FUSION IN A STAGED Z-PINCH

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Accelerating Low-Cost Plasma Heating and Assembly –
ALPHA

Fusion Challenges

- Heating

  Ohmic, wave, beams, shocks

- Stability

  Magneto-Rayleigh Taylor

  Sausage, kink, and others

- Confinement

  Lawson criteria: $nt > 10^{14}$ cm^{-3}-s

  $\tau \sim$ s for low density (e.g., 10^{14} cm^{-3})

  $\tau \sim$ ns for high density (e.g., $>10^{23}$ cm^{-3})

- Ignition

  α-particle trapping is required!

  Large containment vessel

  High magnetic field.

PINCH INSTABILITIES

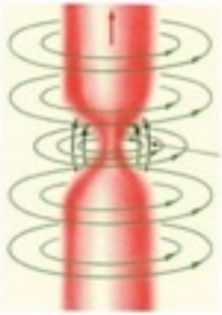
KINK
$m = 1$

incompressible, sharp boundary:

$$\gamma = \frac{C_A}{r_o}(kr_o)\,\frac{I_m(kr_o)}{I'_m(kr_o)}\left[1 + \frac{m^2}{kr_o}\,\frac{K_m(kr_o)}{K'_m(kr_o)}\right]$$

~ (150 ns)$^{-1}$

SAUSAGE
$m = 0$



$m = 0$;

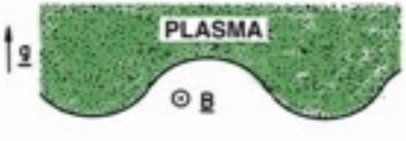
$$\gamma = \frac{C_A}{r_o}(kr_o)\,\frac{I_m(kr_o)}{I'_m(kr_o)}\ ;$$

compressible, $k \to 0$;

$$\gamma = \frac{C_A}{r_o}(2 - \alpha^2)^{-1/2}\,kr_o$$

~ (50 ns)$^{-1}$

RAYLEIGH TAYLOR



$$\gamma^2 = -kg + \frac{(\bar{k}\cdot\bar{B}_o)^2}{4\pi\rho_o}$$

~ (10 ns)$^{-1}$

Z-Pinch PhDs at UCI

(Norman Rostoker and Amnon Fisher)

Joseph Shiloh (1978), High Density Z-Pinches

James Bailey (1983), Effects of Radiation Cooling and Plasma Atomic Number on Z-Pinch Dynamics

Irving Weinberg (1985), X-Ray Lithography and Microscopy using a Small Scale Z-Pinch

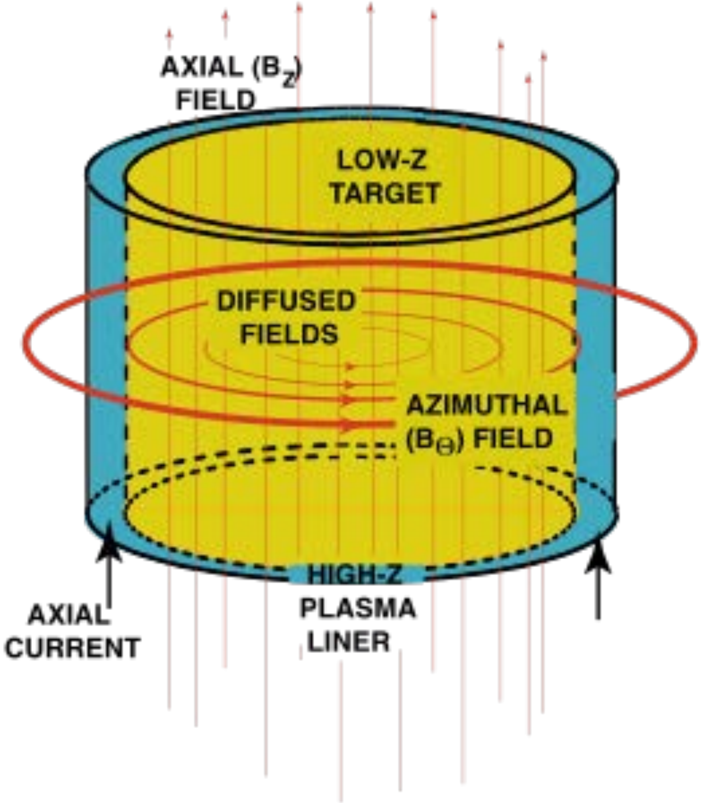
Edward Ruden (1988), Magnetic Flux Compression with a Gas-Puff Z-Pinch

Gus Peterson (1994), Effects of Initial Conditions on a Gas-Puff Z-Pinch Dynamics

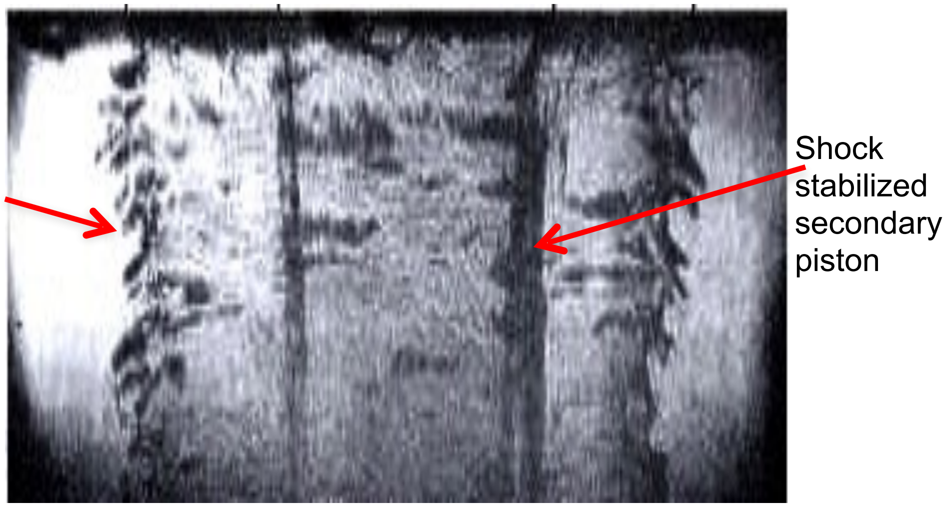
Brian Moosman (1997), Diagnostics of Exploding Wires.

Alan Van Drie (2001), Thermonuclear Fusion in Staged Z-pinch

STAGED Z-PINCH



Stable Liner on Target Implosion:
Kr Liner imploding on Deuterium target (UCI)



Shock stabilized secondary piston

Overview

- Shock waves in multi-stage, *Z*-pinch implosion
- Plasma liner implodes onto a plasma target
- Radiative liner: Xe, Kr, Ag, Cu, etc.
- Control and mitigation of the RT-instability
- Target pre-heating and compression, by shocks
- Formation of high-energy-density, stable plasma
- Trapping of α-particles, leading to ignition
- Prospects for production of high-gain fusion

Physical Phenomena Associated with Compression

$$\xi = \xi_0 \exp^{\gamma t} \qquad \gamma = \sqrt{gk} \qquad k = 2\pi/\lambda$$

