Clearly Print Name:____________________________

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Classical Mechanics
Qualifying Exam
Fall 2017

You may consult only *Classical Mechanics* by Goldstein, Poole, and Safko. Do all four (4) problems. The exam is worth 50 points. Problem No. 4 is relativistic; the other three problems are non-relativistic. Write directly on this exam; do not use a blue book. Every other page has been left blank to provide extra work space. In order to receive credit you must show all of your work.

DO NOT OPEN THIS EXAM UNTIL YOU ARE TOLD TO DO SO

FOR ADMINISTRATIVE USE ONLY:

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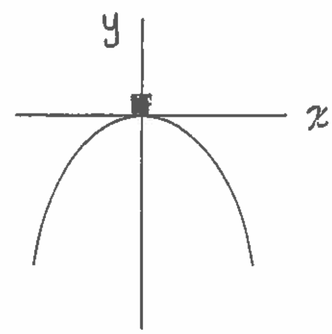
Total: ________________________

1. **[12 points]** A point mass is placed at the origin, where it rests on a two-dimensional track described by the equation:

$$y = -x^4/a^3$$

If the point mass is given an infinitesimal nudge to the right, find the value of x at which it leaves the track.



2. **[14 points]** A wire is bent into a semi-circular shape, described in terms of the usual spherical coordinates by:

$$\begin{aligned} r &= a\sin\theta \\ 0 \le &\ \theta \le \pi/2 \\ \phi &= 0 \end{aligned}$$

The wire is then rotated about the z-axis at the constant angular velocity ω_0 where:

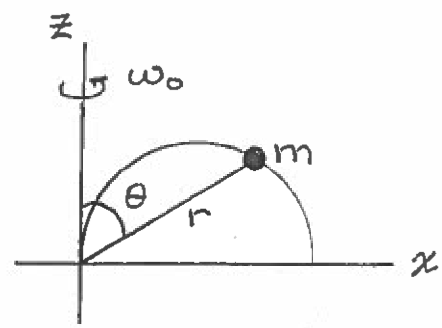
$$\omega_0^2 = \frac{25}{27}\frac{g}{a}$$

where g is the acceleration due to gravity. A bead of mass m slides on the wire. In this problem we are only concerned with initial conditions and times such that the bead remains on the wire and does not fly off (or has not yet flown off). The initial conditions are:

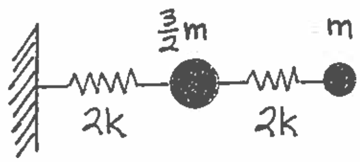
$$\begin{aligned} r &= 3a/5, \quad & \sin\theta = 3/5, \quad & \phi = 0 \\ \dot{r} &= 0, & \dot{\theta} = 0, \quad & \dot{\phi} = \omega_0 \end{aligned}$$

(a) Find the initial value of the second derivative $\ddot{r}$ in terms of g only.

(b) Find the initial value of the second derivative $\ddot{\theta}$ in terms of a and g only.



3. **[14 points]** All motion in this problem takes place on a horizontal table and is restricted to one dimension. The oscillations are small and there is no friction. Two springs and two masses are connected to each other and to a wall in the order: wall, spring ($2k$), mass ($\frac{3}{2}m$), spring ($2k$), mass (m), where the quantities in parentheses are the spring constants or masses as appropriate. Suppose the first mass ($\frac{3}{2}m$) is displaced a small distance a to the right of its equilibrium position and the second mass (m) is displaced a small distance x_2 from its equilibrium position (positive if to the right, negative if to the left). Both masses are released from rest. For each of the following values of x_2 state whether the system oscillates at a single, well-defined frequency, or at a superposition of two frequencies. If a single frequency, evaluate that frequency.



(a) $x_2 = a$

(b) $x_2 = -a$

(c) $x_2 = -3a/2$

(d) $x_2 = 3a/2$

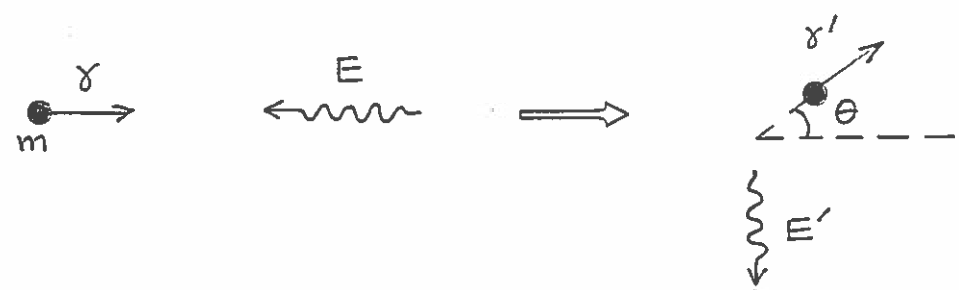
(e) $x_2 = 2a/3$

4. **[10 points]** An electron (mass m) traveling to the right along the x-axis with Lorentz factor $\gamma = 13/5$ collides with a photon of energy $E = 2m/5$ that is traveling nominally along the x-axis to the left. The collision is not head-on. After the collision there is a photon of energy E' traveling along the negative y-axis and an electron with Lorentz factor γ', moving in the x-y plane at an angle θ measured counter-clockwise from the positive x-axis.

   (a) Find the energy E' of the photon after the collision in terms of m and c.

   (b) Find (numerically) the Lorentz factor γ' of the electron after the collision.

   (c) Find (numerically) the angle θ associated with the electron after the collision.



