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Classical Mechanics
Qualifying Exam
Spring 2017

You may consult only *Classical Mechanics* by Goldstein, Safko, and Poole. Do all four (4) problems. The exam is worth 50 points. Problem No. 4 is relativistic; the other three problems are non-relativistic. Write directly on this exam; do not use a blue book. Every other page has been left blank to provide extra work space. In order to receive credit you must show all of your work.

DO NOT OPEN THIS EXAM UNTIL YOU ARE TOLD TO DO SO

FOR ADMINISTRATIVE USE ONLY:

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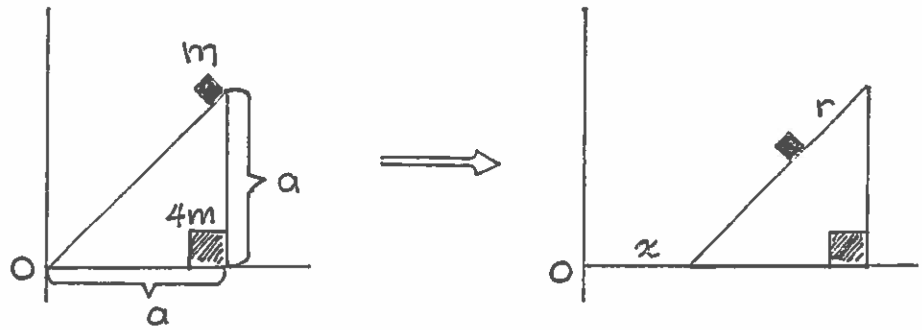
Total: ______________________________

1. **[14 points]** A triangular block has sides of length a, a, and $\sqrt{2}a$, respectively. One of its short sides rests on the x-axis, with a 45° vertex initially at the origin and the right angle vertex initially at $x = a$. The block is not uniform; all of the mass, which is equal to $4m$, is located at the right angle vertex. A particle of mass m starts out at the upper 45° vertex and slides down the hypotenuse of the block as the block slides to the right. For the variables in this problem we use x, the location on the x-axis of the vertex that was originally at the origin, and r, the distance along the hypotenuse from the upper vertex to the point mass. As noted, initially we have $r = 0$; r increases as the particle slides down the block. There is no friction in the problem and the system is released from rest.

   (a) Find the initial value of $\ddot{r}$, the second derivative of r.

   (b) Find the initial value of $\ddot{x}$, the second derivative of x.

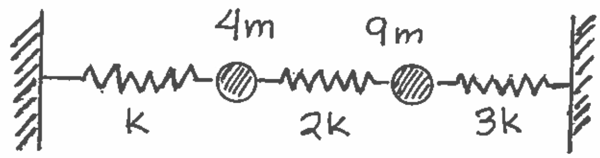
   (c) What is the x-coordinate of the particle when it hits the x-axis?



2. **[14 points]** The motion in this problem all takes place on a horizontal table and is restricted to one dimension. Three springs and two masses are connected between two walls in the order: wall, spring (k), mass ($4m$), spring ($2k$), mass ($9m$), spring ($3k$), wall, where the quantities in parentheses are the spring constants or masses as appropriate. At equilbrium all of the springs are unstretched.

(a) Find the frequencies of the normal modes.

(b) Suppose the mass $4m$ is displaced to the right from its equilibrium position by a small distance a. By what distance should the mass $9m$ be displaced from its equilibrium position so that the system, when released from rest, will oscillate at the *higher* frequency only? Indicate the direction of this latter displacement by the sign of your answer: positive for right, negative for left.



3. **[12 points]** For simplicity, dimensions have been suppressed throughout this problem. Numerical answers are expected.