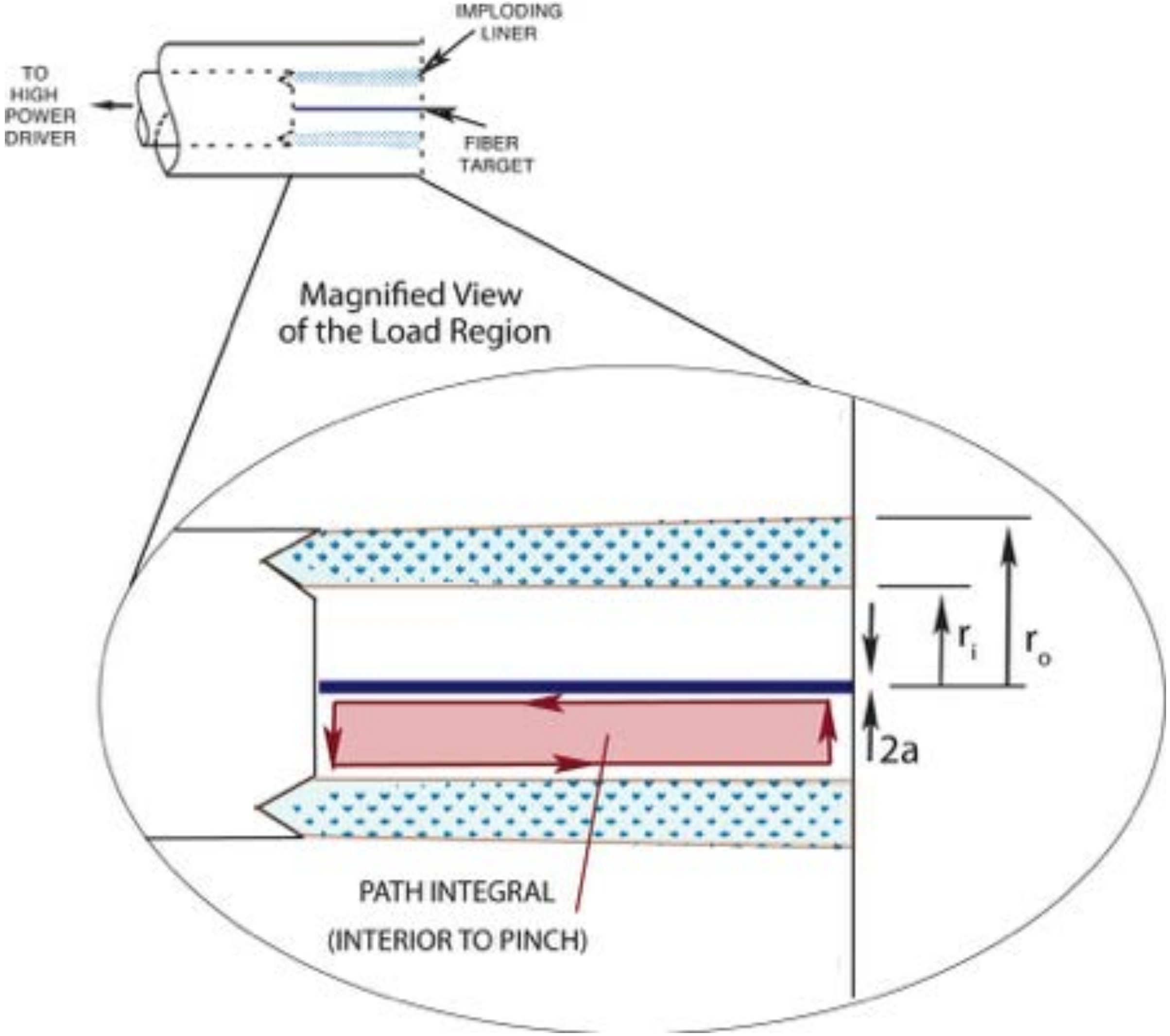
$$s = \tfrac{1}{2} g t^2$$

$$\xi = \xi_0 \exp^{\sqrt{4\pi s/\lambda}}$$

Physical Phenomena Associated with Compression (contd.)

$$W = \int \vec{F}.d\vec{r} = \int_{r_m}^{r_i} \frac{B_\theta^2}{8\pi} 2\pi r l dr = \frac{I_m^2\, l}{c^2} \ln \frac{r_i}{r_m} \qquad B_\theta = \frac{2I_m}{cr}$$

Current Amplification in a Staged Z-Pinch



Current Amplification in a Staged Z Pinch (contd.)

$$\Phi = \int \vec{B} \cdot d\vec{A} = \int_a^{r_i} \frac{2Il}{cr} dr = \frac{2Il}{c} \ln \frac{r_i}{a}$$

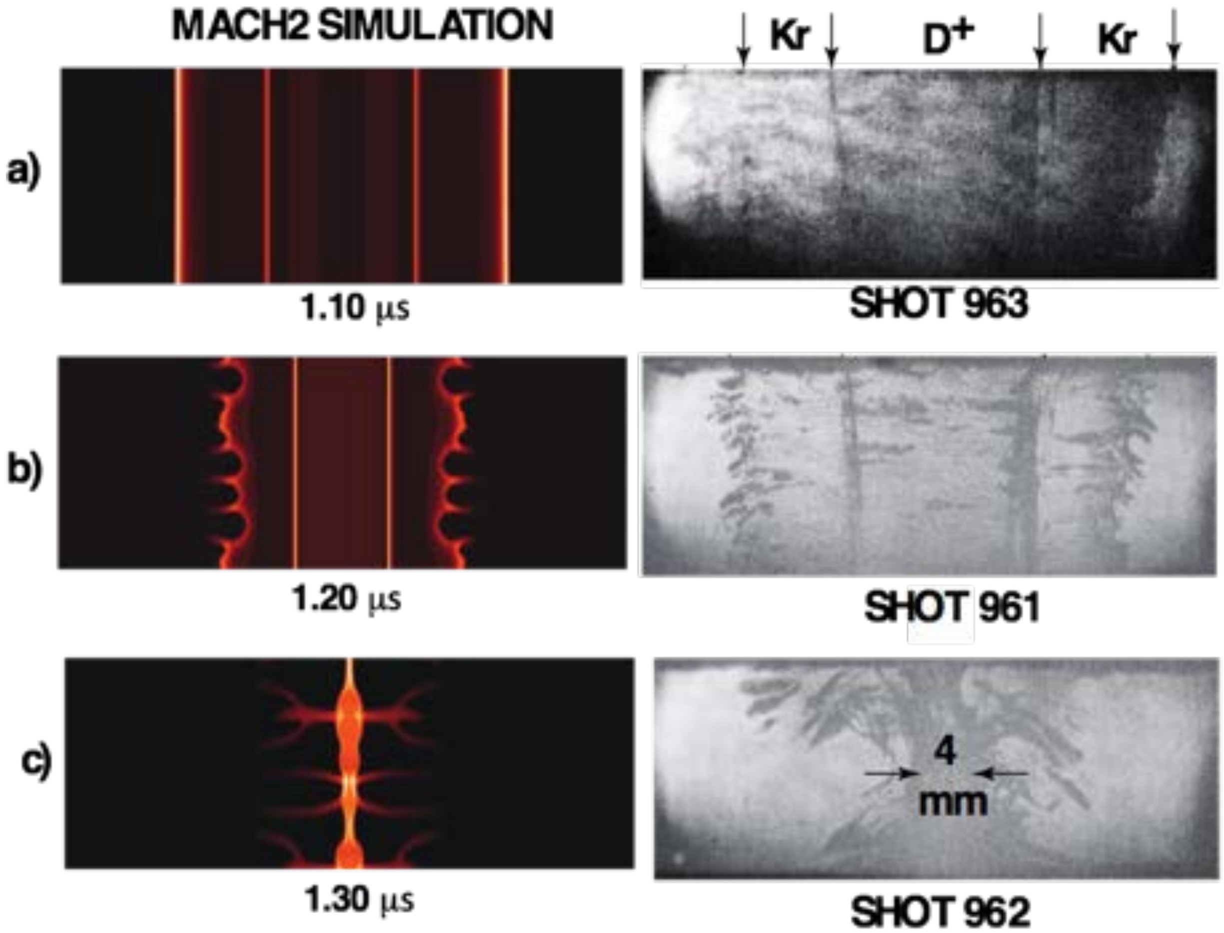
$$\frac{1}{c}\frac{\partial \Phi}{\partial t} = \oint \vec{E} \cdot d\vec{S} = 0$$

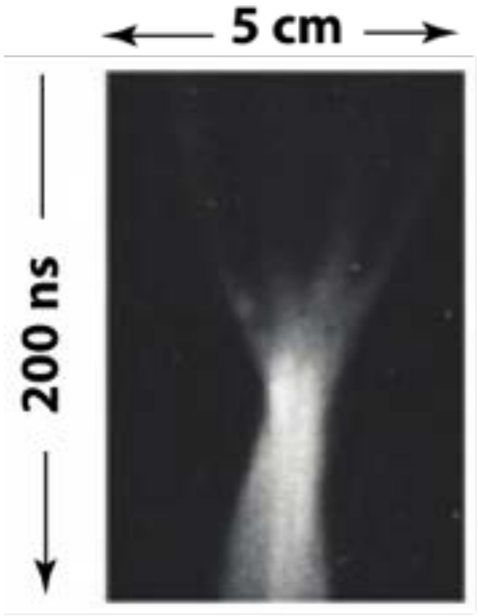
$$I = I_0 \frac{\ln r_i(0)/a(0)}{\ln r_i(t)/a(t)}$$

and $r_i(t) \rightarrow a_i(t)$

Role of shock waves

UCI experiments, at 1 MA, 1.2 μs, 10^{-2} T, provide evidence and benchmarked MACH2

