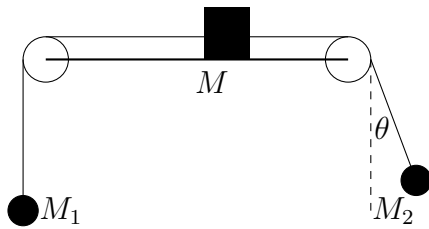


Classical Mechanics Qualifying Exam

1. A proton at rest (mass m_p) is struck by another proton travelling along the x-axis with energy E . The interaction produces an extra pion (mass m_π) so that the final state is two protons plus a pion. All the final state particles are travelling along the x-axis.

Find the minimum and maximum possible energy of the pion.

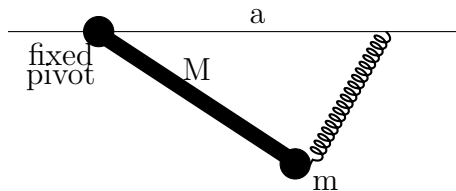
2. A frictionless table has two pulleys on it. A mass M sits on the table. It is connected by a rope through one hole to another mass M_1 , and is connected by another rope through the other hole to another mass M_2 . The mass M_1 can move up and down, and the mass M_2 can oscillate in a plane like a pendulum. Assume all motion happens in a plane.



- (a) Find the Lagrangian. Explain your choice of coordinates clearly.
- (b) Write the equations of motion using any method you choose.

3. A rod of mass M , length L has one end fixed at a pivot which we take to be the origin. There is a mass m attached to the second end of the rod.

A spring is then attached to the mass and the other end of the spring is fixed at a point $(a, 0)$. The spring constant is k and the unstretched length of the spring is b .



- (a) Find the length of the spring in the equilibrium position.
- (b) Find the frequency of small oscillations.

Statistical Mechanics (2019 Spring)

1. Consider a system with N non-interacting particles each with spin $S=1$ in an external magnetic field H . If $H=0$, all single particle states with different S_z are degenerate in energy $\epsilon=0$.
 - a. Plot $\epsilon(H)$ for all single particle states.
 - b. Calculate the partition function, $Q_N(T,H)$
 - c. Compute the average energy and entropy of the system at T and H .
 - d. Compute the specific heat at constant H .
2. Wave motion, having the dispersion formula $\omega = Aq^n$ ($n=1$ for the Debye model) exists in solid and can be excited at nonzero temperature. Show that the specific heat is proportional to $T^{3/n}$ at low temperature.
3. For N ideal Fermi gas particles ($\epsilon=p^2/2m$) moving in a two-dimensional plane within an area A at $T=0$, find
 - (a) The density of state
 - (b) The Fermi energy.
 - (c) Total energy.
4. Consider equilibrium between a solid and a vapor made up of monatomic molecule. It is assumed that the energy ϕ is required per atom for transforming the solid into separate atoms. For simplicity, take the Einstein model for the vibration of atoms in the solid, i.e., assume that a three-dimensional harmonic oscillator performing vibration with a frequency ω represents each atom independently. Evaluate the vapor pressure as a function of temperature.

Quantum Mechanics

(Ph.D. Qualifying Exam Spring 2019)

I. The unperturbed Hamiltonian for a two-state system is represented by

$$H_0 = \begin{pmatrix} \epsilon_1 & 0 \\ 0 & \epsilon_2 \end{pmatrix} .$$

A time-dependent perturbation is added of the form

$$V_t = \begin{pmatrix} 0 & \lambda \cos \omega t \\ \lambda \cos \omega t & 0 \end{pmatrix} ,$$

with λ a real parameter.

(a) Assuming that at $t = 0$ the system is in the state $(1, 0)$ with energy ϵ_1 , find, using time-dependent perturbation theory, an expression for the probability that the system transitions to the state $(0, 1)$ for $t > 0$. Do the calculation assuming $\epsilon_1 - \epsilon_2$ is *not* close to $\pm \hbar \omega$.

(b) What problem arises if $\epsilon_1 - \epsilon_2$ is close to $\pm \hbar \omega$?

II. Determine the commutation relations for the 3×3 matrices

$$[G_i]_{jk} = -i\hbar \epsilon_{ijk} .$$

Compute the eigenvalues of $\sum_i G_i^2$. What physical observable do you think it represents?

III. Consider two spatially localized spins $\frac{1}{2}$ coupled by a transverse exchange interaction and in an inhomogeneous magnetic field. The Hamiltonian is

$$H = h_1 S_1^z + h_2 S_2^z - J (S_1^+ S_2^- + S_1^- S_2^+)$$

Here h_1 and h_2 are proportional to the magnetic fields at the two sites, and J measures the strength of their exchange coupling.

a) Find the eigenvalues of H .

b) If both spins are up at time $t = 0$, what is the probability that they will both be up at a later time t ?

c) If $h_1 = h_2$ and S_1 is up and S_2 is down at time $t = 0$, what is the probability that S_1 will be down and S_2 up at a later time t ?

IV. In a simplified nuclear model for the Deuteron the potential energy part of the Hamiltonian is $V = V_1(r) + V_2(r) \mathbf{S}_p \cdot \mathbf{S}_n$. Here V_1 and V_2 are some functions of the separation r , the proton and neutron masses are given by m_p and m_n , and \mathbf{S}_p and \mathbf{S}_n are their respective quantum mechanical spin operators.

(a) The quantum-mechanical eigenvalue problem can be reduced to a one-dimensional problem in the variable r . Write out explicitly the Schrödinger equation in this variable.

(b) Assuming $V_2 < 0$, discuss the ground state config. of the Deuteron.

Spring 2019 EM Qualifying Exam

1. A spherical shell of inner radius a and outer radius b has a volume charge $\rho = \rho_0 \cos \theta$.

(a) Find the potential everywhere, completing any integrals that arise.

(b) Find the charge and dipole moment, completing any integrals that arise.

(c) Find the total energy. You may leave integrals unevaluated for this part.

2. A circular loop of radius R is in the x-y plane and is centered at the origin. It carries a current I . Find the magnetic field at $(0,0,a)$. Only consider the limit $a \gg R$.

Another identical loop of the same shape, orientation and current is placed at $(b,0,c)$ where $b,c \gg R$. Find the force and torque on the second loop.

3. An antenna of length L has its ends at the origin and at $(0,0,L)$. It carries a current $I = \sin(\omega t)\hat{z}$. Find the radiation pattern (i.e. $\frac{dP}{d\Omega}$) to lowest order in the frequency.