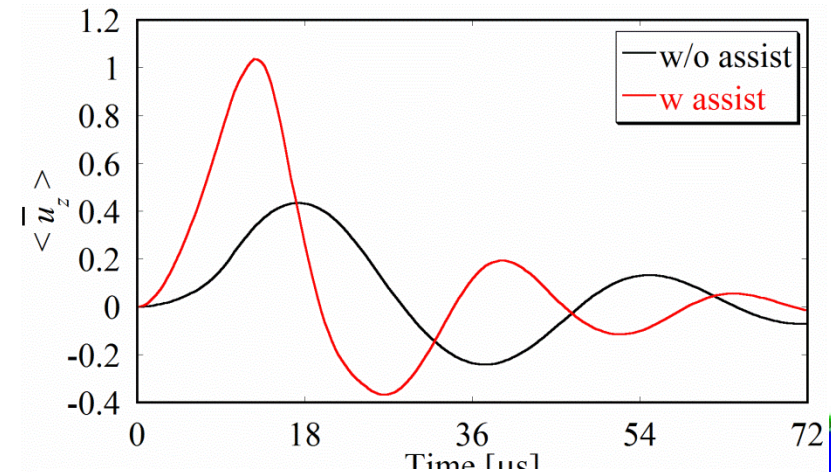
Collision simulation with magnetic assist, 0-72[ $\mu$ s].

Magnetic reconnection occurs in the separatrix field line.

Larger translation velocity is observed

➤ About two times larger

Our result is different from the experiment

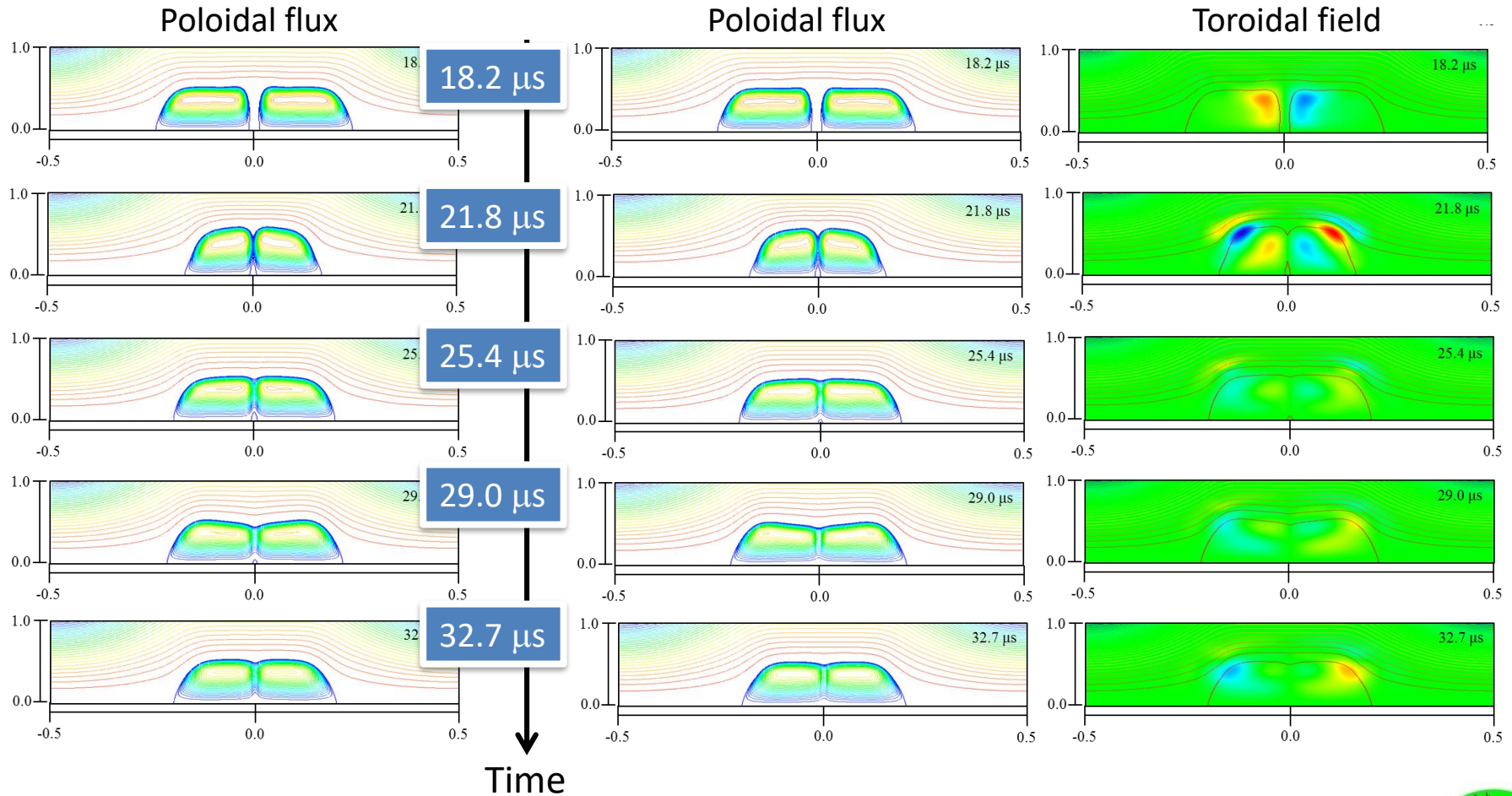


Volume-averaged translation velocity

# Comparison of field structure

Resistive MHD

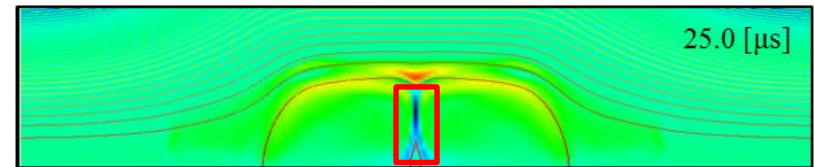
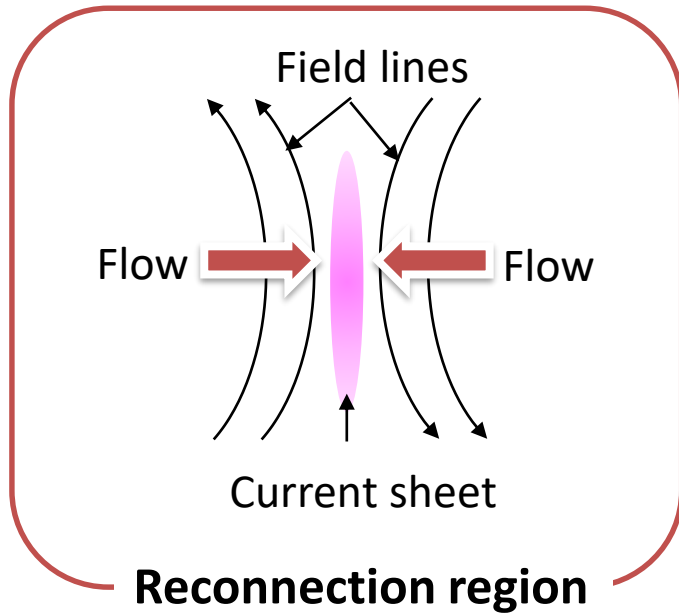
Hall MHD