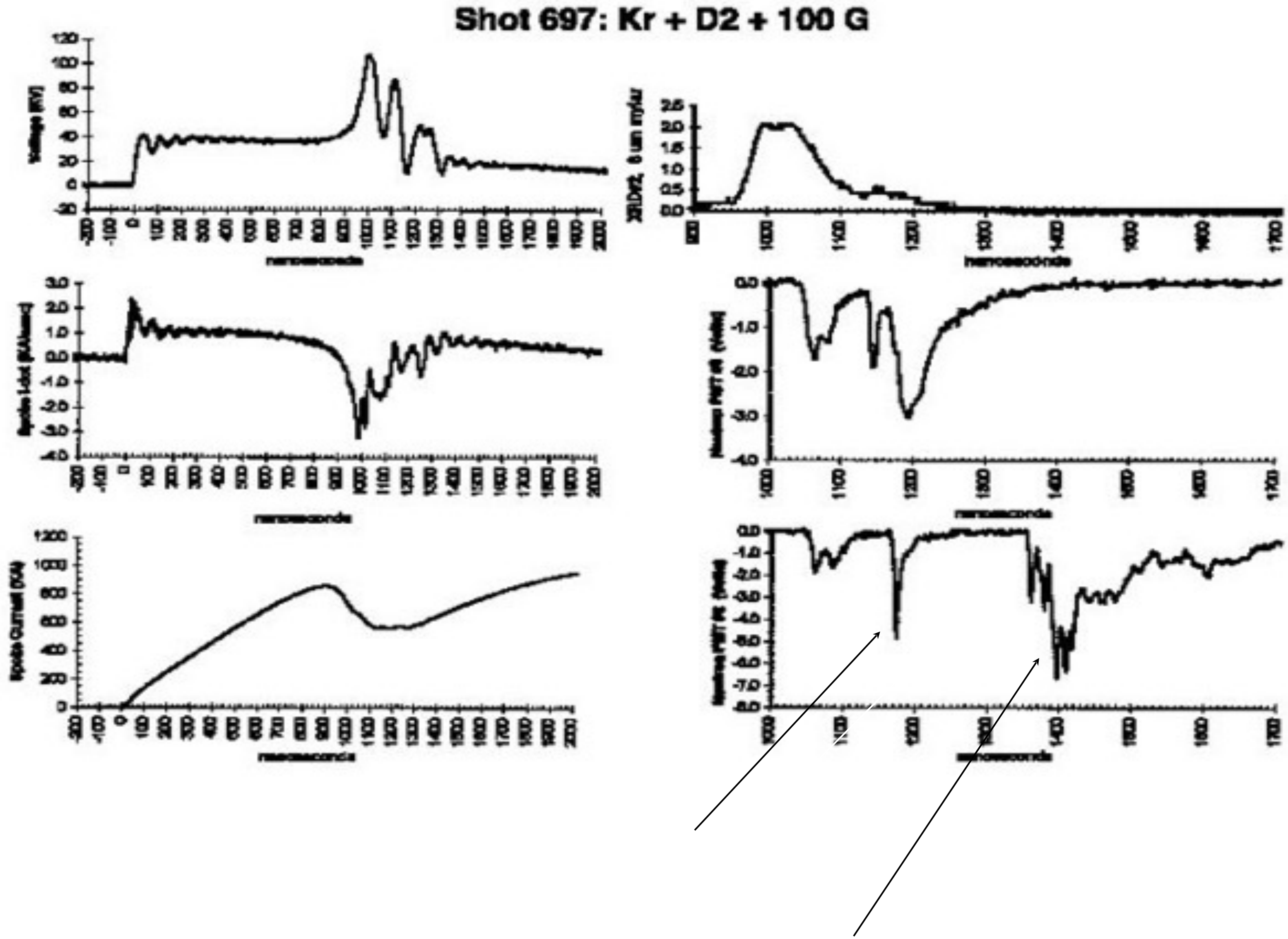




Neutron yield ~ 0.5×10^{10}
Evidence for d-t reactions

From: Thesis of Alan Van Drie

Electrical Signals



Numerical Simulation using MACH2

- 2-1/2 D, time-dependent, single fluid, MHD
- Eulerian mode
- External capacitor-bank circuit
- Tabular (SESAME) equations of state
- Implicit MHD, with **B** and **U**
- Multi-species plasma
- Flux-limited, single group, implicit radiation diffusion
- Benchmarked against several experiments

MACH2 Equations

Continuity Equation:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \vec{u})$$

Momentum Equation:

Radial and axial components:

$$\rho \frac{\partial v^i}{\partial t} = -\rho v^j \nabla_j v^i + \nabla_j[-(P + Q + \tfrac{1}{3}u_R)\delta^{ji} + \frac{1}{\mu_0}(B^j B^i - \tfrac{1}{2}B^2\delta^{ji}) + \sigma^d_{ji}]$$

Azimuthal component:

$$m_i \frac{dv_{i\theta}}{dt} = +eE_\theta - \eta e J_\theta$$

E_Θ, from flux compression produces ion rotation

Electron Specific Energy Equation:

$$\rho \frac{\partial \epsilon_e}{\partial t} = -\rho \vec{v} \cdot \nabla \epsilon_e - P_e \delta^{ji} \nabla_i v_j + \eta J^2 - \vec{J} \cdot (\frac{\nabla P_e}{e n_e}) + \nabla \cdot (\kappa_e \nabla T_e) - \\ ac\rho\chi_{planck}(T_e^4 - T_R^4) - \rho c_{v_e} \frac{(T_e - T_i)}{\tau_{ei}}$$

Ion Specific Energy Equation:

$$\rho \frac{\partial \epsilon_i}{\partial t} = -\rho \vec{v} \cdot \nabla \epsilon_i + [-(P_i + Q)\delta^{ji} + \sigma^d_{ji}]\nabla_i v_j + J_\theta E_\theta + \nabla \cdot (\kappa_i \nabla T_i) + \\ \rho c_{v_e} \frac{(T_e - T_i)}{\tau_{ei}}$$

Radiation Energy Density:

$$\frac{\partial u_R}{\partial t} = -\rho \vec{v} \cdot \nabla u_R - \frac{4}{3} u_R \nabla \cdot \vec{v} + \nabla \cdot (\rho \chi_{ros} \nabla u_R) + ac\rho\chi_{planck}(T_e^4 - T_R^4)$$

Magnetic Induction:

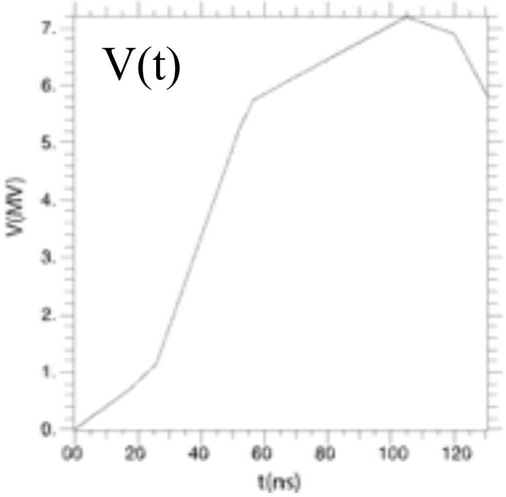
$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) - \nabla \times (\eta \vec{J}) - \nabla \times (\frac{\vec{J} \times \vec{B}}{e n_e}) + \nabla \times (\frac{\nabla P_e}{e n_e})$$

