Name:________________________________

Student ID:____________________________

Classical Mechanics Comprehensive Exam

Spring 2018

You may use any intermediate results in the textbook. No electronic devices (calculator, computer, cell phone etc) are allowed.

For Administrative use only:

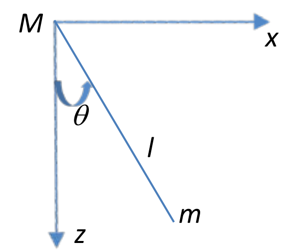
#1:____________________

#2:____________________

#3:____________________

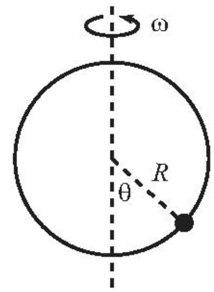
Total:__________________

1. (20pts) **Moving pendulum--** A pendulum of point mass m and massless rod of length l is attached to a point mass M that can move freely on a horizontal x-y plane. At time $t=0$, both particles are at rest and the rod forms an angle $\theta=\theta_0$ with the z-axis on the x-z plane. The gravitational acceleration g is in the positive z-direction. Derive (A) equation of motion for $\theta(t)$, (B) solution of $\theta(t)$ if $\theta_0 \ll \pi$, and (C) pendulum frequency if $m \ll M$ in part (B).



2. (15pts) $\boldsymbol{\Psi}$ **particle--** The particle called Ψ was discovered in an electron-positron collision: $e^- + e^+ \rightarrow \Psi$. The mass of electron (or positron) is m_e and Ψ particle is $6.2 \times 10^3 m_e$. Calculate the minimal electron energy (in terms of m_e and speed of light c), (*A*) if positron and electron approach each other with the same speed, and if (*B*) positron is at rest.

3. (15pts) **Bead on a rotating hoop--** A bead of mass m is free to slide along a frictionless hoop of radius R. The hoop rotates with constant angular speed ω around a vertical diameter. The gravitational acceleration g points downward. (A) Find the Hamiltonian H in terms of the angle θ shown in the figure and its conjugate momentum p, and then write down Hamilton's equations. (B) Is H the energy? Is H conserved? Why?



Physics Qual - Statistical Mechanics

(Spring 2018)

I. Experiments show that the specific heat of some (three-dimensional) substance can be fitted quite closely to a law $C_V = a\,T^n$ at very low temperatures. A model is proposed describing a gas of collective excitations which obey Bose-Einstein statistics. The long-wavelength dispersion for these excitations is $\omega(k) = b\,k^\alpha$.
(a) What is the value of α required to obtain the power n?
(b) Under what conditions does Bose-Einstein condensation occur?

II. Metallic sodium can be thought of as being made up of a lattice of sodium atoms and a gas of free electrons (each sodium atom contributes on the average one free electron).

a) What is the classical value for the specific heat of a piece of sodium containing N atoms?

b) Explain why the classical expression for the contributions of the lattice and the electron gas to the specific heat are both incorrect at room temperature.

c) What is the correct expression for the specific heat at room temperature?

III. Many organic molecules can form very large rings which act like one dimensional "raceways" for nearly free electrons. Assume such an idealized ring has a radius R which is fairly large compared to a single atom. Suppose this ring has N electrons. Find the Fermi energy E_F of the electrons on the ring as a function of N, R, and other necessary physical constants (such as the mass of the electron m).

Quantum Mechanics Ph.D. Qualifying Exam (Spring 2017)

Show all your work. Make sure to explain all your answers, especially if the answer does not require a calculation.

Some useful integrals

$$\int_{-\infty}^{\infty} e^{iax} e^{-bx^2/2} dx = \sqrt{\frac{2\pi}{b}} e^{-\frac{a^2}{2b}} , \qquad \int_{-\infty}^{\infty} e^{iax} x e^{-bx^2/2} dx = \frac{i(2\pi)^{1/2} a}{b^{3/2}} e^{-\frac{a^2}{4b}}$$

1. (25 points) In this problem we will consider a harmonic oscillator of mass m and frequency ω. You may want to recall wave-functions of the two lowest energy eigenstates:

$$\psi_0 = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega x^2}{2\hbar}}, \qquad \psi_1 = \sqrt{2}\left(\frac{m\omega}{\pi^{1/3}\hbar}\right)^{3/4} x e^{-\frac{m\omega x^2}{2\hbar}}$$

   (a) Multiple measurements of energy are performed. At the start of each measurement the oscillator is in the state $\psi(x) = \frac{\sqrt{3}}{2}\psi_0(x) + \frac{1}{2}\psi_1(x)$. What values of energy can be observed in each individual measurement? Find the average value of energy measured in a series of experiments.

   (b) Multiple measurements of momentum are performed. At the start of each individual measurement the oscillator is in the ground state. What values of momentum can be observed in each individual measurement? Find the probability that a single measurement observes the momentum in an interval between p_0 and $p_0 + dp$ (for some fixed p_0 and small dp). Find the average value of the momentum observed in multiple experiments.

   (c) Assume that the oscillator is in the ground state at time $t = 0$. At time $t_1 > 0$ the momentum is measured to be p_0 with an exremely high accuracy. Later, at time $t_2 > t_1$ a measurement of energy is performed on the same oscillator. Find probabilities that the value of energy measured in this second experiment is each of the following: $E = \frac{\hbar\omega}{2}$; $E = \frac{3\hbar\omega}{2}$; $E = \frac{p_0^2}{2m}$.