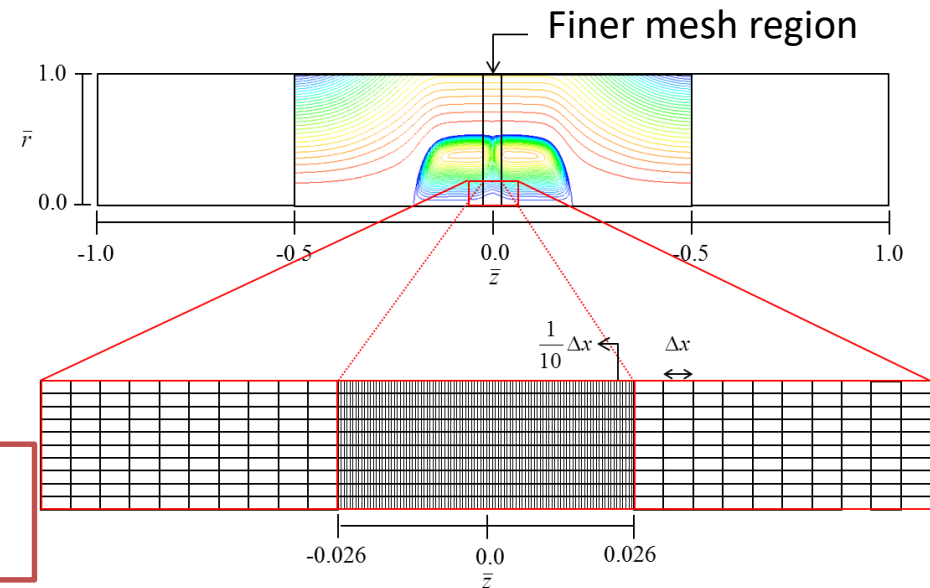
- Only slight difference between two models is observed
- No complete core merging can be reproduced



# High resolution resistive MHD simulation



2D profile of the current density



Calculation mesh structure

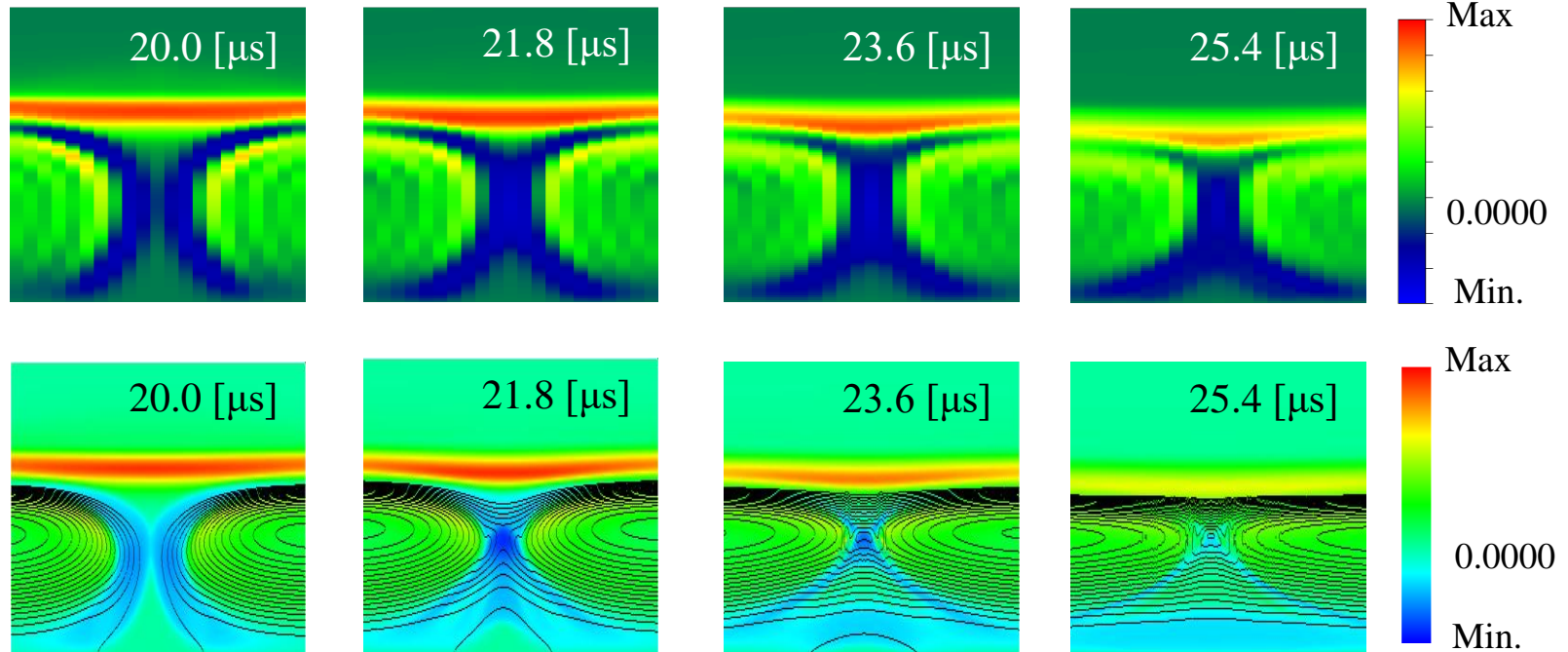
Current sheet width  
~ 4-5 axial meshes  $\Delta z$

Fine magnetic structure in the reconnection region can not be calculated by MHD model.

**1/10 mesh size** region is prepared in high-resolution MHD model.

# High resolution simulation results

*Current density profile in the reconnection region*



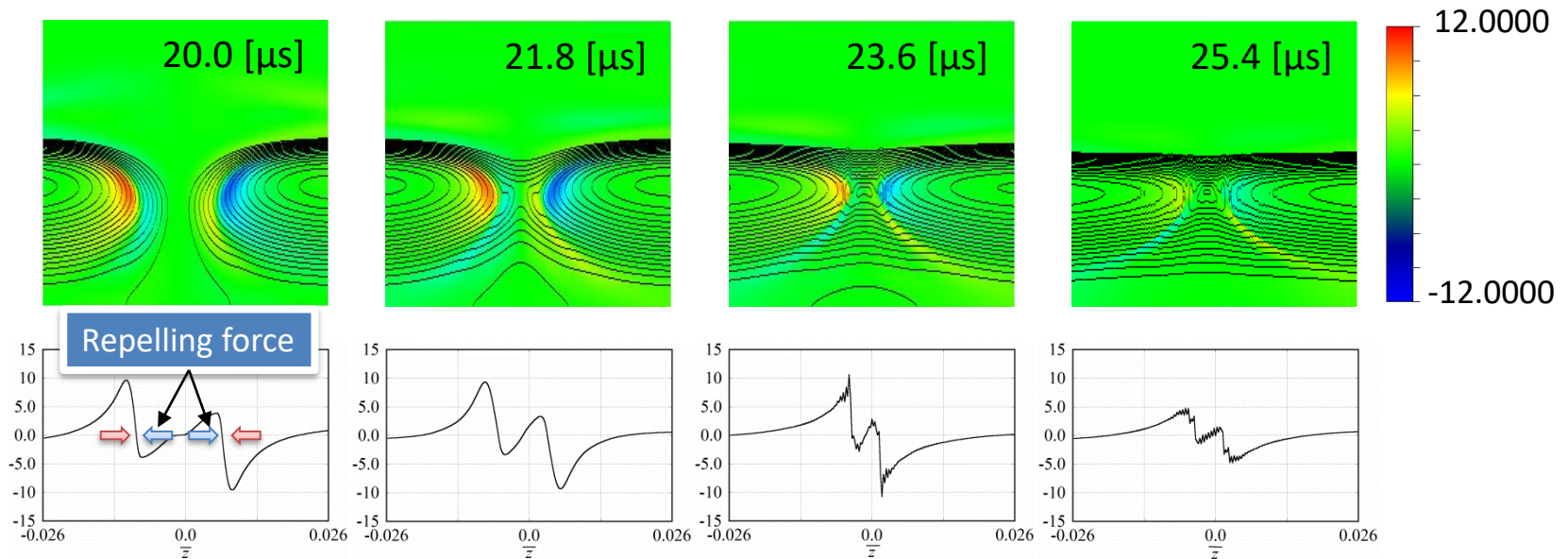
2D current density profile. (Top) Conventional model (bottom) high-resolution model

