Classical Mechanics Qualifying Exam

1. (a) A particular particle decays in 10 ns in its own rest frame. Suppose a particle accelerator produces the particle with total energy E=100 mc^2, where m is its mass. (a) How far in meters (say within a few percent) will it travel before decaying? (b) A different particle has a kinetic energy equal to its mass energy. If it travels a distance D before decaying, how long did it live in its own rest frame?

2. A flat object has a 2D mass density $\rho(x, y)$ so that its total mass is $M = \int dxdy\ \rho(x, y)$. It is known that $\int dxdy\ \rho(x, y)x = 0 = \int dxdy\ \rho(x, y)y$.

(a) Find the location of the body's center of mass.

(b) For rotations in the xy plane about the origin, the relevant moment of inertia is I_{zz}, and it is known that $I_{zz} = B$, where B is a measured number. We now move the object in the plane by a translation (a, b), which means that $\rho \to \rho(x - a, y - b)$. Find the new value of I_{zz}^{new}, still relative to the origin, in terms of B, a, b, and M.

(c) Suppose the body is tilted up to lie in a vertical plane and allowed to rotate in that plane about a horizontal axle $\perp$ to the plane and through some point which is not its center of mass. It has a stable equilibrium point where the center of mass is directly below the axle point. For small oscillations, find the frequency, in terms of its moment of intertia about the axle point I, its mass M, and the distance d between the axle and center of mass. (Part (c) does not depend on the results of (a) and (b)).

(d) Now using the results of (a) and (b), find the frequency of our object's rotation in terms of B, m, d, and the gravitational acceleration g.

Qualify Exam

1. N non-interacting particles each with magnetic moment $m\mu_B$ (m=-J,-J+1,…,J-1,J, and J is an integer) are placed in a magnetic field B at temperature T. Find
   a) the partition function Q_N,
   b) the magnetization M and magnetic susceptibility χ
   c) and asymptotic form of M and χ and high T ($kT>>\mu_B B$).
   d) entropy as a function of B (make a plot).

2. Develop the 2D Debye model to calculate the C_v of layered materials such a single carbon sheet (graphene). Analyze the high-T and low-T results.

3. (a) Find the chemical potential of a classical ideals gas that contains N indistinguishable particles. The mass of each particle is m, the volume of the system is V and the temperature is T.

   (b) Find the chemical potential of M indistinguishable classical particles that are trapped on a surface as a two-dimensional ideal gas with an energy-momentum relationship $\varepsilon=p^2/2m-\varepsilon_0$, where ε_0 is the adsorption energy.

   (c) Find the equilibrium coverage (i.e., M/A) of this gas on the surface under a temperature T and a pressure p_0 on top of the surface.

4. Consider an ideal Bose gas confined to a region of area A in *two* dimensions. Express the number of particles in the excited states, N_e, and the number of particles in the ground state, N_0, in terms of z, T, and A, and show that the system does not exhibit Bose–Einstein condensation unless $T \rightarrow 0$ K.

Quantum Qualifying Exam Fall 2024

1. A particle is in a square well potential; the potential is

$$V = \begin{cases} \infty & x \leq 0 \\ 0 & 0 < x < L \\ \infty & x \geq L \end{cases} \tag{1}$$

At time $t = 0$, the particle is in the left half of the box; the wavefunction of the particle is

$$\psi(x) = \begin{cases} N \sin(2\pi x/L) & 0 < x < L/2 \\ 0 & L/2 < x < L \end{cases} \tag{2}$$

where N is a normalization constant.

Find an expression for the probability that the particle is at $x = 3L/4$ at time $t = t_0$. **You do not need to perform the sums or integrals that may occur.**

2. Consider a harmonic oscillator in two dimensions, with a perturbation. The Hamiltonian is

$$H = \frac{p_x^2 + p_y^2}{2m} + \frac{m\omega^2}{2}(x^2 + y^2) + \epsilon x^2 y^2 \tag{3}$$

Treat ϵ as a small parameter. Find the ground state energy to linear order in ϵ. Perform all integrals and sums that may appear.

3. Consider a hydrogen atom in the limit where the proton mass is infinite, and the proton is at the origin. The electron is initially in the ground state, It is subjected to a time dependent perturbation

$$H_1 = \begin{cases} \epsilon E x^2 t & 0 < t < 1 \\ 0 & \text{else} \end{cases} \tag{4}$$

What is the probability that at $t = 2$, the electron is in

(a) the first excited state?

(b) the second excited state?

Name:_____ ID#_____

Electrodynamics Comprehensive Exam
Fall 2024

You may use any intermediate results in the textbook. No electronic devices (calculator, computer, cell phone etc) are allowed.

For Administrative use only:

#1:_____

#2:_____

#3:_____

#4:_____

#5:_____

Total:_____

1. (20pts) A conducting sphere of radius *a* carries a total charge of *q*. It is surrounded by a dielectric material of permittivity $\epsilon = 2\epsilon_0$ out to radius *b*. Find the charge density at the outer surface *r=b* and the potential on the conducting sphere.

2. (20pts) An infinitely long, thin wire parallel to the *z*-axis at a position of *x=d>0* and *y=0* carries a current of *I* flowing in the positive *z*-direction. The *y*-*z* plane at *x=0* is a conducting wall. Find the magnetic field at *x>0* and the force per unit length acting on the wire.

3. (20pts) A hollow infinite cylinder of radius *a* carries a time-dependent, uniform, surface current density: $\mathrm{K} = K_0 e^t \mathrm{e}_\phi$, where K_0 is a constant, *t* is time, e_ϕ is the unit vector in the azimuthal direction. The time variations are slow so that you can ignore the displacement current. Find the magnetic field and electric field inside and outside of the cylinder.

4. (20pts). Electromagnetic wave oscillates in the interior of a cylindrical resonant cavity with a length of *L* and a radius of *R*, and $\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right) f = 0$ with boundary conditions of $\partial f / \partial \mathrm{z} = 0$ at *z=0* and *z=L*, and *f=0* at *r=R*. Find the allowable wave frequency.

5. (20pts) A uniform magnetic field of magnitude B_0 points in the +z direction. Into this field is placed an infinitely long solid cylinder of radius *a* and magnetic permeability $\mu = 2\mu_0$ centered on the x-axis. There are no free currents. Find the magnetic field inside the cylinder.