Mathematica Workshop
Assignment 1

(1) Look up the N function. Use it to evaluate the following numbers to 30 digits:

(a) $e^{\pi^{1/2}} \ln(128!)$

(b) $\cos(e^{32})$

(2) Compute 50! (that's 50 factorial). Now, evaluate it to 5 digits using the function N. Notice that even though 50! is an integer, applying N on it represents it as a real number. This is true even if you use enough digits of precision. Try it with the integer 1 using 10 digits of precision.

(3) Which is bigger: e^{π} or π^{e} ? Rather than numerically evaluating the two expressions and then comparing the resulting numbers yourself, try having *Mathematica* compare them by using an inequality to derive a truth value (for instance, $1 < 2$ returns True). Also try testing the following equality (which is false): $e = 2.718281828459$. (Note that = stands for assignment and == stands for equal in *Mathematica*.) Can you explain the result? Try looking up Equal for help.

(4) Look up Expand, Factor, Simplify, and ExpandAll.

(a) Fully expand $\frac{(x-a)(x-b)}{(x-c)(x-d)}$ and then factor the result to recover the original input.

(b) Fully expand $(x+y)^2 + (x+y)^{-2}$. Then recover the initial result.

Finally transform it into a single fraction.

(5) Look up the D function for differentation. Evaluate the following expressions:

(a) $\frac{d}{dx}(x^y)$

(b) $\frac{d}{dy}(x^y)$

(c) $\frac{d}{dx} f(x)\, g(x)$

(d) $\frac{d}{dx} f(g(x))$

Notice that for the last two expressions *Mathematica* returns the product rule and the chain rule for differentiation.

(6) Let $u = e^x \cos y$. Verify the following:

(a) $\frac{d^2 u}{dx\, dy} = \frac{d^2 u}{dy\, dx}$

(b) $\frac{d^2 u}{dx^2} + \frac{d^2 u}{dy^2} = 0$

(7) Look up the Integrate function. Evaluate the following expressions:

(a) $\int x^y \, dx$

(b) $\int x^y \, dy$

(c) $\int x^2 e^{-x} \cos(x) \, dx$

(d) $\int_0^1 x^2 (1-x)^3 \, dx$

Verify the results of the first three parts by checking that differentiation returns the integrands.

Mathematica Workshop Assignment 2

(1) Look up ExpToTrig and TrigToExp. Check the following formulas by applying the appropriate function to the left hand sides of the equations. (Recall that in *Mathematica*, we use I rather than i to represent the square root of -1.)

(a) $e^{iz} = \cos z + i \sin z$
(b) $\cos z = \frac{e^{iz}+e^{-iz}}{2}$
(c) $\sin z = \frac{e^{iz}-e^{-iz}}{2i}$

(2) Use TrigExpand to find formulas for $\cos(5\,\theta)$ and $\sin(5\,\theta)$ in terms of $\cos\theta$ and $\sin\theta$. You can also derive trig identities such as the one involving $\cos(x+y)$.

(3) Look up the Series function. Find the third-order Taylor series about x = 0 for the following functions:

(a) f(x)
(b) f(g(x))
(c) arctanh(x)
(d) $(1+x)^n$
(e) $\int_0^x e^{-t^2}\,dt$

(4) Look up the Plot function. Then plot the following functions:

(a) $\sin(1/x)$ for $x \in (-1, 1)$
(b) $\sin(x)\cos(2\,x)\sin(3\,x)\cos(4\,x)$ for $x \in (0, \pi)$
(c) $e^{-x}\cos(2\,x)$ for $x \in (0,\pi)$

(5) Look up section 1.7.1 of the *Mathematica* book on defining functions. It gives the squaring function as an example: `f[x_] := x^2`. Let phi = $\frac{1+\sqrt{5}}{2}$ and define a function fib(n) to be $\frac{1}{\sqrt{5}}$ (phi^n − (−1 / phi)^n). What are the values of this function for n from 0 to 10 ? They should be integers.

(6) Look up the ReplaceAll function. It is usually used as an operator (/.) consisting of a slash and then a period. Here is an example: `Sin[x]/.x->5` evaluates `Sin[5]`. Define a variable fib2 to be the same expression as above: $\frac{1}{\sqrt{5}}$ (phi^n − (−1 / phi)^n) where phi = $\frac{1+\sqrt{5}}{2}$. Now you can evaluate the expression by substituting for n in fib2. Do so for some values between 0 and 10 and make sure they agree with the values from the last problem.

(7) Look up the Solve and FindRoot functions. Find the roots of the equation $x^5 + x + 1 = 0$. You can also evaluate them numerically to see more comprehensible results. Compute the sum and product of the roots; how do these relate to the equation? Now try finding the roots of $x^5 + x + 3 = 0$. *Mathematica* cannot find them symbolically but can approximate them.

Mathematica Workshop
Assignment 3

(1) Look up NIntegrate. Numerically evaluate $\int_0^1 \int_0^1 \int_0^1 \frac{1}{(1+x^2+y^2+z^2)^2}\, dx\, dy\, dz$.

(2) Read about lists (and vectors and matrices) in section 1.8 of the *Mathematica* book. Also look up the functions Det, Inverse, and Eigenvalues. Consider the general 2-by-2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Find its determinant, inverse, and eigenvalues.

(3) Look up DSolve and follow the link to the *Mathematica* book for more information. Consider the differential equation $y''(x) + k^2\, y(x) = 0$.

(a) Solve the equation for the general solution.

(b) Solve it with the initial conditions $y(0) = \text{y0}$ and $y'(0) = \text{v0}$

(c) Solve it with the boundary conditions $y(0) = \text{y0}$ and $y(1) = \text{y1}$

(4) Look up NDSolve and follow the link to the *Mathematica* book for more information. Solve the differential equation $y'(x) - 2\sin(x)\, y(x) = 0$ with $y(0) = 1$. Evaluate $y(x = 8)$. Plot the solution in the interval (0, 10).

(5) Solve the nonlinear equation for the pendulum

$$\frac{d^2}{dt^2}\theta(t) + \frac{g}{l}\sin\theta(t) = 0$$

for $g/l = 1$ and for time t from 0 to 20. For initial conditions, choose $\theta(0) = \pi/2$ and $\theta'(0) = 0$. Plot the solution. Superimpose it with the plot of the solution for the linear equation that is good for small oscillations:

$$\frac{d^2}{dt^2}\theta(t) + \frac{g}{l}\theta(t) = 0.$$

(6) Look up the package Calculus`VectorAnalysis` in the Help menu under Add Ons, Standard Packages. Load the package. Work in Cartesian coordinates unless otherwise noted.

(a) Let $f = x^2 y + x z$. Find its gradient. Find the directional derivative of f at $(1, 2, -1)$ in the unit direction parallel to the vector $(2, -2, 1)$. Find the Laplacian of f.

(b) Let $V = (x, y, z)$. Find its divergence and curl.

(c) Calculate the volume contained by a sphere in 3-space using spherical coordinates.

(d) Find the Jacobian determinant for going from spherical to Cartesian coordinates and from cylindrical to Cartesian coordinates. Do they look familiar?