Width of the current sheet {

- Up to 5 meshes in conventional model
- Up to 12 meshes in high-resolution model

# Force in reconnection region

$\mathbf{j} \times \mathbf{B} - \nabla p$  : Dominant force components in our MHD model



(Top) 2D Force profile, (Bottom) Axial force profile ( $r = R$ )  $R$ : major radius

- Attracting force acts on two FRC plasmas
- Repelling force, however, is found near the separatrix surface

Difficulty in FRC merging by resistive MHD model

# ST plasma merging

Required subject of fusion research = **Core Fueling**



Merging fueling method<sup>[1]</sup>

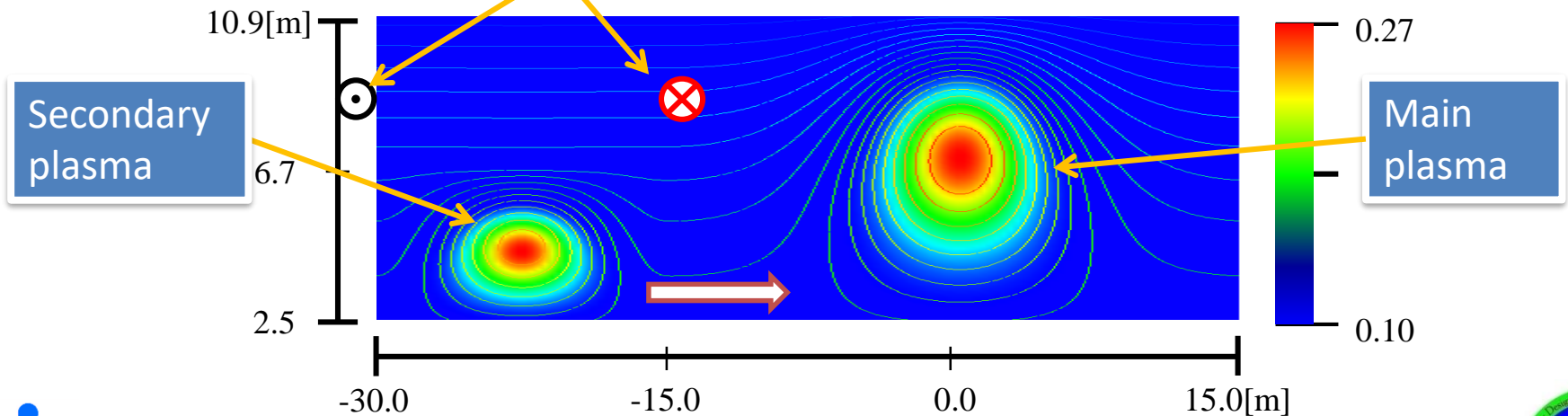
[1] O. Mitarai *et al.*, Fusion Eng. Des. **109-111** Part B, 1365 (2016).

Feasibility study

3D MHD simulation code -MIPS<sup>[2]</sup>-

[2] Y. Todo *et al.*, PFR **5**, S2062 (2010).

**Assist coils** (to accelerate the secondary plasma)



Initial poloidal flux and pressure profiles of ST plasmas

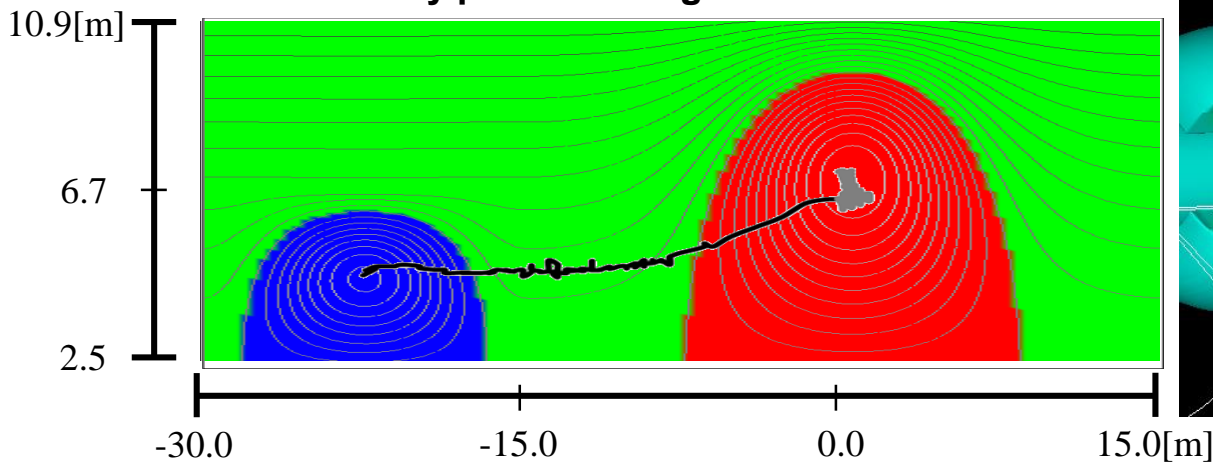
# Translation and merging process

● Main plasma

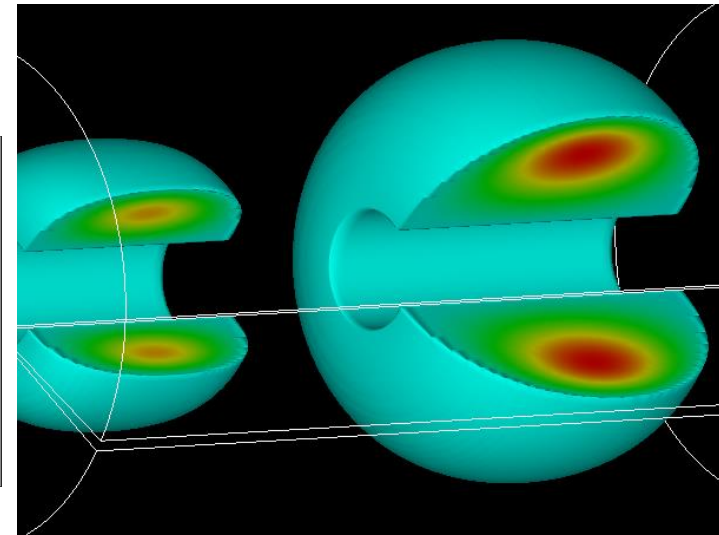
● Secondary plasma

— Main plasma's magnetic axis

— Secondary plasma's magnetic axis



Trace of two ST plasmas



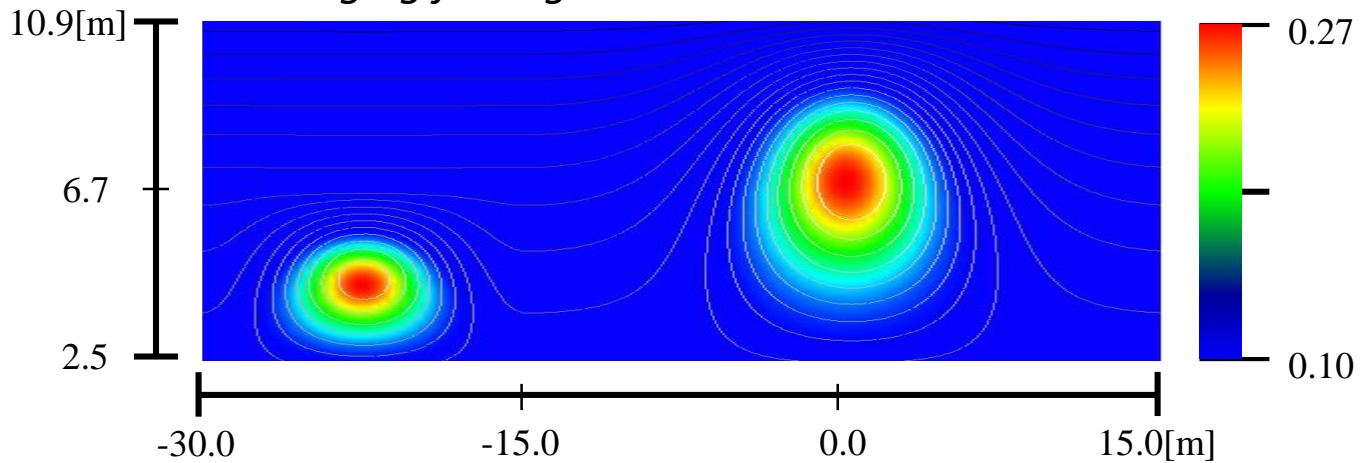
3D poloidal flux structure

1. Collision occurs at  $260 t_A$ , and then the reconnection process starts.
2. During  $600-630 t_A$  the magnetic axes of two plasmas approach rapidly, and the secondary plasma is absorbed into the main plasma.