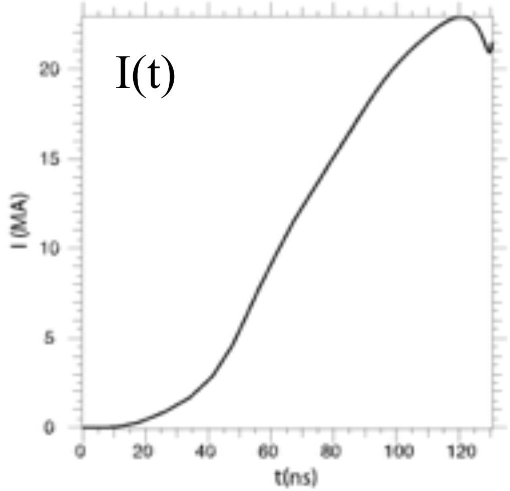
MACH2 simulations using Sandia Z Facility parameters

$R_0 = 3.0$ mm
$\Delta R_L = 100$ µm
$Z_0 = 1.5$ mm
$\rho_{Be} = 10490$ kg/m^3
$\rho_{DT} = 6$ kg/m^3

T_0(liner) = 0.002 eV
T_0 (target) = 2.0 eV
$B_{Z0} = 0$ T
$\Delta n_0 = 5\%$

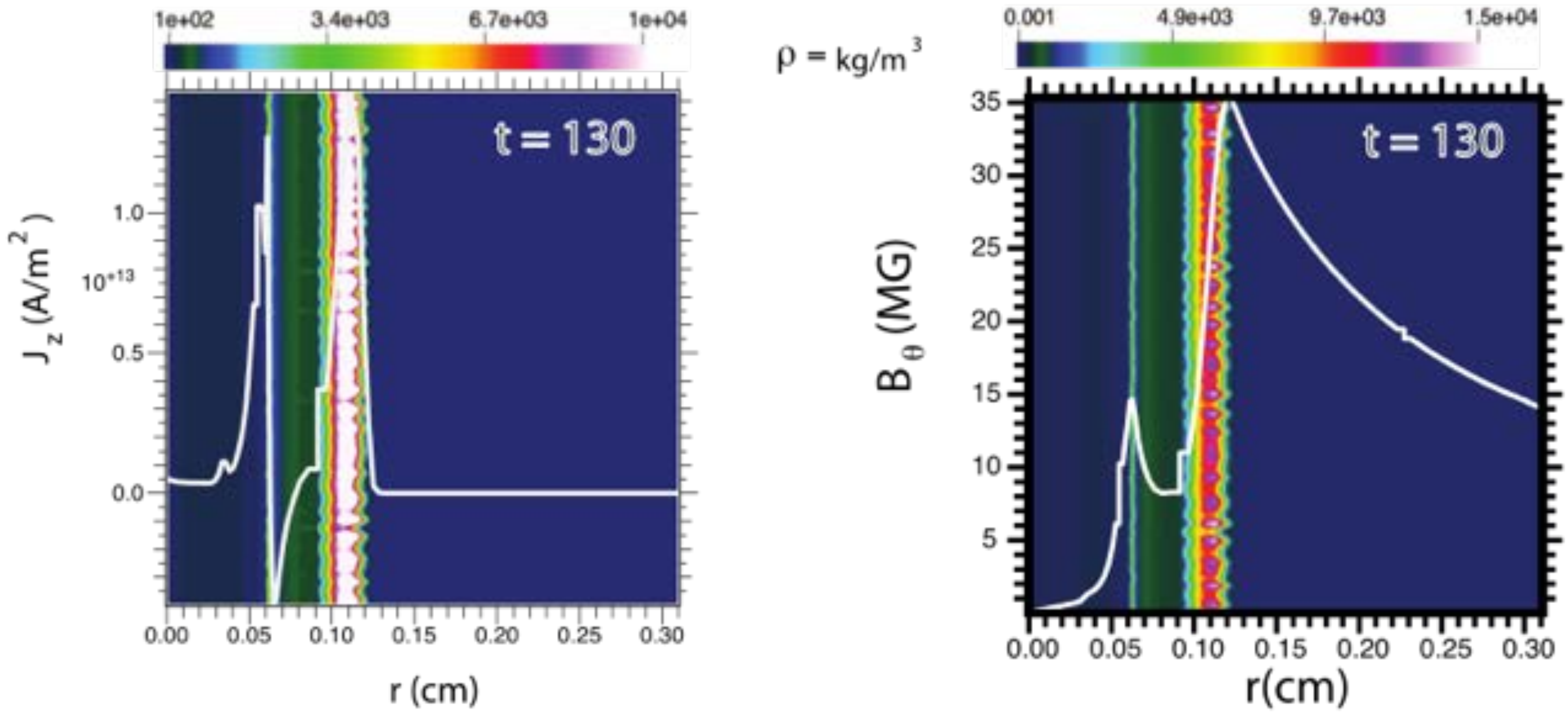
- Discretization:
  - Axially - 124 zones
  - Radially - 84 zones & 5 sub-regions
- Silver
  - R = 2.9 – 3.0 mm, 16 zones
  - R = 3.0 – 3.1 mm, 4 zones
- Deuterium:
  - R = 0.0 - 0.5 mm, 16 zones
  - R = 0.5 -1.0 mm, 32 zones
  - R = 1.0 – 2.9 mm, 32 zones



