Classical Mechanics Qualifying Exam

1. (a) A particular particle decays in 26 ns in its own rest frame. Suppose a particle accelerator produces the particle with total energy E=100 mc^2, where m is its mass. (a) How far in meters will it travel before decaying? (b) A different particle has a kinetic energy equal to its mass energy. If it travels a distance D before decaying, how long did it live in its own rest frame?

2.(a) A satellite in orbit has a height over the earth of $r_e/5$ at perigee, and a height of $2r_e$ at apogee. (Here r_e is the Earth's radius.) Find the orbit's eccentricity. (b) If the Earth is at the origin, and the orbit takes place in the x-y plane, and the major axis of the ellipse is along the x axis, find the height of the satellite above the earth when it crosses the y-axis. (c) Assume Earth's orbit around the sun to be circular. If, hypothetically, the sun's mass is suddenly decreased by half, what orbit will the Earth have after this happens? What will its eccentricity be? Will the Earth escape the solar system?

Statistical Mechanics (2021 Spring)

1. (30 points) A system of gas of N indistinguishable particles with zero rest mass is contained in volume V at temperature T. The energy-momentum relation is $E= pc$. Find the equation of state and internal energy of the gas, and compare it to non-relativistic ideal gas (Note: one can use $\int_0^\infty x^2 e^{-x} dx = 2$ and Stirling's approximation $lnN! \approx NlnN - N$).

2. (30 points) Calculate the root-mean-square fluctuation $\sqrt{\langle \tilde{N}_k{}^2 \rangle} \equiv \sqrt{\langle N_k^2 \rangle - \langle N_k \rangle^2}$ of the occupancy N_k at a certain energy level ε_k for: (i) a classical particle; (ii) a boson; and (iii) a fermion.

3. (40 points) For an ideal non-relativistic gas confined in volume V at temperature T, the difference between the specific heats at a constant pressure P and at a constant volume V is written as:

$$C_P - C_V \equiv T[\left(\frac{\partial S}{\partial T}\right)_P - \left(\frac{\partial S}{\partial T}\right)_V]$$

(i) With the equation of state $P=P(V,T)$ of the system, show that

$$C_P - C_V = -T\frac{(\partial P/\partial T)_V^2}{(\partial P/\partial V)_T} \equiv \frac{VT}{K}\left(\frac{\partial P}{\partial T}\right)_V^2,$$

with $K \equiv -V(\frac{\partial P}{\partial V})_T$ he reciprocal compressibility

(ii) For the degenerate ideal 3D Fermi gas, use the relation from (i) to show $C_P - C_V \propto T^3$ for low temperature T in the leading or first order approximation ($k_B T \ll \varepsilon_F$ with ε_F the Fermi energy; ignore the weak temperature dependence of K), and compare the results with that from the ideal classical gas.

1. Consider a two-level system with the Hamiltonian $H = E_0 \begin{bmatrix} 1 & \lambda \\ \lambda & -1 \end{bmatrix}$ where $\lambda \ll 0$.

   (a) Why should λ be real? [2]

   (b) When $\lambda = 0$, what are the energy eigenvalues and eigenvectors? [8]

   (c) Calculate the corrections to the energy eigenvalues and eigenvectors to the lowest *non-vanishing* order in λ? [12]

   (d) Find the exact energy eigenvalues for $\lambda \neq 0$. Show that an expansion in λ correctly recovers the answer you found in the previous part. [8]

2. Consider a two-particle state of Spin-1 and Spin-1/2 particles given by $|s_1 = 1, m_1 = 1, s_2 = 1/2, m_2 = -1/2\rangle$, i.e., the spins of the two particles are anti-parallel. Total spin is given by $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$. Denoting the eigenvalue of S^2 by $s(s+1)\hbar^2$, what are the possibilities for s when the total spin is measured? What are the probabilities of obtaining those s values? [12]

3. Consider a particle of mass m with Hamiltonian $H_0 = p_x^2/(2m) + (1/2)m\omega^2 x^2$ at $t < 0$. Assume that a perturbation of the form $V_1 = F_0 x$ is applied for $t \geq 0$.

   (a) Calculate the correction to the energy of the first excited level to the lowest *non-vanishing* order in perturbation? [8]

   (b) Find the exact energy levels at $t > 0$, i.e., the exact energy eigenvalues for the Hamiltonian $H = H_0 + V_1$. Compare to the answer you found in the previous part and briefly discuss what this tell you? [8]

   (c) Assume that the particle is in the ground state at $t < 0$. What are the probabilities of finding the system in the first and second excited states after the perturbation V_1 is turned on? [8]

   (d) Discuss the condition under which first-order time-dependent perturbation theory that you used in the previous part is valid. [4]

EM Qual Spring 21

1. A magnetic field occupies the region $x > 0$ and has the form $B = B_0\hat{z}$ in that region. A screen lies in the y-z plane at $x = x_0$ where x_0 is positive (i.e. the screen is in the region where the B-field is nonzero).

An electron (charge e, mass m) is travelling along the x-axis at a speed v when it enters the region where the B-field is nonzero.

(a) Show that there is a minimum speed so that the electron hits the screen.

(b) If the speed is large enough, where does it hit the screen?

(c) What happens if the speed is lower than the minimum speed required to hit the screen?

2. A sphere of radius R carries a charge distribution $\rho = \rho_0(1 + \cos\theta)^2$.

(a) Find the charge and dipole moment.

(b) A dipole $\vec{p} = p_1\hat{x} + p_2\hat{y}$ is placed far from the sphere at the point $(a, 0, b)$ where a, b are much larger than R. Find the force on the dipole to leading order in the inverse distance.

(c) Find the exact potential outside the sphere.

(d) The charge on the sphere is rearranged so that the charge distribution becomes spherically symmetric. What is the change in energy?

3. A charge oscillates along the x-axis; its position is $(x_0\cos(\omega t), 0, 0)$. Find the radiated power in the direction of the vector (1,1,1).