A particle of mass $m = 1$ moves in the x-y plane under the action of an attractive central force. The corresponding potential energy is:

$$V = -k/r$$

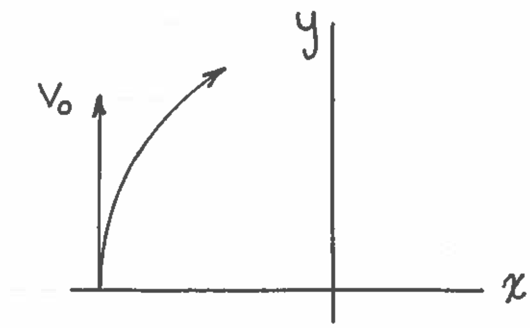
where $k = 36$ and r is the distance from the origin. At time $t = 0$ the particle is at $x = -8$, $y = 0$, and its velocity is $v = 3$ in the positive y-direction.

(a) At what value of y does the particle cross the y-axis (for $t > 0$)?

(b) What is the value of the velocity component v_x corresponding to Part (a)?

(c) What is the value of the velocity component v_y corresponding to Part (a)?

(d) Either find the distance from the origin to the point where the particle crosses the line $y = x$ (for $t > 0$) or prove that it never crosses this line.



4. [10 points] An X-particle at rest decays into particles A, B, and C:

$$X \rightarrow A + B + C$$

The mass of the X-particle is $6m$, the mass of the A-particle is $3m$, and the mass of the B-particle is $2m$. The C-particle is massless and travels at the velocity of light.

(a) If after the decay the B-particle is at rest, find the energy E_C of the C-particle and the Lorentz factor γ_A of the A-particle.

(b) If instead after the decay the A-particle is at rest, does the C-particle have more or less energy than in Part (a)? Explain briefly or calculate explicitly.

Physics Qual - Statistical Mechanics

(Spring 2017)

I. Determine the specific heat of a system of N independent, two - dimensional harmonic oscillators each of which has $(n+1)$-fold degenerate energy levels

$$\epsilon_n \;=\; (n+1)\hbar\omega \quad (n = 0, 1, 2, \ldots)$$

II. It is possible, in a ferromagnet, to have what are called spin waves. These correspond to waves where the individual atomic spins oscillate in time. As in the case of lattice vibrations, which give rise to phonons, these waves can be quantized and also give rise to propagating particle-like objects. For low frequencies spin waves have a dispersion law $\omega(k) = c\,k^2$, with c some positive constant. Here we will consider the case of N such (non-interacting) particles in a large box of volume V and temperature T.

(a) Compute the average energy, entropy and specific heat for such a system. How do these quantities behave for very low temperatures?

(b) If the ferromagnet in question is a metal, how does the spin wave specific heat compare to the electronic contribution at low temperatures ?

III. Find the photoelectric emission current provided the electrons obey the Fermi-Dirac statistics, and assuming that the work function for an electron escaping the metal is W. Describe the constraints that arise due to the fact that the electrons are emitted from the metal's surface, and how this affects the resulting probability integrals. Are there any sensible approximations that one could apply to a metal at room temperature? What does the answer look like if you assume that $W - \mu \gg kT$ where μ is the chemical potential? Would the answer change if the electrons were treated relativistically?

Quantum Mechanics Ph.D. Qualifying Exam (Spring 2017)

1. Consider two identical spin 1/2 particles with mass m in a one dimensional infinite well with length L. The particles interact weakly with a potential $V = V_0\delta(x_1 - x_2)$.

   (a) What is the energy and wave-function of the ground state in the absence of the interaction V? Include both position and spin dependence in your answer.

   (b) What is the energy of the first excited level? What is the degeneracy of the first excited level? Write down wave-functions (including position and state dependence) for the states with this energy.

   (c) Using the first order perturbation theory, find the corrections to the ground state energy found in (a) due to potential V. (You do not have to evaluate the integrals explicitly.)

   (d) Does the perturbation V lift degeneracy of the first excited level? If so, what is the remaining degeneracy? Give the detailed explanation.

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2. A spin-1/2 particle is moving in a harmonic oscillator potential with frequency ω (i.e. $H = \hbar\omega(a^\dagger a + 1/2)$). The eigenstates of the system are doubly degenerate and can be described by $|n,\uparrow\rangle$ and $|n,\downarrow\rangle$, where n is the energy level of the harmonic oscillator. This double degeneracy is lifted by the perturbation

$$V = \frac{\delta}{2}(aS_+ + a^\dagger S_-)\,,$$

where $S_\pm = S_x \pm S_y$ are spin raising and lowering operators.