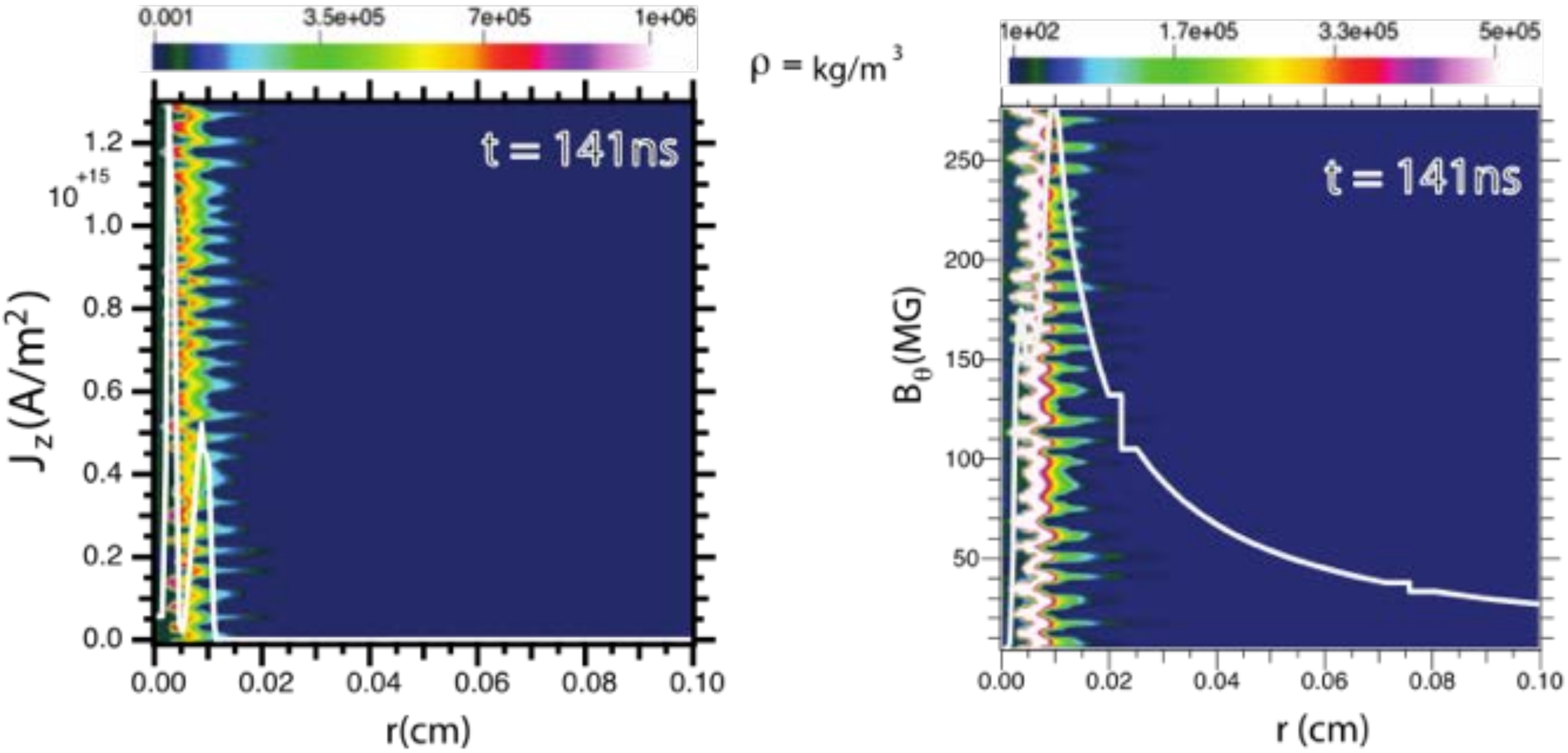


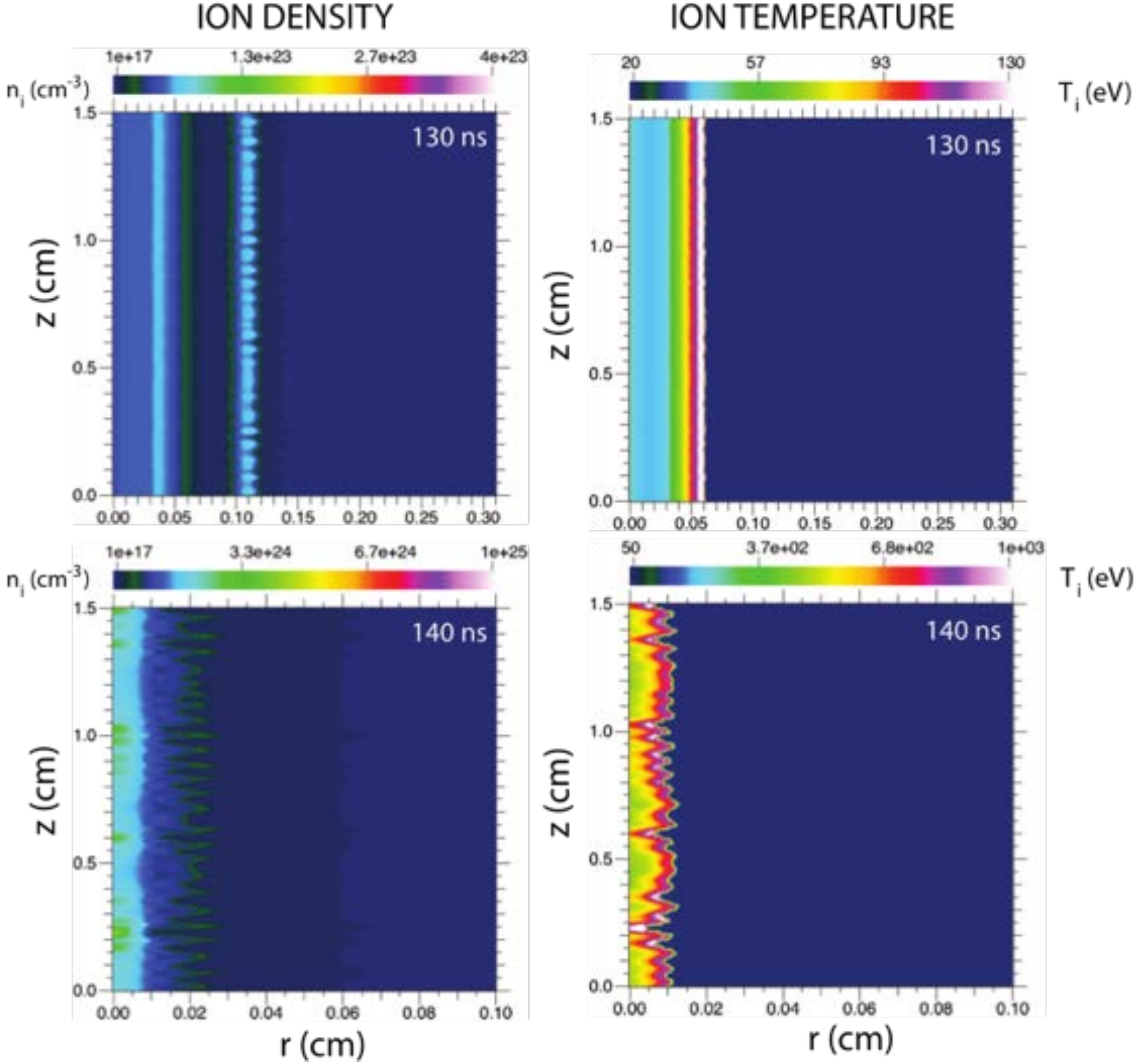
L = 6.64 nH, R = 0.18 Ω, E = 20 MJ

Mass density at 130 ns with J_z & B_θ



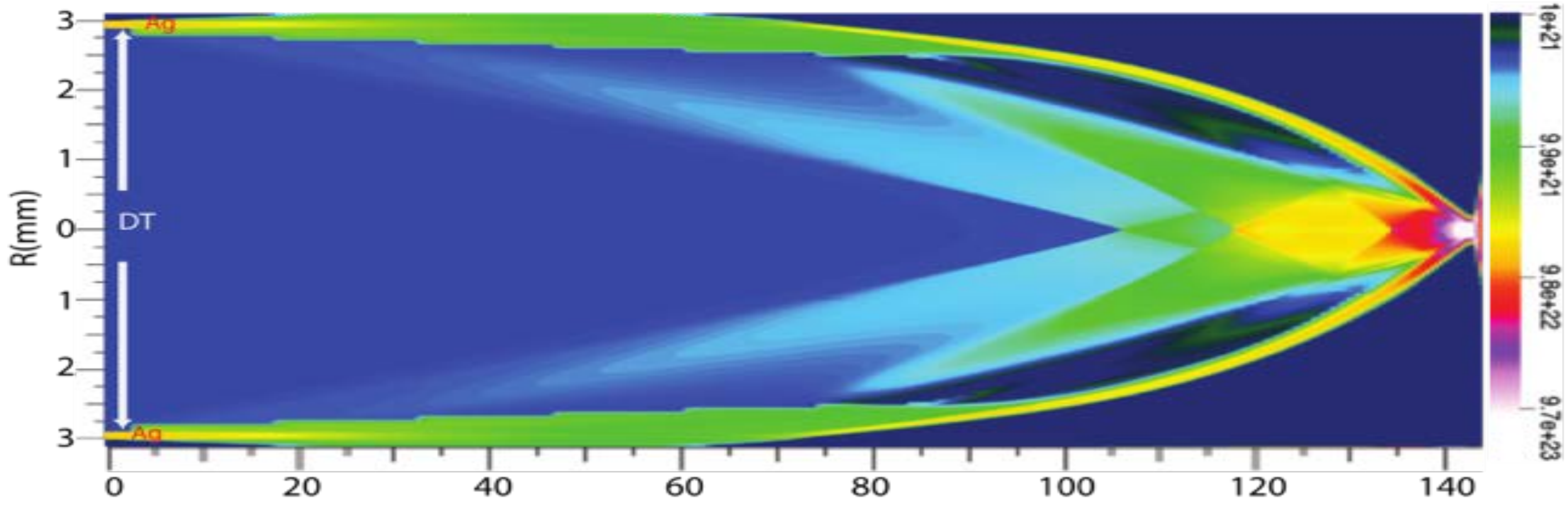
Mass density at 141 ns with J_z & B_θ



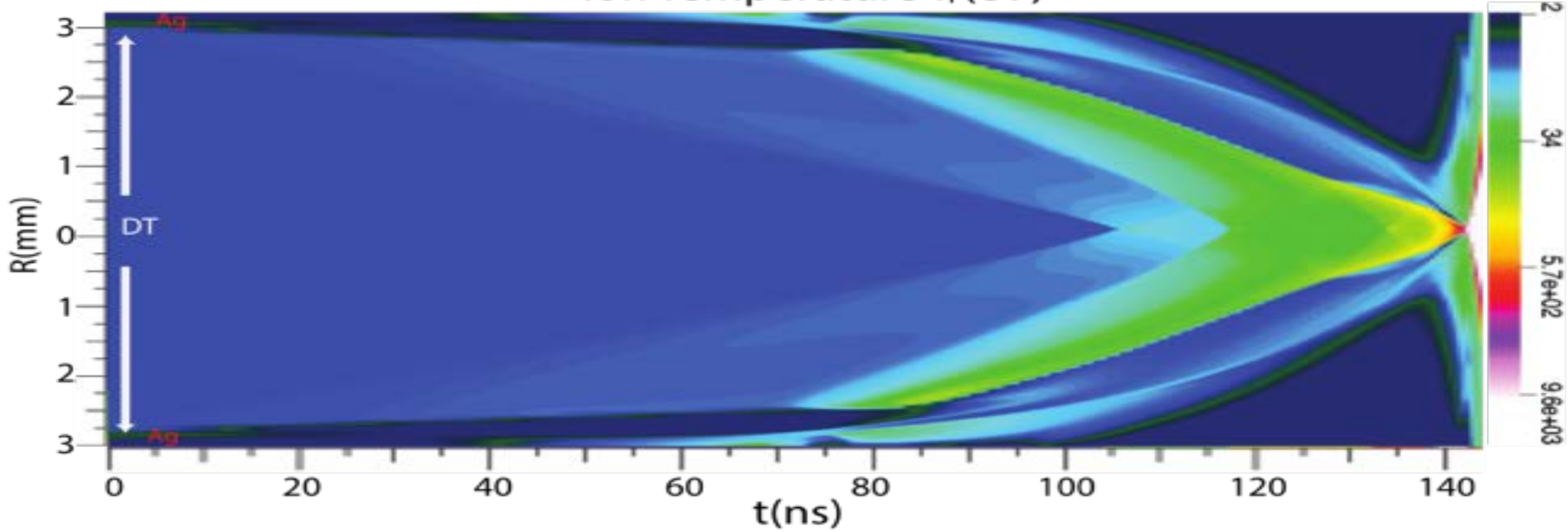

ION DENSITY
ION TEMPERATURE
n_i (cm^-3)
1e+17
1.3e+23
2.7e+23
4e+23
130 ns
T_i (eV)
20
57
93
130
130 ns
n_i (cm^-3)