2. (45 points) Two particles of the same mass move in an infinite square well with walls at $x = \pm L/2$. We will consider three different systems: A – two distinguishable spin 0 particles; B – two indistinguishable spin 1/2 particles; C – two indistinguishable spin 1 particles.

   (a) What are the possible values of the total spin (total angular momentum) J in system B? How many different spin states does the system B have? Write down all spin states in the total spin basis (i.e. eigenstates of J^2 and J_z) in terms of states in the two particle basis (i.e. in terms of eigenstates of s_1^2, s_{1z}, s_2^2, and s_{2z}. (*Hint:* Start with the highest J and J_z state and obtain other states by acting with spin lowering operators.)

   (b) Repeat part (a) for system C.

   (c) Derive the ground state energy and wave-function of system A.

   (d) What is the degeneracy of the ground state of the system B? Which spin-wave-function(s) in part (a) is (are) spin wave-function(s) of the ground state?

   (e) Repeat part (d) for system C. (*Hint:* Carefully consider symmetry properties of the spin and position space wave-functions when both particles are in the lowest eigenstate of the square well potential.)

   (f) What is the degeneracy of the first excited energy level in systems A? Write down the wave-function(s) for all states of the first excited energy level. Repeat this for systems B and C. For systems B and C write down position space wave-functions and identify which spin wave-functions from parts (a) and (b) come with each of the position space wave-functions.

   (g) We now turn on perturbation $V = k(x_1 - x_2)^2/2$. For all three systems determine whether degeneracy of the first excited energy level is lifted. For all three systems determine the remaining degeneracy (if any). You do not need to calculate corrections to energy. (*Hint:* here symmetry properties of the wave-functions are useful once again.)

3. (30 points) Consider a system two spin 1/2 particles in an external magnetic field described by the Hamiltonean

$$H = A\boldsymbol{S}_1 \cdot \boldsymbol{S}_2 + \left(\frac{e}{m_e c}\right) \boldsymbol{B} \cdot (\boldsymbol{S}_1 - \boldsymbol{S}_2) \ .$$

We will treat the second term as a small perturbation.

(a) Assuming that $\boldsymbol{B} = 0$ find eigenstatates and eigenvalues of the Hamiltonean. Describe eigenstates in terms of the eigenvalues of $\boldsymbol{J}^2$ and J_z (where $\boldsymbol{J} = \boldsymbol{S}_1 + \boldsymbol{S}_2$).

(b) Assuming that the magnetic field is static, $\boldsymbol{B} = B_0\hat{\mathbf{z}}$, use first order degenerate time-independent perturbation theory to find energy levels and eigenstates of the perturbed system. Label these eigenstates by J and m values of states into which perturbed states turn in the $B_0 \to 0$ limit.

(c) At time $t = 0$ an additional a time-dependent perturbation is turned on, so that the full magnetic field for $t \geq 0$ is $\boldsymbol{B} = (B_0 + b\cos(\omega t))\hat{\mathbf{z}}$. Will this magnetic field cause the transitions between $J = 0$, $m = 0$ and $J = 1$, $m = 0$ levels found in part (b)? What about transitions between $J = 0$, $m = 0$ level and $J = 1$, $m = \pm 1$ levels? Explain your answers. Using first order perturbation theory calculate the transition probability as a function of time for allowed transitions. (*Hint:* Be careful to distinguish between time-dependent and time-independent parts of the perturbation.)

(d) Repeat part (c) if the magnetic field at $t > 0$ is $\boldsymbol{B} = B_0\hat{\mathbf{z}} + b\cos(\omega t)\hat{\mathbf{x}}$. (Do not calculate the transition probability here but give detailed answers to all other questions of part (c).)

EM Qualifying Exam Spring 2018

1. Three fixed charges are situated as follows: (i) there is a charge $4q$ at the origin (ii) there is a charge $2q$ at $(a, 0, 0)$ (iii) there is a charge $-3q$ at $(0, a, 0)$.

(a) Find the first three multipole moments ($l = 0, 1, 2$) of this configuration.
(b) What is the energy of this configuration?
(c) A fourth charge of magnitude q is initially at the point $(b, 0, 0)$. What is the force on the charge? Give all components.
(d) The fourth charge is moved from the point $(b, 0, 0)$ to the point $(0, b, 0)$. How much work is done?

2. Two charges q and $-q$ are initially sitting at $(0, 0, a)$ and $(0, 0, -a)$ respectively. At $t = 0$ they begin to oscillate; their positions after $t = 0$ are $(0, 0, a + b\sin(\omega t))$ and $(0, 0, -a - b\sin(\omega t))$ respectively.

One observer is at the location $(d, 0, 0)$ and a second observer is at $(0, \frac{d}{\sqrt{2}}, \frac{d}{\sqrt{2}})$. You may assume that d is large compared to all other distances.

At sufficiently large times, each observer will see some radiation. Calculate the energy flux per unit angle (i.e. $\frac{dP}{d\Omega}$) for each observer to leading order in the frequency. State which kind of radiation (electric dipole/ magnetic dipole/electric quadrupole) is produced.

3. A sphere of radius a has magnetic permeability μ. There is a surface current

$$\vec{J} = K_0 \sin(2\theta)(-\sin\phi\hat{i} + \cos\phi\hat{j})\delta(r - a) \tag{1}$$

Note that the delta function already enforces that this is a surface current. Find the vector potentials and the B-fields both inside and outside the sphere. Evaluate the integrals to the best of your ability.

4. A very thin insulating spherical shell of radius a is held at a surface potential $V = V_0 \sin^2\theta \sin^2\phi$. Find the total charge on the spherical shell. You may assume that all the charge is on the surface of the shell.