Classical Mechanics Qualifying Exam

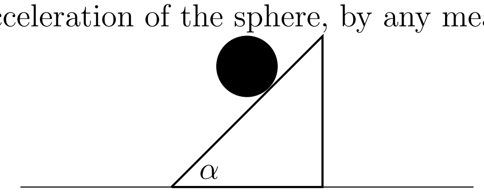
1. A pion ($mc^2$=140 MeV) decays to a muon ($mc^2$=106 MeV) and a neutrino ($mc^2$=0) with a lifetime of $26 \times 10^{-9}$ s.

(a) In the rest frame of pion, what are the momenta and energies of the muon and neutrino?

(b) In the lab frame, the pion has a momentum 500 MeV/c. What will be its average lifetime?

(c) What are the maximum and minimum energies of the neutrino?

2. A triangular wedge of mass m, and slope angle $\alpha$ sits on a frictionless table. A solid sphere of radius $R$, mass $M$ ($I_{CM} = \frac{2}{5}MR^2$) rolls down the wedge (there is friction between the sphere and the wedge). Find the angular acceleration of the sphere, by any means you like.



# Qualify Exam: Statistical Mechanics

1. For a system with N identical molecules that have three internal motion energy levels: 0, $\varepsilon$, and $10\varepsilon$, find the contributions of internal motions to the
   a. average internal energy, $<E>$,
   b. specific heat, $C_v$.

2. Consider a system of $N$ localized atoms, each of which has an intrinsic magnetic moment $m$. The system is in equilibrium with a thermal reservoir at the temperature $T$ and the interaction between atoms is negligible. When an external uniform magnetic field $\boldsymbol{H}$ is applied, the Hamiltonian of the system is

$$-\mu H \sum_{i=1}^{N} \cos\theta_i$$

where $\theta$ is the angle between a magnetic moment and the magnetic field.
a) Write out the partition function of the system.
b) Show that the mean magnetic moment

$$M_z = N < \mu\cos\theta >= N\mu\{\coth(\frac{\mu H}{kT}) - \frac{kT}{\mu H}\}$$

c) At high temperature, show the magnetic susceptibility $\chi$ satisfies Curie's law $\chi \propto T^{-1}$.
(Hints 1: Langevin function $L(x) \equiv \coth x - \frac{1}{x} \approx \frac{x}{3} - \frac{x^3}{45} + \ldots$ at small $x$; 2: the magnetic susceptibility $\chi = \frac{\partial M_z}{\partial H}$)

3. For an ideal gas with N non-interacting relativistic particles $\varepsilon$=pc in a box with a volume V and temperature T,
   a. Calculate its entropy
   b. Find its internal energy and equation of state
   c. Compute its chemical potential as a function of T and P

4. Show that the total zero-point energy of a Debye solid is equal to $9Nk\Theta_D/8$ (show you work from the Debye model).

# Quantum Mechanics Qualifying Exam – Fall 2019

The number of points the problem is worth is in brackets.

1. Consider the Sommerfeld gas model for electrons in a solid, but in the case of a two-dimensional (2D) solid. (This can exist, for example, single layer graphene, which is of interest in solid state physics.) In this case,

$$V(x,y) = \begin{cases} 0, & \text{if } 0 < x < l_x,\ 0 < y < l_y; \\ \infty, & \text{otherwise.} \end{cases}$$

   **(a)** Write down the normalized wave function for this case. [5]

   **(b)** What are the allowed quantized energies? [5]

   **(c)** What is the area ("volume") of each state in $k$-space? [5]

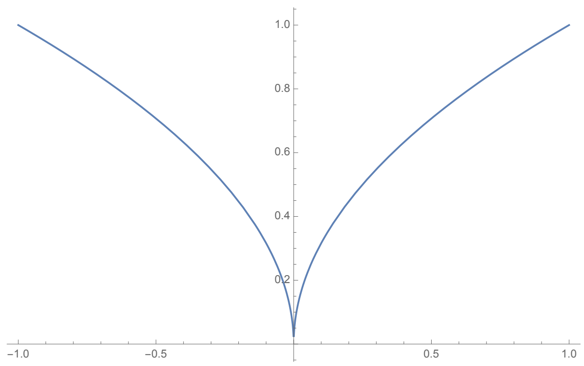
   **(d)** The energy for a given wavenumber is still $E = \hbar^2 k^2/2m$, where $k$ is the magnitude of the wave vector $\mathbf{k}$. Calculate the *total* energy of the 2D gas (in its lowest energy state). [10]