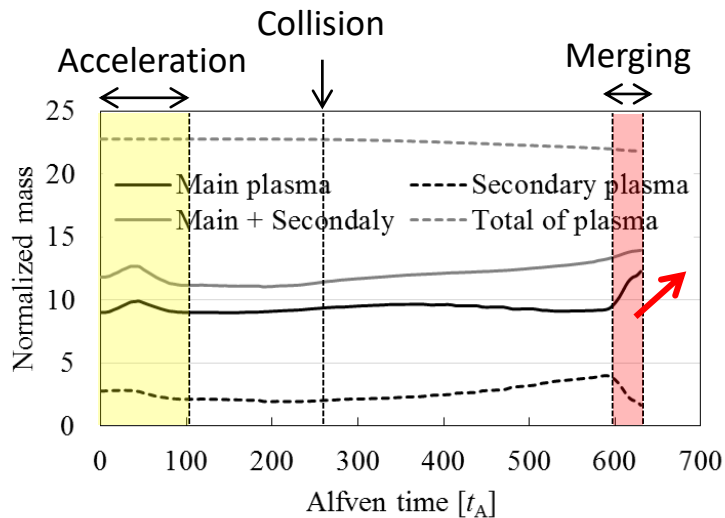


# Core fueling and current drive

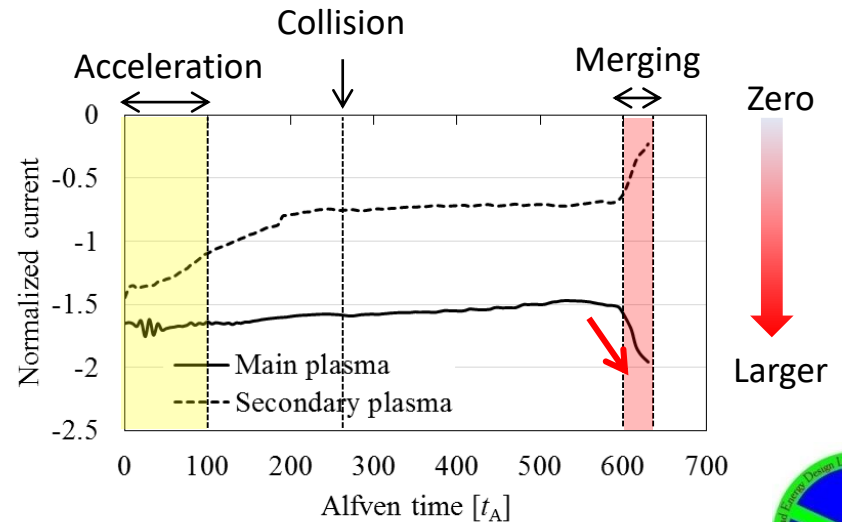
-Merging fueling and current drive will work-



