*Clearly Print* Name:____________________________

Signature:____________________________

# Classical Mechanics
# Qualifying Exam
# Fall 2016

You may consult only *Classical Mechanics* by Goldstein, Safko, and Poole. Do all four (4) problems. The exam is worth 50 points. Problem No. 4 is relativistic; the other three problems are non-relativistic. Write directly on this exam; do not use a blue book. Every other page has been left blank to provide extra work space. In order to receive credit you must show all of your work.

## DO NOT OPEN THIS EXAM UNTIL YOU ARE TOLD TO DO SO

FOR ADMINISTRATIVE USE ONLY:

# 1 : ____________________________

# 2 : ____________________________

# 3 : ____________________________

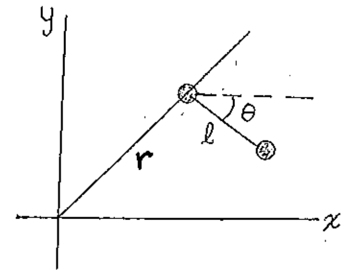
# 4 : ____________________________

Total: ____________________________

1. **[14 points]** All motion in this problem takes place in the $x$-$y$ plane. A tightly stretched wire extends from the origin upward and to the right along the line $y = x$. A bead of mass $m$ slides freely along the wire. A second bead also of mass $m$ is attached to the first by means of a massless rigid rod of length $\ell$ that pivots freely about the first bead. Let $\theta$ be the angle from the horizontal to the rod, measured clockwise, and let $r$ be the distance from the origin to the bead on the wire. The system is released from rest with the rod horizontal ($\theta = 0$). The acceleration of gravity is $g$.

(a) Find the initial value of the second deriviative $\ddot{r}$.

(b) Find the initial value of the second derivative $\ddot{\theta}$.

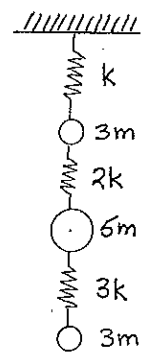


**2.** [**14 points**] All motion in this problem takes place along the $y$-axis. Three springs and three point masses are connected to each other and suspended from the ceiling so that they hang vertically. The order, starting from the ceiling, is: spring ($k$), mass ($3m$), spring ($2k$), mass ($5m$), spring ($3k$), mass ($3m$), where the quantities in parentheses are the spring constants or masses as appropriate.

(a) Find the angular frequencies of the normal modes.

(b) Suppose the system is observed to oscillate such that the coordinates of the masses are as follows, where $y_1$ the coordinate of the top mass, $y_2$ is the coordinate of the middle mass, and $y_3$ is the coordinate of the bottom mass, where all coordinates are measured *from their equilibrium positions*:

$$\begin{aligned} y_1 &= a\cos\omega t \\ y_2 &= 0 \\ y_3 &= -\frac{2}{3}a\cos\omega t \end{aligned}$$




Find $\omega$.

**3.** **[12 points]** A particle of mass $m$ moves in the field of a central force given by:

$$\mathbf{f}(r) = (-\beta + \alpha r^3)\,\hat{\mathbf{r}}$$

where $\hat{\mathbf{r}}$ is the usual radial unit vector and $\beta$ and $\alpha$ are *positive* constants.

(a) Find the condition on the angular momentum $\ell$ which will admit circular orbits. The radius $r$ of the orbit should not appear in your answer.

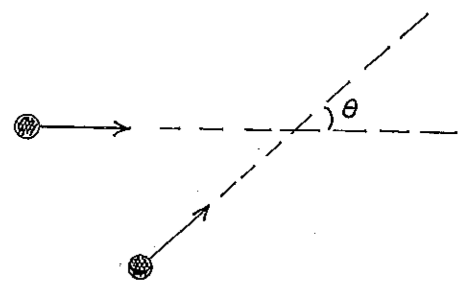
(b) For a given value of $\ell$ that satisfies this condition, and ignoring considerations of stability, how many different circular orbits are possible? What are their corresponding radii?

(c) Are any of the above circular orbits stable? If so, which one(s)?

4. **[10 points]** All motion in this problem takes place in the $x$-$y$ plane. Two protons, each with Lorentz factor $\gamma = 13/5$, collide at an angle $\theta$; that is, $\theta$ is the angle between their velocity vectors. As a result of the collision, an anti-proton and an additional proton are created, so that there are 4 particles after the collision.

   (a) Assuming that $\gamma$ is fixed at the above value, find the threshold *angle* $\theta_0$ at which the reaction will just barely take place.

   (b) For which range of angles does the reaction occur: $0 < \theta < \theta_0$ or $\theta_0 < \theta < \pi$?



# Physics Qual - Statistical Mechanics

(Fall 2016)

**I.** Describe what is meant by:

(a) A quasi-static process
(b) The second law of thermodynamics
(c) A throttling process and the function that is conserved therein
(d) A microcanonical ensemble
(e) The Gibbs paradox

**II.** A system is in thermal equilibrium with a reservoir at temperature $T$. Find an expression for the mean square fluctuation in energy as a function of the heat capacity of the system. Now consider a system of classical magnetic spins under the same conditions. Compute the mean square fluctuation in the magnetization as a function of the magnetic susceptibility.

**III.** In a three dimensional solid explain why the heat capacity due to lattice vibrations differs in the Einstein and Debye approximations at low temperatures. Which one is correct? Does the dispersion law $\omega(\mathbf{k})$ affect the results?