(a) Using the second order perturbation theory, find the corrections to energies of the states $|0,\uparrow\rangle$ and $|0,\downarrow\rangle$.

(b) Now find the exact eigenstates of the system in terms of $|n,\uparrow\rangle$ and $|n,\downarrow\rangle$. While you can find general expressions, it is sufficient to find two eigenstates that turn into $|0,\downarrow\rangle$ and $|0,\uparrow\rangle$ as $\delta \to 0$. Find the energies of these two states and determine which them corresponds to the ground state of the system.
Note: you may find eigenvalues and eigenvectors of a matrix

$$\begin{pmatrix} (2n+1)a & b \\ b & (2n+3)a \end{pmatrix}$$

to be useful. These eigenvalues and eigenvectors are given respectively by

$$\lambda_\pm = 2a(n+1) \pm \sqrt{a^2+b^2}$$

and

$$v_\pm = \mathcal{N}_\pm(a \mp \sqrt{a^2+b^2}, -b)\,,$$

where $\mathcal{N}_\pm = 2((a^2+b^2) \mp a\sqrt{a^2+b^2})$ are normalization coeffficients.

(c) Assuming that the perturbation is turned on at $t = 0$ and at that time the system is in the state $|0,\uparrow\rangle$ find the probability of observing the system at a later time t in the state $|1,\downarrow\rangle$. Use the first order perturbation theory.

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3. In class we discussed the upper bound on ground state energy of the hydrogen atom obtained by using a variational method with a trial function $\exp(-r^2/a^2)$. Choosing a reasonable modification of this trial wave-function find an upper bound on the energy of $(n, l, m) = (2, 1, 0)$ state of the hydrogen atom. Explain why variational method can be used for this state but could not be used for any other $l = 1$ state with $n > 2$ nor why it could be used for $n = 2$, $l = 0$ state.

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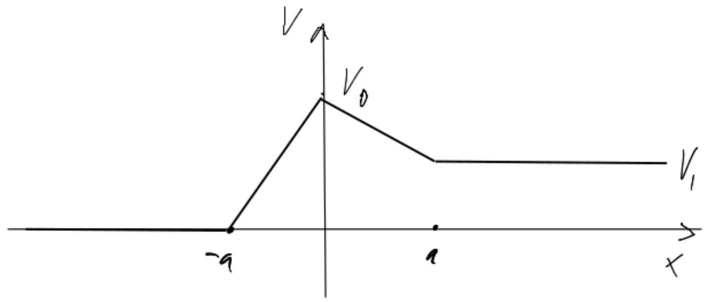
4. A particle of mass m is incident on the barrier given by (see the Figure)

$$V(x) = \begin{cases} 0, & x < -a \\ V_0(1 + \frac{x}{a}), & -a < x < 0 \\ V_0 - (V_0 - V_1)\frac{x}{a}, & 0 < x < a \\ V_1, & x > a \end{cases},$$

from the left with energy E such that $V_0 > E > V_1$. Using WKB approximation determine the transmission coefficient.



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EM Qualifying exam

1. A rod of length L is oriented along the x-axis and centered at the origin. It carries a time varying current of the form $I = I_0 \sin(\frac{2\pi x}{L}) \cos(\omega t)$. Find

(a) the charge density as a function of time.

(b) an exact expression for the scalar and vector potentials at the location $(0, L, 0)$ at the time $t = 0$

(c) the power radiated as a function of angle at long wavelengths.

2. There is an infinite grounded conducting plane at $x = 0$. An electric dipole of strength $p = qa$ is centered at $(x_0, 0, 0)$ and oriented along the $-\hat{i} + \hat{j}$ axis. Find

(a) the charge distribution on the plane

(b) the torque on the dipole.

3. An insulating sphere carries charge $\rho = \rho_0 \cos\theta \sin\phi$. It is then rotated slowly at angular frequency ω about the z-axis. Find the magnetic fields everywhere.