2D plot of the plasma pressure on a poloidal cross-section



Particle

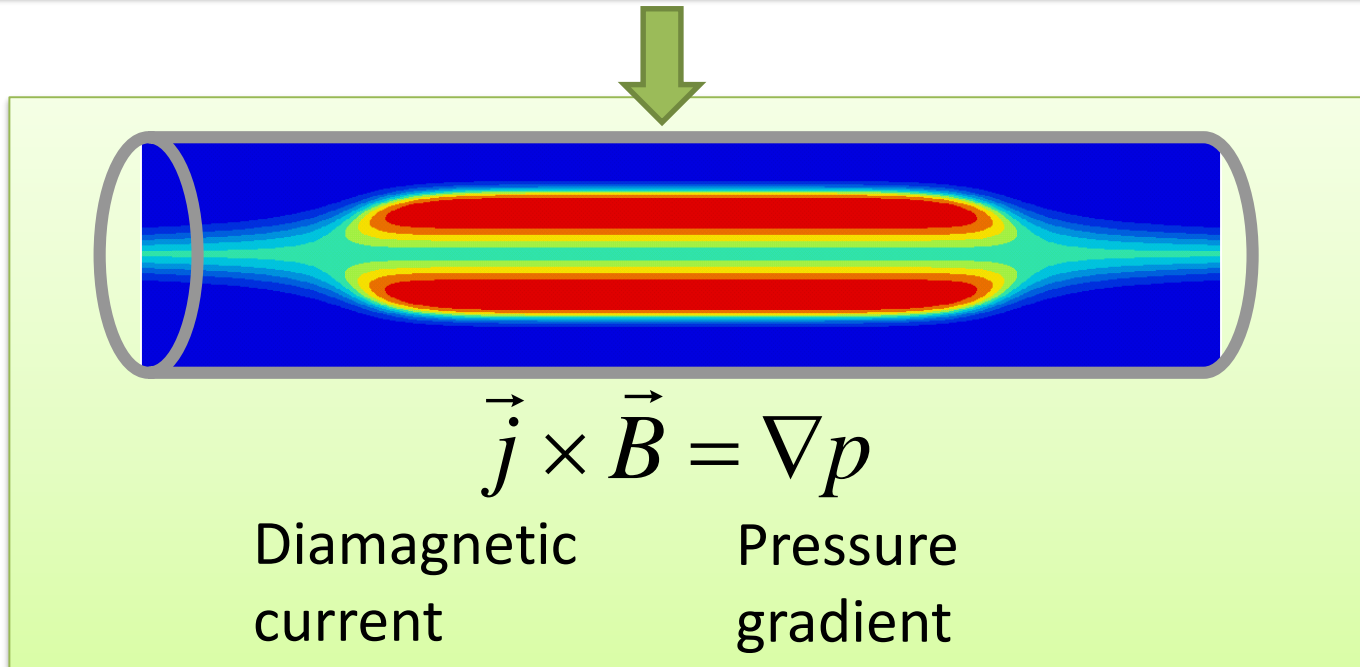


Current



# Can we produce FRC by giving the pressure gradient?

Formation of FRC: Field Reversed Theta Pinch (F RTP) method



*Is it possible to generate an FRC by the following scenario?*

The pressure gradient

→ the diamagnetic current

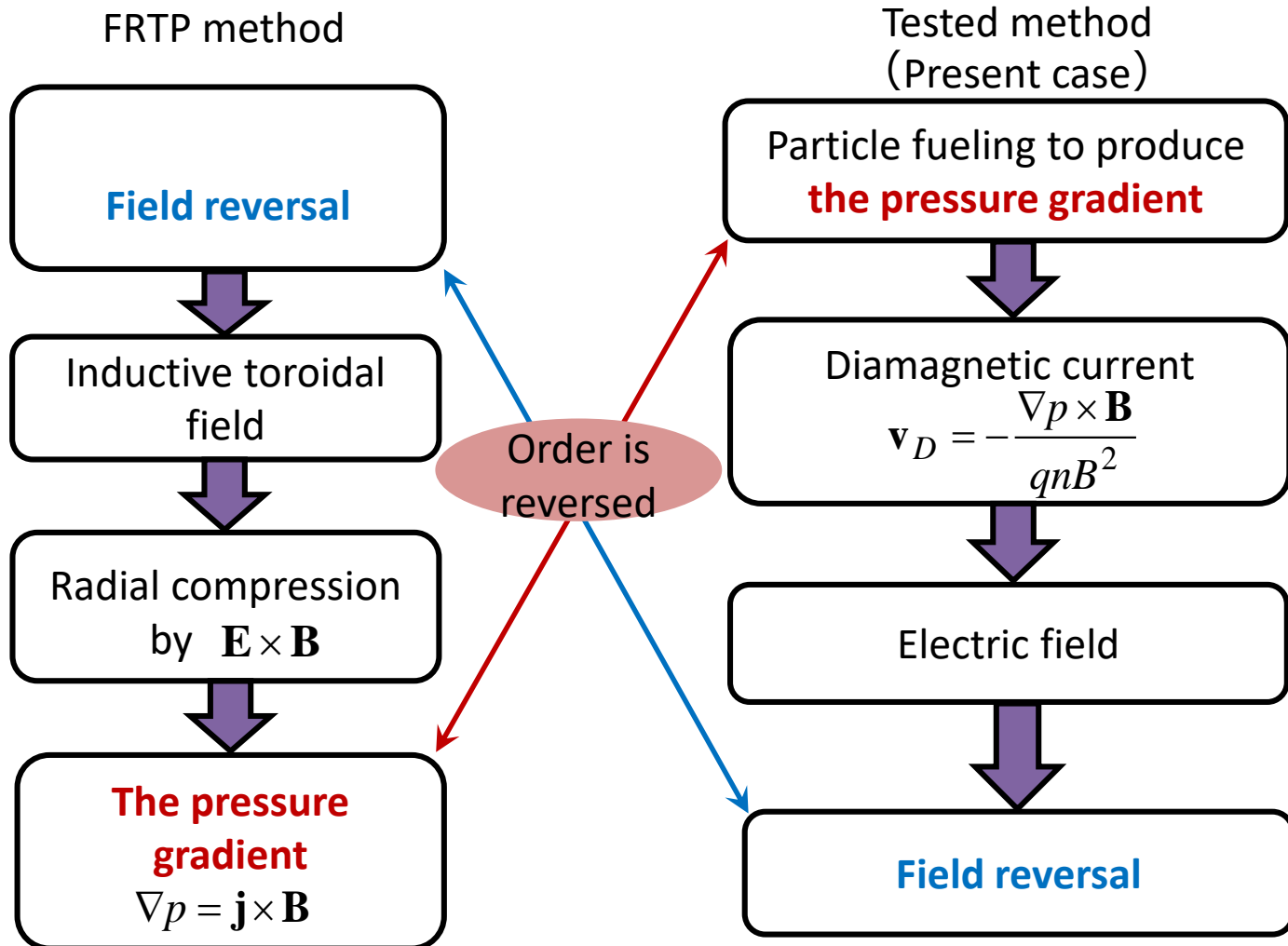
→ a field-reversed configuration

# 3D hybrid simulation

Goal

We clarify the possibility to occur field reversal without field-reversed the pinch method by using 3D hybrid simulation

Difference of sequence



# Hybrid simulation model

Equation of motion for  $\alpha$  species

$$m_{\alpha} \frac{d\mathbf{v}_{\alpha}}{dt} = q_{\alpha} (\mathbf{E} + \mathbf{v}_{\alpha} \times \mathbf{B}) - \sum_{\beta} m_{\alpha} \nu_{\alpha\beta}(\mathbf{v}_{\alpha}) (\mathbf{v}_{\alpha} - \mathbf{u}_{\beta})$$

Collisional pitch angle scattering can be also considered by the Monte-Carlo method

Equation of motion for massless electron fluid

$$-en_e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) - \nabla p_e + \mathbf{R}_{ei} = \mathbf{0}$$

$$\mathbf{R}_{ei} = -\mathbf{R}_{ie} = \int m_i \nu_{ie} (\mathbf{v}_i - \mathbf{u}_e) f_i(\mathbf{v}_i) d\mathbf{v}_i$$

Faraday's law  $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$       Ampere's law  $\mu_0 \mathbf{j} = \nabla \times \mathbf{B}$

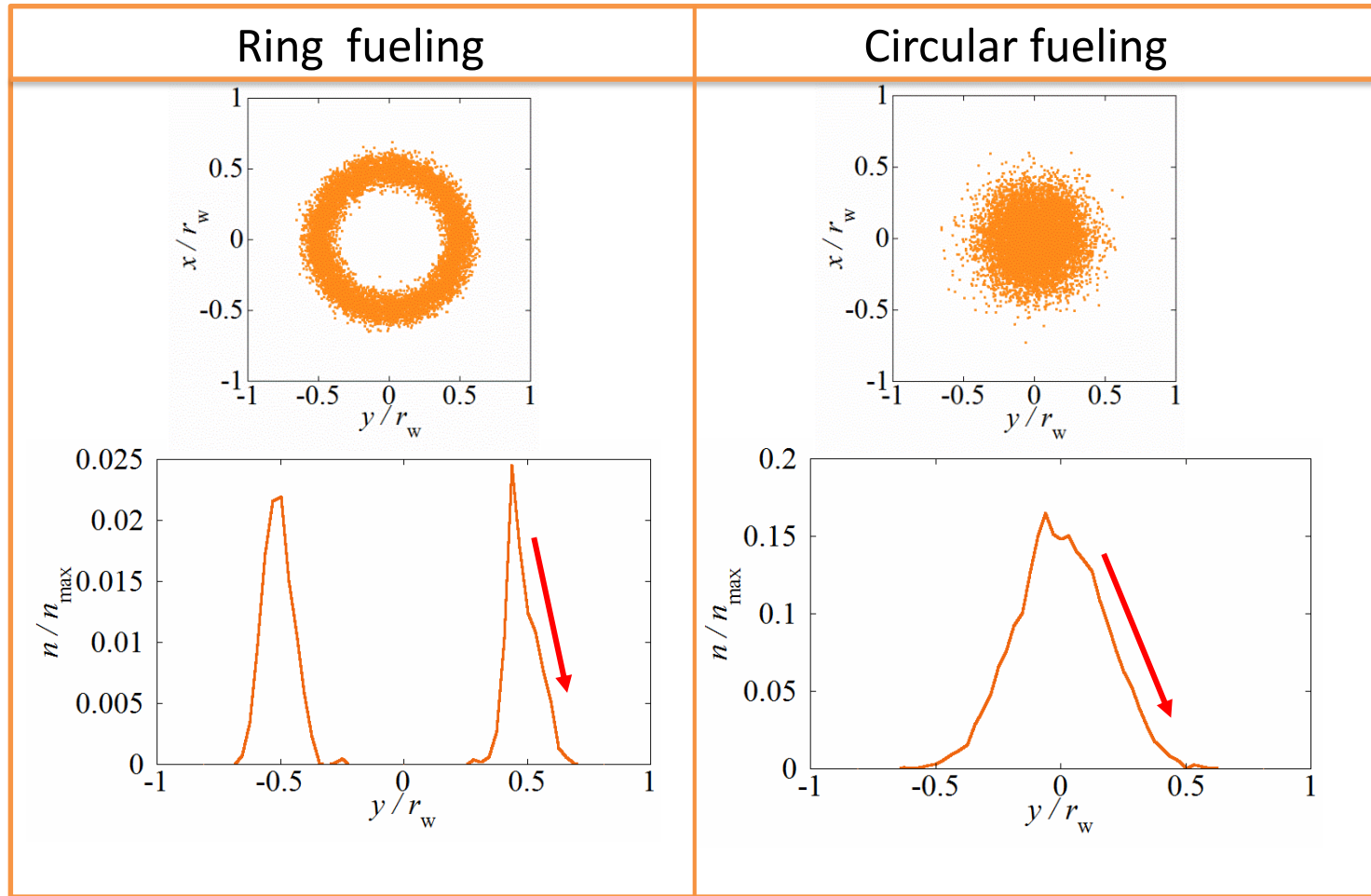
Definition of current density  $\mathbf{j} = en_e (\mathbf{u}_i - \mathbf{u}_e)$