1e+17
3.3e+24
6.7e+24
1e+25
140 ns
T_i (eV)
50
3.7e+02
6.8e+02
1e+03
140 ns
z (cm)
0.0
0.5
1.0
1.5
r (cm)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.00
0.02
0.04
0.06
0.08
0.10

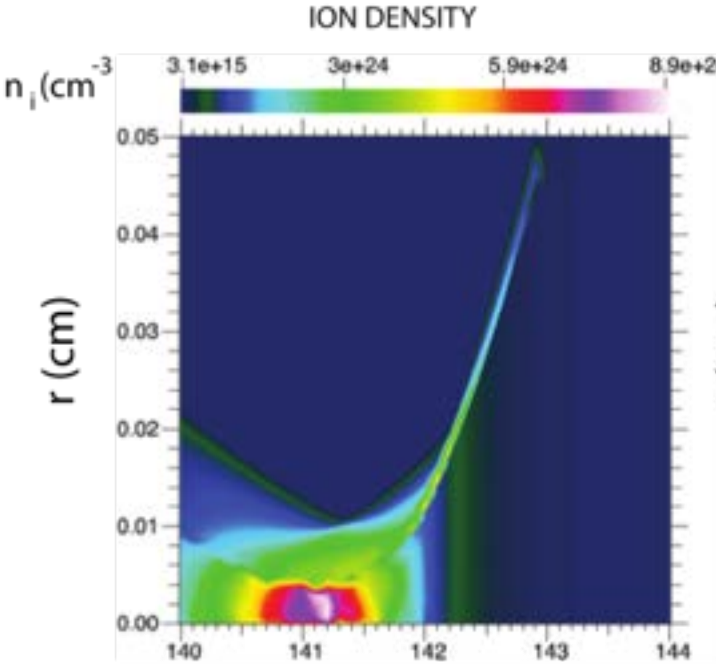
100-μm Ag liner-DT target

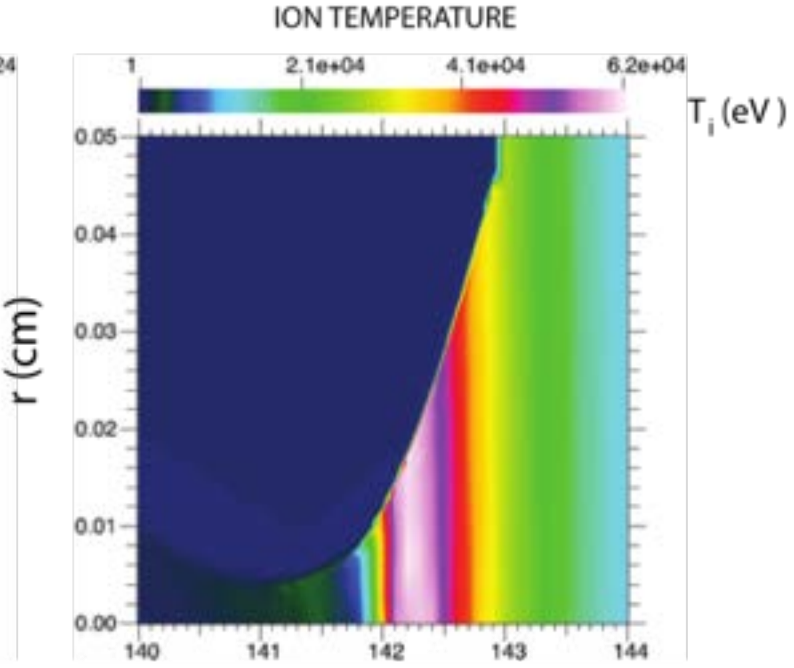
Ion Density n_i (cm^{-3})



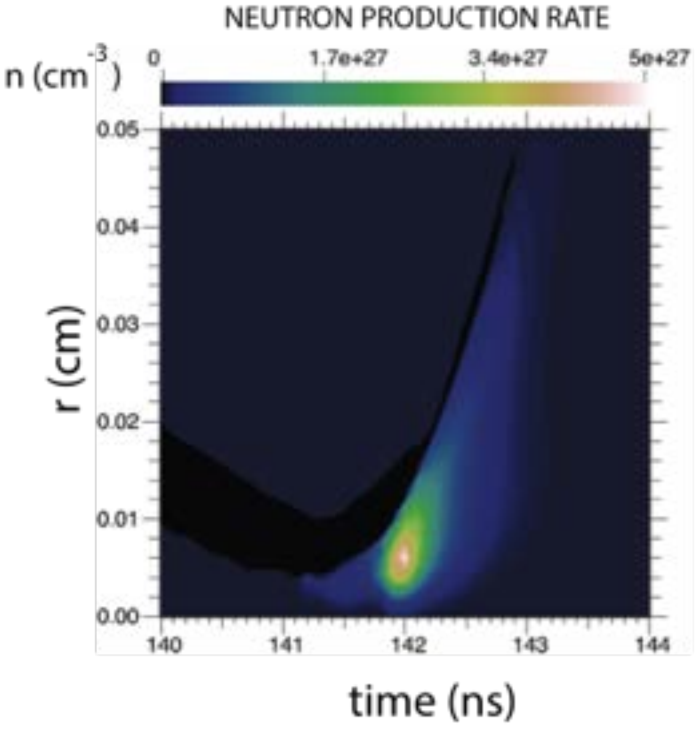
Ion Temperature T_i (eV)