Physics Qual - Statistical Mechanics

(Fall 2017)

I. Consider a harmonic oscillator with frequency ω.

a) Find the volume in classical phase space, $\Gamma_0(E)$, with energy less than E.

b) Find the number of quantum mechanical states, $\Omega_0(E)$, with energy less than E.

c) Show that for large E $\Gamma_0(E) \approx c\,\Omega_0(E)$. Determine the constant c.

II. Consider a gas made out of diatomic Nitrogen molecules. How does the specific heat behave as a function of temperature, and to what extent is the classical prediction incorrect? Would the same reasoning and formulas apply to a gas of ammonia molecules, and if not explain why.

III. Can the theory of a free Fermi gas be used to describe a neutron star? Describe the most important physical parameters that would enter into such a physical description, and how the results would differ from those obtained in a classical statistical mechanics treatment. Write down the relevant equation of state and discuss how it differs from the one appropriate for free bosons. Would a relativistic treatment make any difference?

Quantum Mechanics Ph.D. Qualifying Exam (Fall 2017)

Some useful integrals

$$\int_{-\infty}^{\infty} dx e^{-x^2} = \sqrt{\pi}\,, \quad \int_{-\infty}^{\infty} dx x^2 e{-x^2} = \sqrt{\pi/2}\,,$$

$$\int_{-\infty}^{\infty} dx x^4 e^{-x^2} = \frac{3}{4}\sqrt{\pi}\,, \quad \int_{-\infty}^{\infty} dx x^6 e^{-x^2} = \frac{15}{8}\sqrt{\pi}$$

1. In this problem we will consider perturbations of two dimensional harmonic oscillator. The unperturbed Hamiltonean is

$$H_0 = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{m\omega^2}{2}(x^2 + y^2)\,.$$

   (a) First, consider a perturbation

$$V_2 = \delta_2 m\omega^2 (x+y)^2\,,$$

   and find *exact* eigenststate and eigenenergies of the system with the Hamiltonean $H_0 + V_2$. (*Hint:* The best way to solve this problem is by diagonalizing the Hamiltonean.)

   (b) Now consider a perturbed system with the Hamiltonean $H_0 + V_4$, where

$$V_4 = \delta_4 (x+y)^4\,,$$

   Using the first order perturbation theory, find the correction to the energy of the ground state. Using the first order degenerate perturbation theory find corrections to the energies of the (doubly degenerate) first excited level. (*Hint:* Once again, a smart choice of coordinates will simplify the problem. The integrals on the title page could be useful.)

   (c) Assume that at time $t = 0$ the system is in the ground state of the Hamiltonean H_0. At time $t = 0$ the perturbation $V_4(t) = \delta_4(x+y)^4\cos(\omega' t)$ is turned on (assume that ω' is not close to ω). Write down the formula for the probability of transition to higher energy levels. Show that the probability of transition to states at the first excited energy level is zero. Show that only for one of the three degenerate states at the second excited energy level the probability of transition non-zero.

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2. Using variational method with the trial wave function

$$\psi = \begin{cases} (x-a)^2(x+a)^2, & |x| \leq a \\ 0, & |x| \geq a \end{cases}$$

calculate the upper bound for ground state energy of the harmonic oscillator and compare your result to $\hbar\omega/2$

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3. Consider a composite system made up of two spin 1/2 objects. For $t < 0$, the Hamiltonean does not depend on spin and can be taken to be zero by suitably adjusting the energy scale. For $t > 0$, the Hamiltonean is given by

$$H = \left(\frac{4\Delta}{\hbar^2}\right) \vec{S}_1 \cdot \vec{S}_2 \, .$$

Suppose the system is in $|+-\rangle$ state for $t \leq 0$. Find, as a function of time, the probability of being found in each of the following states $|++\rangle$, $|+-\rangle$, $|-+\rangle$, $|--\rangle$ by

(a) Solving the system exactly.

(b) Solving the problem by assuming validity of first order time-dependent perturbation theory with H as a perturbation switched on at $t = 0$. Under what condition does (b) give the correct results?

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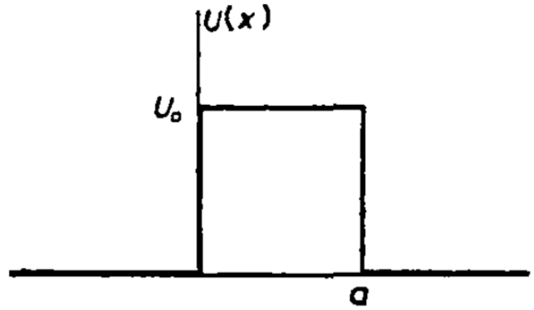
4. Determine the transmission coefficient for a rectangular barrier shown in the Figure (both when $E > U_0$ and when $E < U_0$).



EM Qualifying Exam Fall 2017

1. A charge Q is placed at the position $(a, 0, 0)$, while the plane $x = 0$ is held at the potential $V = V_0 e^{-(y^2+z^2)/b^2}$. Find the potential everywhere. Find the charge distribution on the plane.

2. An infinite wire passes through the origin and lies along the z-axis. An observer is at a point $(x, 0, 0)$. Initially, there is no current through the wire. At $t = 0$ a uniform current $I = I_0$ starts to flow and does not change further with time.

Find the magnetic field seen by the observer as a function of time. Do not ignore the speed of light.

3. A charge q is at $(0, 0, b + a\cos(\omega t))$, while another charge q is at $(0, 0, -b - a\cos(\omega t))$. Find the differential power distribution to lowest order in the frequency.

4. A time dependent dipole $p = p_0 \cos(\omega t)$ sits at the origin. Find the scalar and vector potentials everywhere.