Thermal energy equation for electron fluid

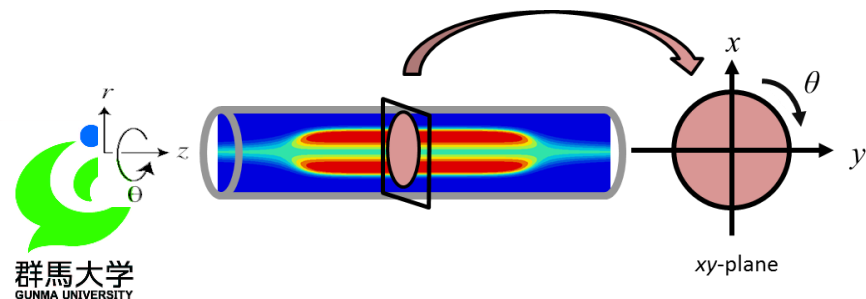
$$\frac{\partial p_e}{\partial t} + \gamma p_e (\nabla \cdot \mathbf{u}_e) + \mathbf{u}_e \cdot \nabla p_e = (\gamma - 1) (\mathbf{u}_e - \mathbf{u}_i) \cdot \mathbf{R}_{ei}$$



# Tested particle fueling model

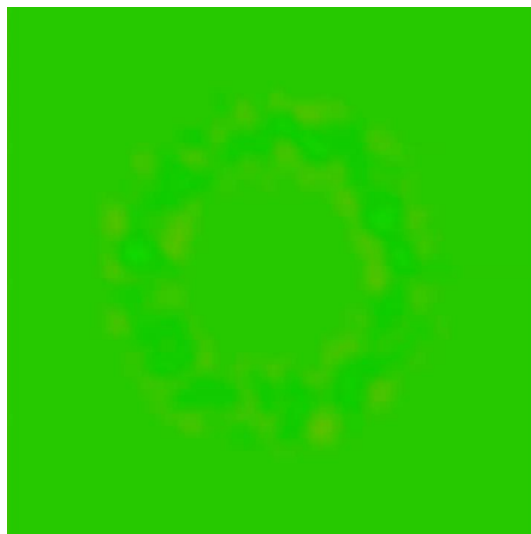
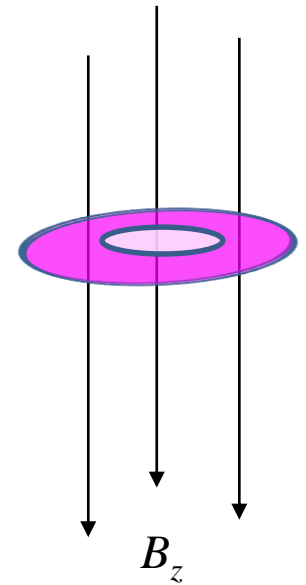
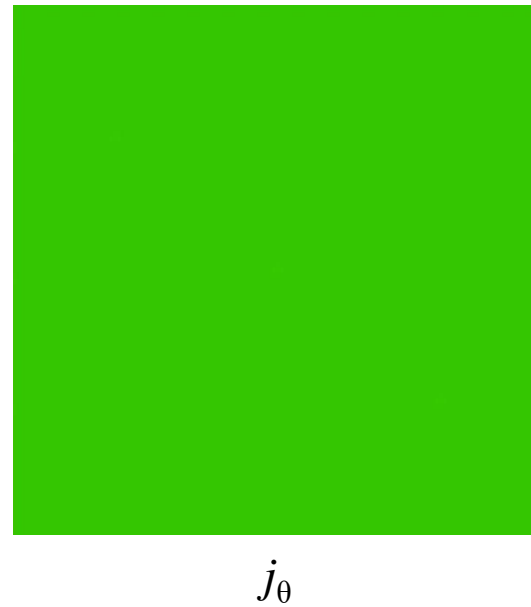
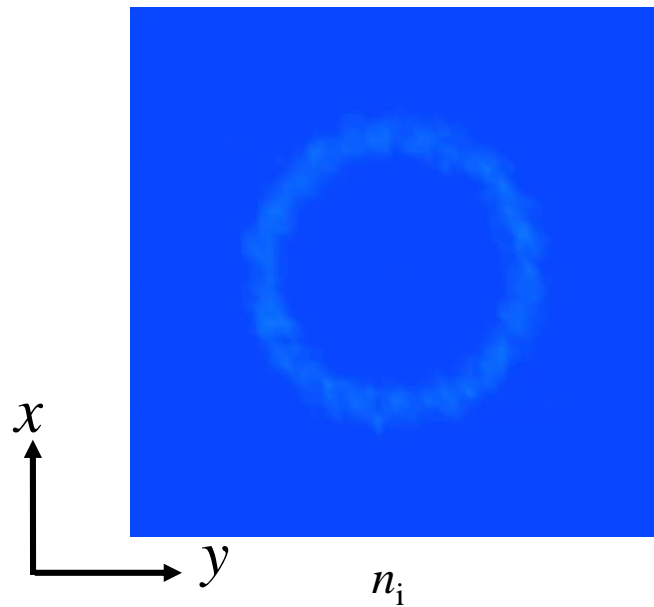


- Particle loading is done by using pseudo random number
- Normal distribution is used for spatial profile
- Velocity distribution is Maxwellian