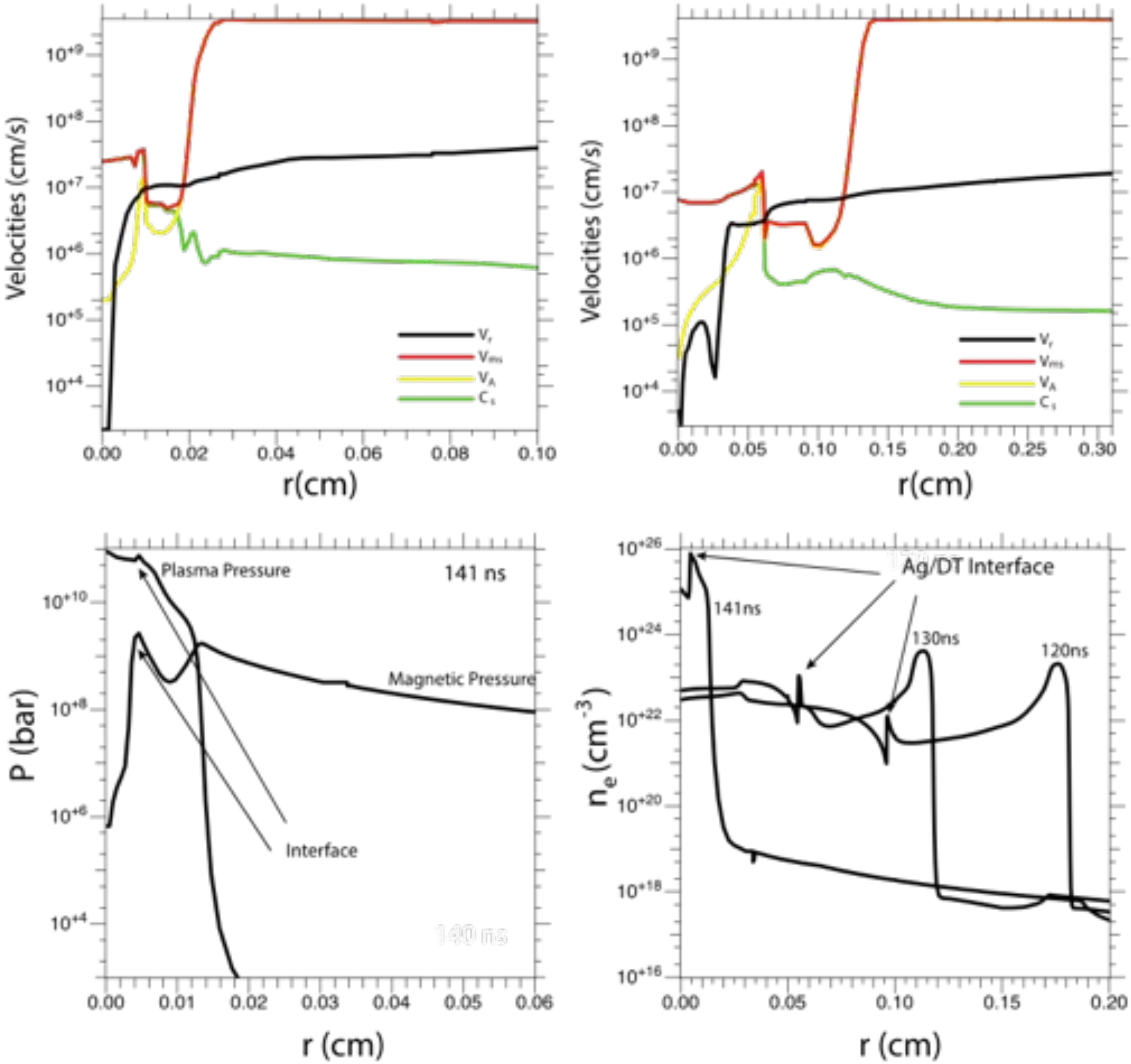

ION DENSITY


ION TEMPERATURE

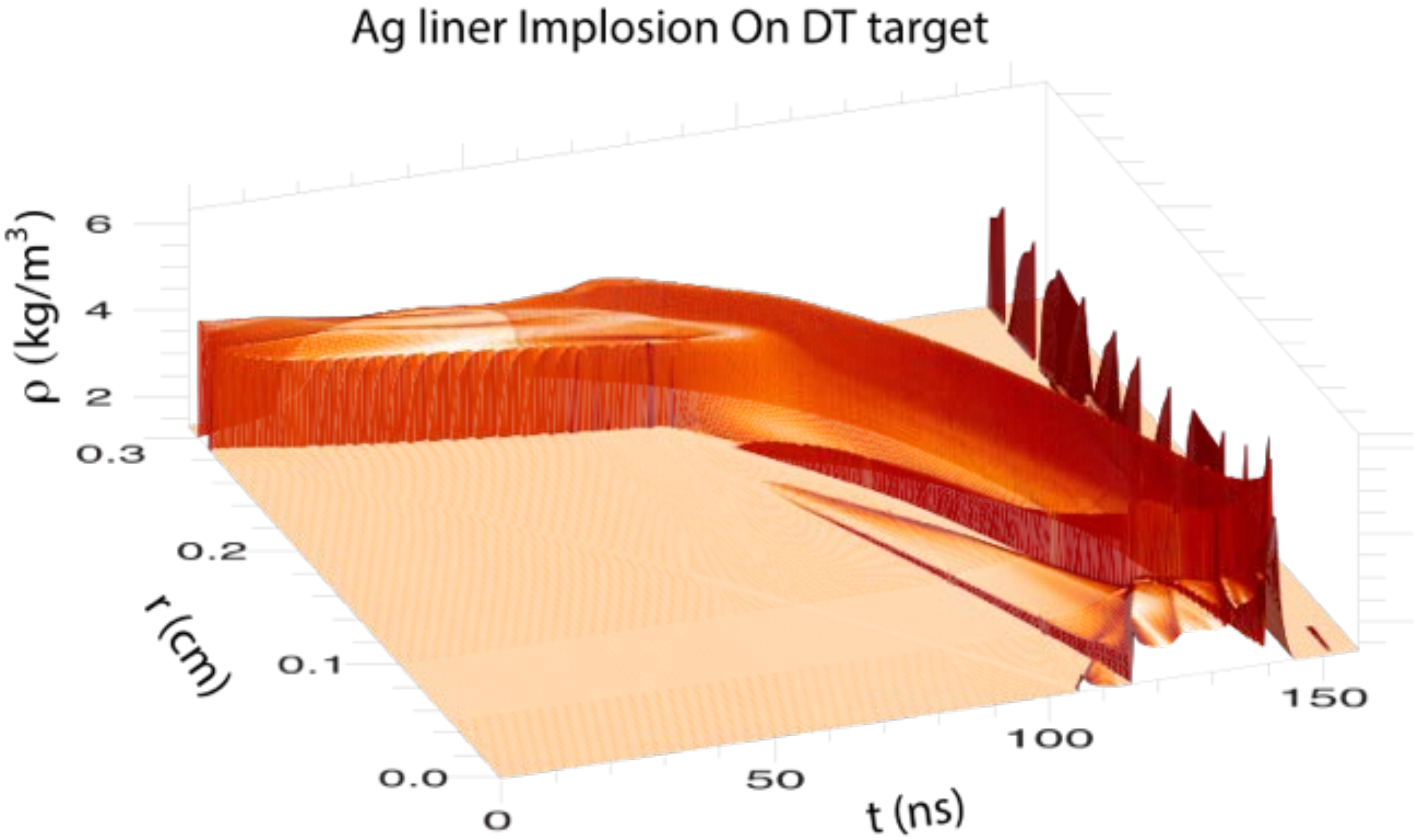
Peak implosion parameters


NEUTRON PRODUCTION RATE

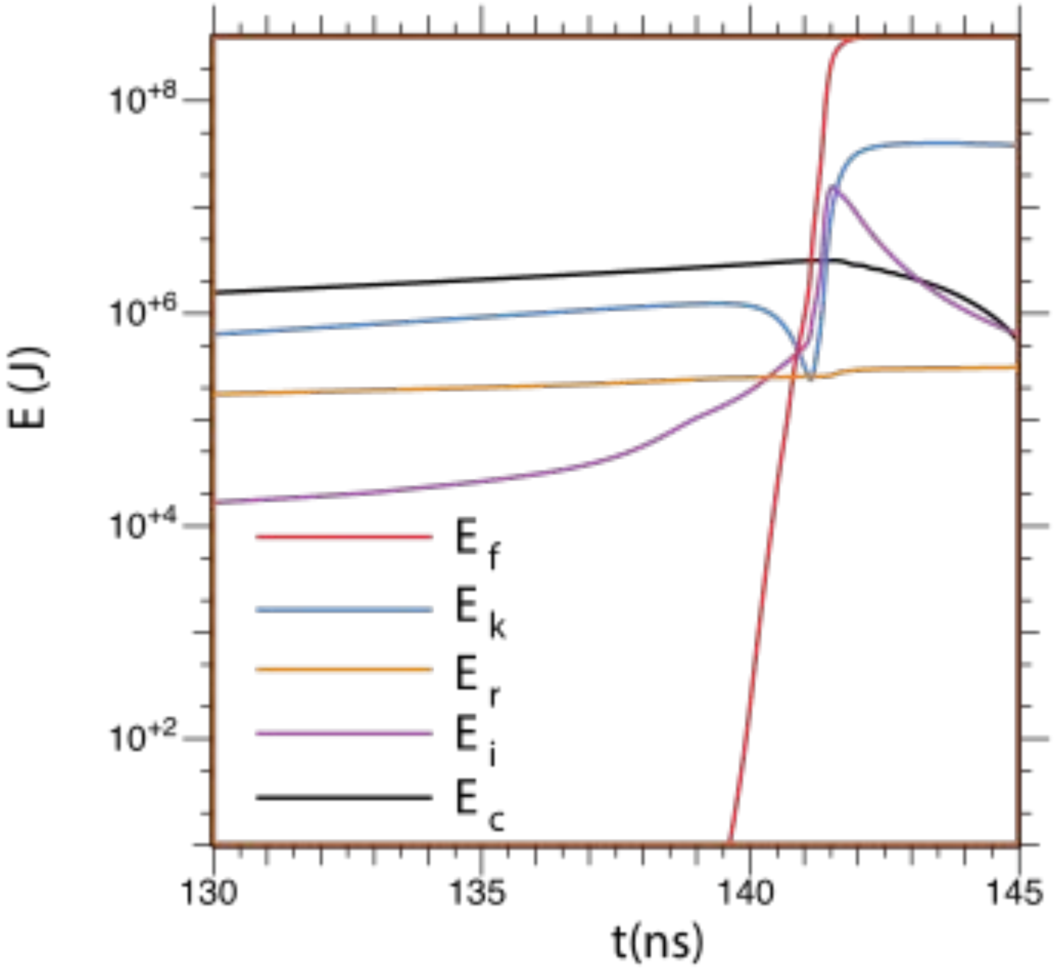
Shocks


Velocities (cm/s)
10+9
10+8
10+7
10+6
10+5
10+4
0.00
0.02
0.04
0.06
0.08
0.10
r(cm)
V_r
V_ms
V_A
C_s
Velocities (cm/s)
0.00
0.05
0.10
0.15
0.20
0.25
0.30
r(cm)
V_r
V_ms
V_A
C_s
P (bar)
10+10