2. Using a Gaussian trial wave function,

$$\psi(x) = Ae^{-bx^2}$$

where $A = (2b/\pi)^{1/4}$ and the variational principle, find the lowest upper bound on the ground state energy of a root potential, plotted below:

$$V(x) = \alpha\sqrt{|x|}.$$




These integrals may be useful.

$$\int_{-\infty}^{\infty} e^{-2bx^2}\,dx = \sqrt{\frac{\pi}{2b}}, \quad \int_{-\infty}^{\infty} x^2 e^{-2bx^2}\,dx = 2\sqrt{2\pi b}, \quad \int_{0}^{\infty} \sqrt{x}e^{-2bx^2}\,dx = \frac{\Gamma(3/4)}{2^{7/4}b^{3/4}},$$

where $\Gamma(z)$ is the gamma function, and $\Gamma(3/4) \approx 1.23$. [10]

3. Consider a quantum system with just three linearly independent states. Suppose the Hamiltonian, in its matrix form, is

$$\mathbf{H} = V_0 \begin{pmatrix} 3 & \epsilon & 0 \\ \epsilon & 1 & 0 \\ 0 & 0 & 1-\epsilon \end{pmatrix}$$

with $V_0$ as a constant term and $\epsilon \ll 1$ being a perturbative small parameter.

**(a)** Write down the eigenvectors and eigenvalues of the *unperturbed* Hamiltonian, with $\epsilon = 0$. Note that there will be two degenerate energies and one unique energy. [10]

**(b)** Solve for the *exact* eigenvalues of $\mathbf{H}$ (with $\epsilon$ non-zero). [5]

**(c)** Write down the perturbation Hamiltonian matrix $\mathbf{H}'$. Remember, $\mathbf{H} = \mathbf{H}^0 + \mathbf{H}'$. [5]

**(d)** Use first and second-order *non*degenerate perturbation theory to find the first $E_1^1$ and second order corrections due to $\mathbf{H}'$ found in the last part of this problem to the non-degenerate state found in part (a) of this problem. [5]

**(e)** Use *degenerate* perturbation theory to find the first-order correction to the two initially degenerate energy eigenstates found in part (a). [5]

4. An electron is in the spin state

$$\chi = A \begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

**(a)** Determine the normalization constant $A$. [5]

**(b)** Find the expectation values of the spin along the axes, $S_x$, $S_y$, and $S_z$. [5]

**(c)** Find the "uncertainties" $\sigma_{S_x}$, $\sigma_{S_y}$, and $\sigma_{S_z}$, where these sigmas are standard deviations, not Pauli matrices. [5]

**(d)** Are these results consistent with all three uncertainty principles encompassed by the relation [5]

$$\sigma_{S_i}\sigma_{S_i} \geq \frac{\hbar}{2}|\langle S_k \rangle|?$$

Fall 2019 E&M Qualifying Exam

1. A hollow spherical shell has inner radius $a$ and outer radius $b$. The shell is filled with a dielectric of dielectric constant $K$. A charge $Q$ is uniformly distributed through the dielectric. Find (a) the electric field (b) the energy of the system.

2. A plane capacitor has a plate area $A$ and a plate separation $h$. It is charged by a battery to a potential difference $V$. It is then disconnected from the battery, and without disturbing the charges, it is filled with a dielectric of dielectric constant $K$. Find (a) the final charge (b) the work done in introducing the dielectric (c) the final force on each plate.

3. A solenoid is aligned along the $z$-axis. It has $n$ turns per unit length, and carries a current $I$. It now moves in the $x$-direction at a relativistic speed $v$. Find the electric and magnetic fields in the solenoid.