# Ring fueling results

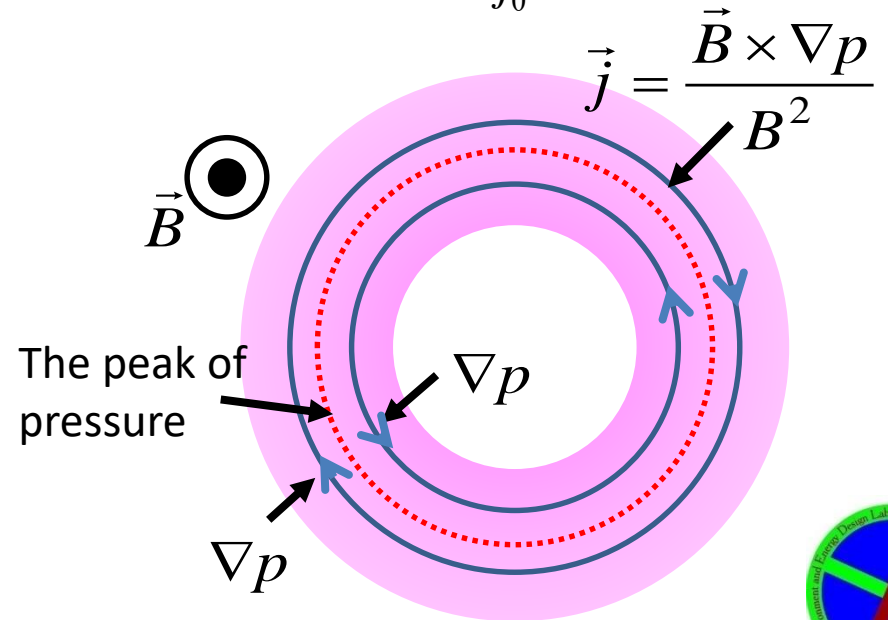
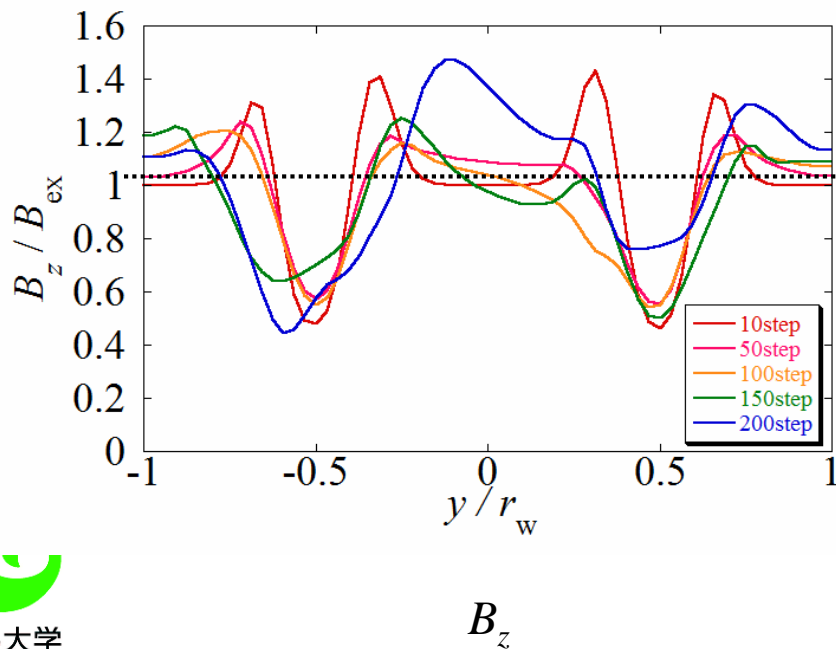
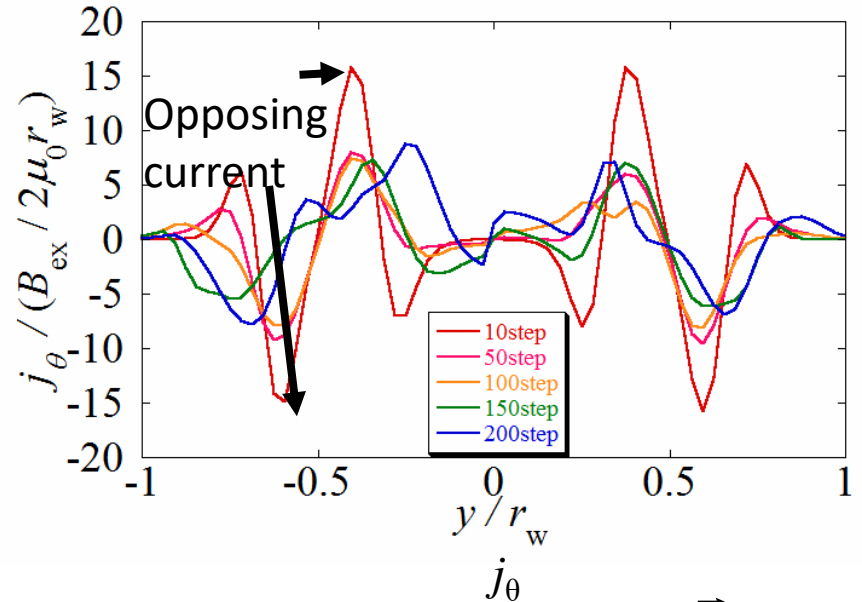
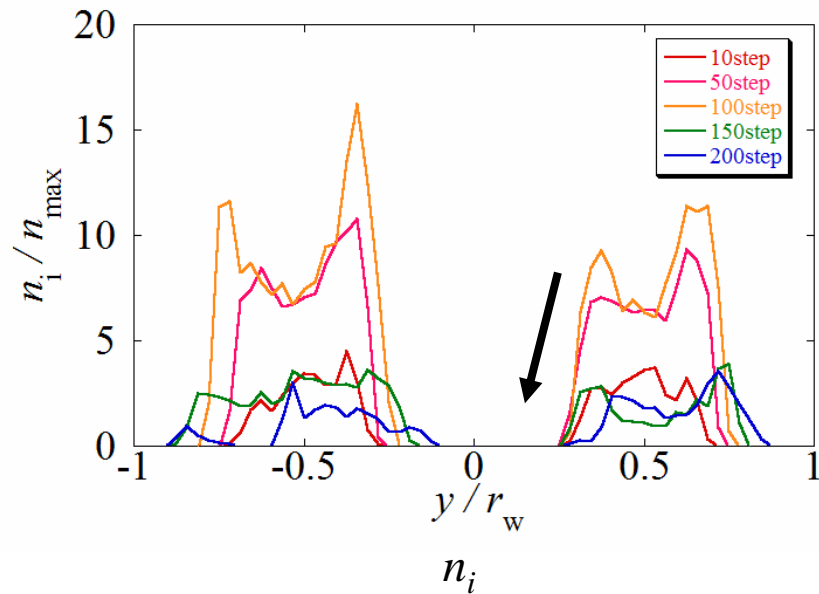


External field:  $B_{ex}=0.4$  T  
Ion temperature:  $T_i=50$  eV  
Ion fueling time  $5 \mu s$   
Calculation time  $15 \mu s$

As soon as fueling stops, ring configuration destroys due to axial diffusion.

# Ring fueling results

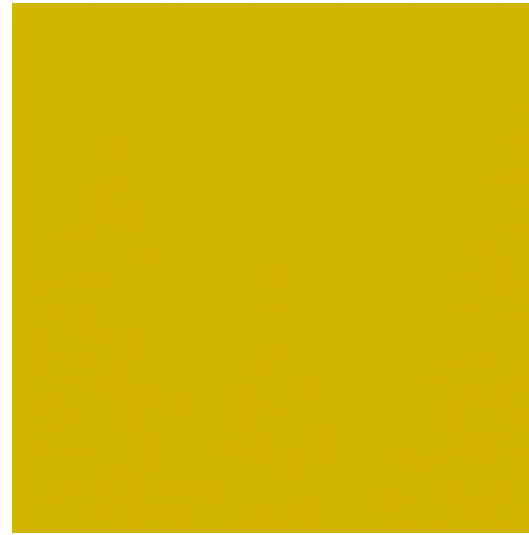
1step =  $5.0 \times 10^{-8}$  [s]