**IV.** Many defects in solids (e.g. excitons) can be simply pictured in terms of the following model: A system consists on $N$ non-interacting elements. Each element has two levels. The lower level has zero energy and the upper level has an energy $\Delta$ above it.

(a) Obtain an expression for the partition function of the system and determine the internal energy ($\langle H \rangle$).
(b) Give an expression for the average number of elements in the excited state. Sketch this occupation number as a function of temperature.
(c) Calculate the specific heat of the system $C_V$. Describe how the value of $\Delta$ could be obtained from measurements of the specific heat.
(d) Calculate the Helmholtz free energy and the entropy. What are the limiting behaviors of the entropy when the internal energy assumes its highest and lowest values?

**Quantum Mechanics Ph.D. Qualifying Exam (Fall 2016)**

1. Consider an isotropic harmonic oscillarot in two dimensions. The Hamiltonean is given by

$$H_0 = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{m\omega^2}{2}(x^2 + y^2) .$$

   (a) What are the energies of three lowest lying states? Is there any degeneracy?

   (b) We now apply a perturbation

$$V_0 = \delta_0 m\omega^2 xy , \tag{1}$$

   where $\delta_0$ is a dimensionless real number much smaller than unity. Find the zeroth-order energy eigenket and the corresponding energy to the first order for each of the three lowest lying states.

   (c) Solve $H_0 + V$ problem *exactly*. Compare with the perturnation results ontained in (b)

   (d) Assume that at time $t = 0$ the system is in the ground state of the Hamiltonean $H = H_0 + V$. At $t = 0$ a new perturbation $V_1(t) = \delta_1(x + y)\cos(\omega' t)$ is turned on (assume that $\omega'$ is not close to $\omega \pm \delta$). Starting with the exact solution of part (c) find the coefficients $c_n(t)$ for the first order corrections to the time-dependent wave-function. Find the transition probability $P_{0\to1}$ for transitions to the first two excited states.

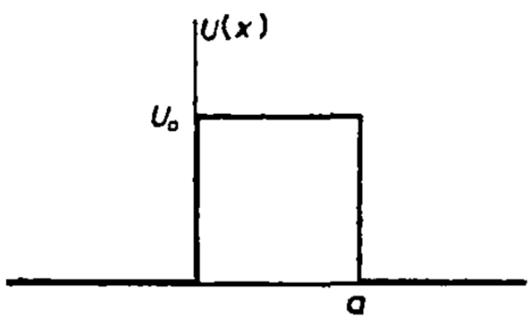
2. In addition to Coulomb interaction the proton and electron in the hydrogen atom interact with the hyperfine interaction which for an electron in the ground state has the form

$$H_{hf} = A\vec{S}_1 \cdot \vec{S}_2 , \qquad A > 0 . \tag{2}$$

   (a) Calculate the corrections to the energy that split the ground state into a triplet and singlet.

   (b) Estimate the probability $P(\text{triplet})/P(\text{singlet})$ of hydrogen atoms in thermal equilibrium at room temperature.

3. Determine the transmission coefficient for a rectangular barrier shown in the Figure (both when $E > U_0$ and when $E < U_0$).




4. A hydrogen atom in its ground state $[(n, l, m) = (1, 0, 0)]$ is placed between the plates of a capacitor. A time dependent potential

$$\vec{E}(t) = \vec{E}_0 e^{-t^2/\tau^2} , \quad \vec{E}_0 = E_z \hat{z} \tag{3}$$

acts from $t = -\infty$ to $t = \infty$. Using time dependent perturbation theory calculate the probability for the atom to be found in each of the three $2p$ states at $t \gg \tau$. You do not need to perform radial integrals but you should perform all other integrations. Repeat the problem for $2s$ states. (*Hint:* You can use $\int_{-\infty}^{\infty} \exp(-t^2/\tau^2)dt = \tau\sqrt{\pi}$; you may need to make a change of variables that results in $t$ having a small imaginary part.)

EM Qualifying Exam Fall 2016

1. An insulating sphere has radius $a$ and is centered at the origin. It is surrounded by another concentric insulating spherical shell of inner radius $b$ and outer radius $c$. The surface of the inner sphere is held at a potential $V_0 \cos^3\theta$ while the inner surface of the outer sphere is at a potential $V_1 \sin^2\theta$ ($\theta$ is the usual coordinate in spherical coordinates which measures the angle from the z-axis.)

Find the potential everywhere between the spheres.

If a charge $q$ is placed exactly halfway between the spheres, what force does it feel?

2. An infinite wire passes through the origin and lies along the z-axis. An observer is at a point $(x, 0, 0)$. Initially, there is no current through the wire. At $t = 0$ a uniform current $I = I_0$ starts to flow and does not change further with time.

Find the magnetic field seen by the observer as a function of time. Do not ignore the speed of light.

3. A square loop of side length $a$ and total resistance $R$ carries a current $I_0 \cos\omega t$ flowing clockwise. What is the total power radiated?

4. A uniform B-field $B_0$ is pointed along the z-axis. A square loop of side length $a$ and total resistance $R$ is initially oriented in the $x - y$ plane and then rotates at angular speed $\omega$ around an axis through its midpoint and parallel to the y-axis. What is the induced current as a function of time?