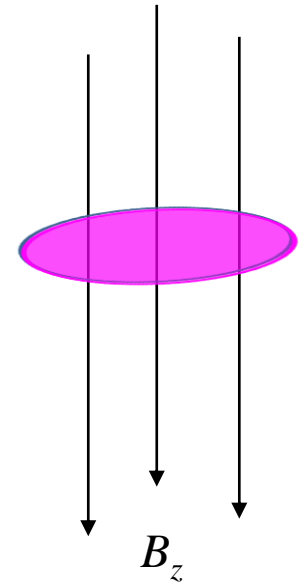
# Circular fueling



$n_i$



$j_\theta$



$B_z$

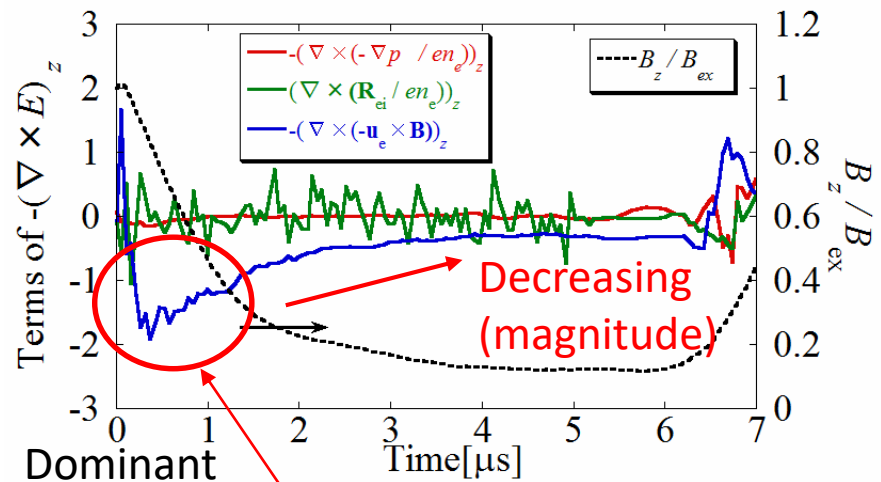
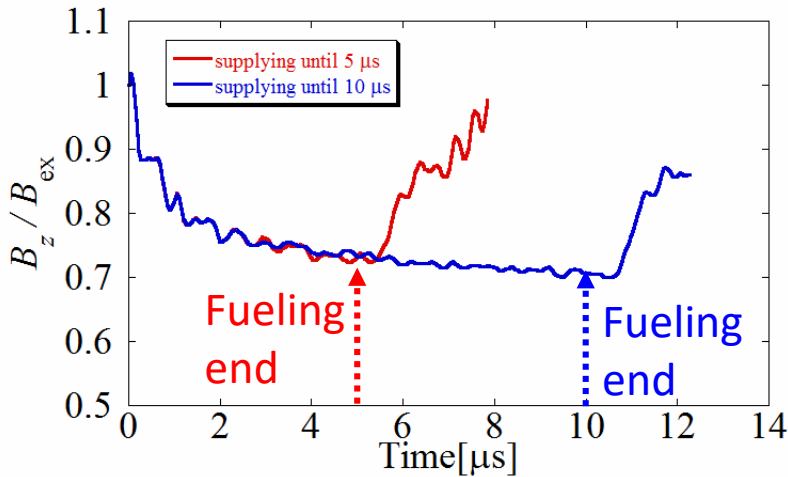
External field:  $B_{ex}=0.4$  T  
Ion temperature:  $T_i=50$ eV  
Ion fueling time  $5 \mu s$   
Calculation time  $10 \mu s$

As soon as fueling stops, ring configuration destroys due to axial diffusion and instability.

# Magnetic field weakening

External field:  $B_{ex}=0.4$  T  
 Ion temperature:  $T_i=50$ eV  
 Electron temperature:  $T_e=50$ eV  
 Ion fueling rate:  $S=1.4 \times 10^{26}$  [1/s]

External field:  $B_{ex}=0.4$  T  
 Ion temperature:  $T_i=50$ eV  
 Electron temperature:  $T_e=50$ eV  
 Ion fueling rate:  $S=1.4 \times 10^{27}$  [1/s]



Dominant component

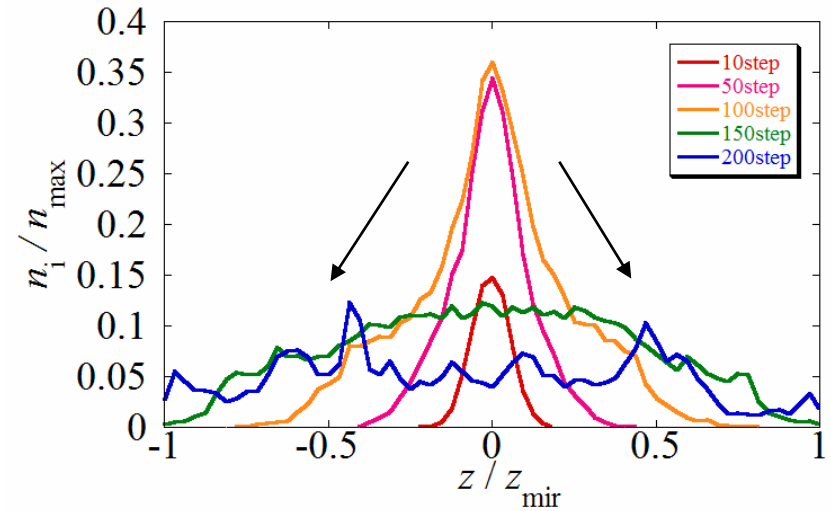
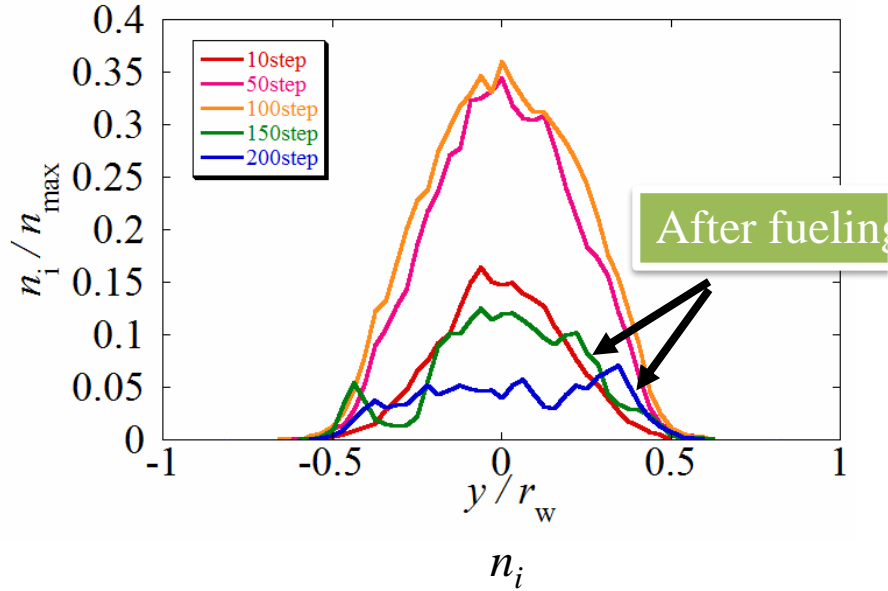
$$\mathbf{E} = \boxed{-\mathbf{u}_e \times \mathbf{B}} - \frac{\nabla p_e}{en_e} + \frac{\mathbf{R}_{ei}}{en_e}$$

1. Magnetic field weakening stops just after the end of fueling.
2. As the ion fueling rate increases, magnetic field weakening enhances. However, it can never produce the field-reversal.



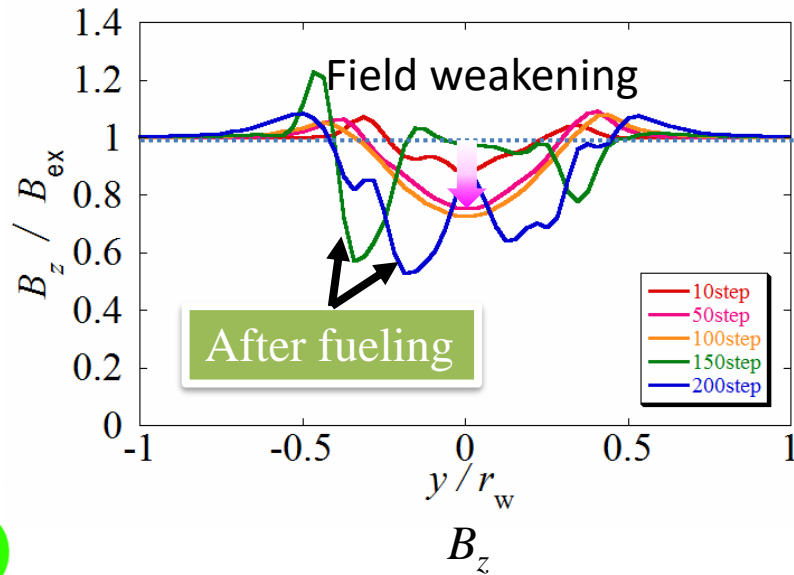
1step =  $5.0 \times 10^{-8}$  [s]

# Circular fueling results



Dispersion by axial diffusion

- Due to no field-reversal and no magnetic reconnection event, the circular plasma configuration destroys by the rapid axial diffusion.





# Summary

- 2D Resistive MHD / Hall MHD simulation has been carried out to study two FRC collision process.
- 3D MHD simulation has also been done for ST merging. After a ballooning instability, the merging process is observed.
- From our high-resolution MHD simulation, the following effects are essential to reproduce FRC merging
  - ✓ Cross-field thermal conductivity
  - ✓ Anomalous resistivity effect  
= requirement of full-particle simulation
- We can not succeed in producing an FRC plasma by giving only the pressure gradient.