10+8
10+6
10+4
Plasma Pressure
141 ns
Magnetic Pressure
Interface
0.00
0.01
0.02
0.03
0.04
0.05
0.06
r (cm)
n_e (cm^-3)
10+26
10+24
10+22
10+20
10+18
10+16
Ag/DT Interface
141ns
130ns
120ns
0.00
0.05
0.10
0.15
0.20
r (cm)

Implosion dynamics



Peak Implosion Energies



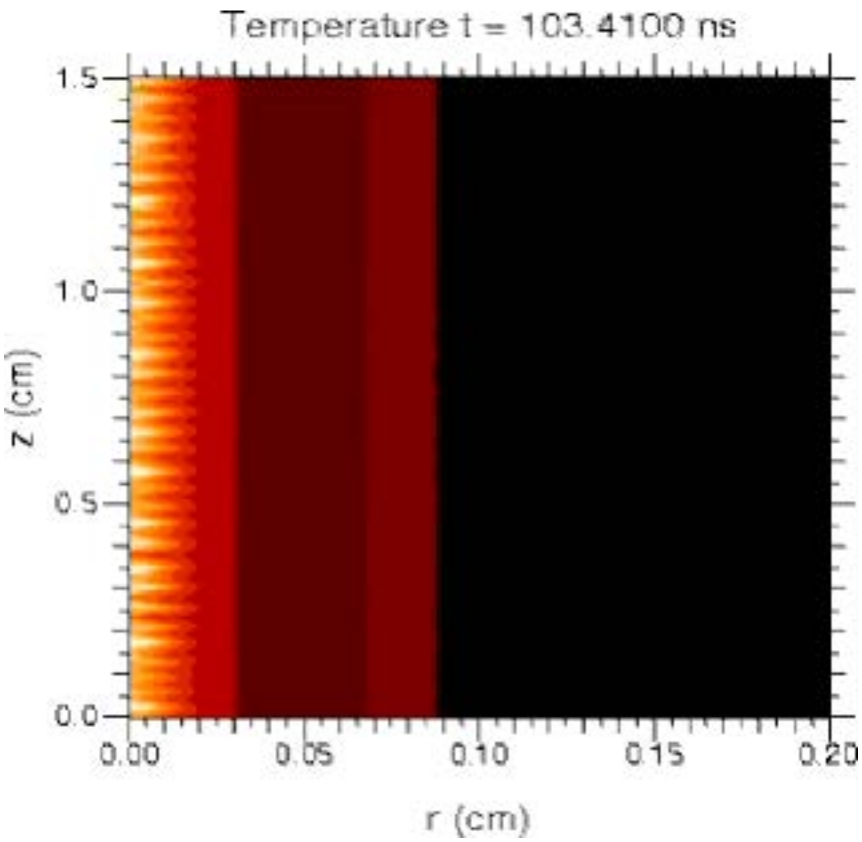
Gain ~ 20

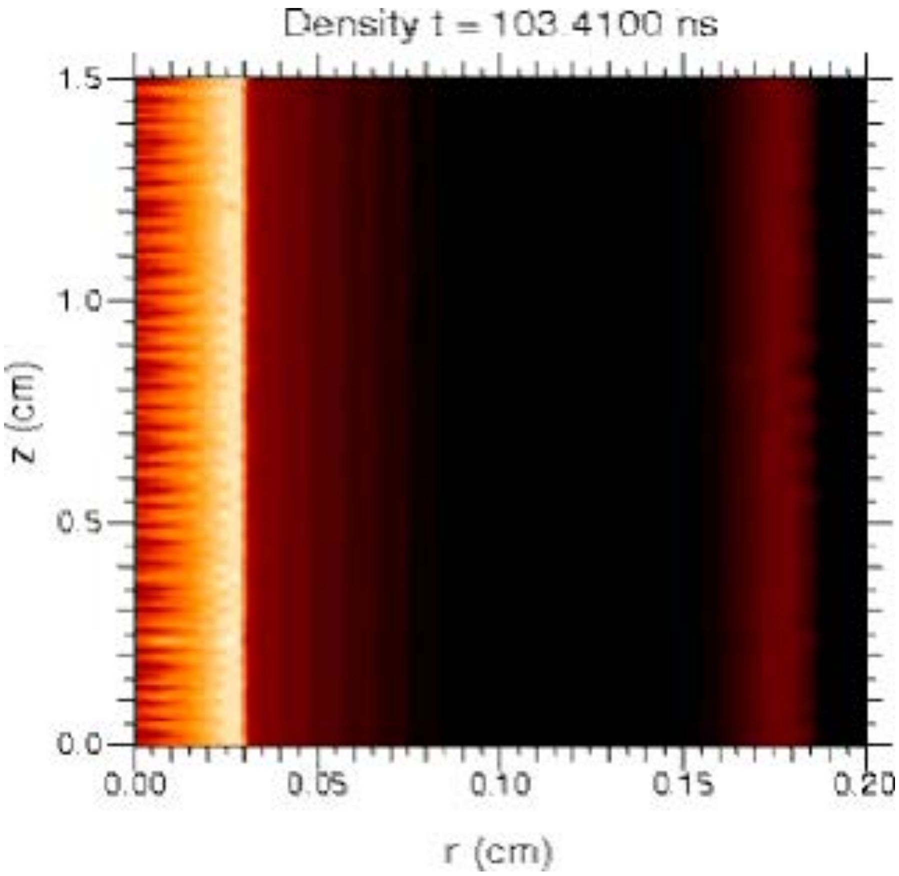
Fusion energy reaches up to 400 MJ

Summary

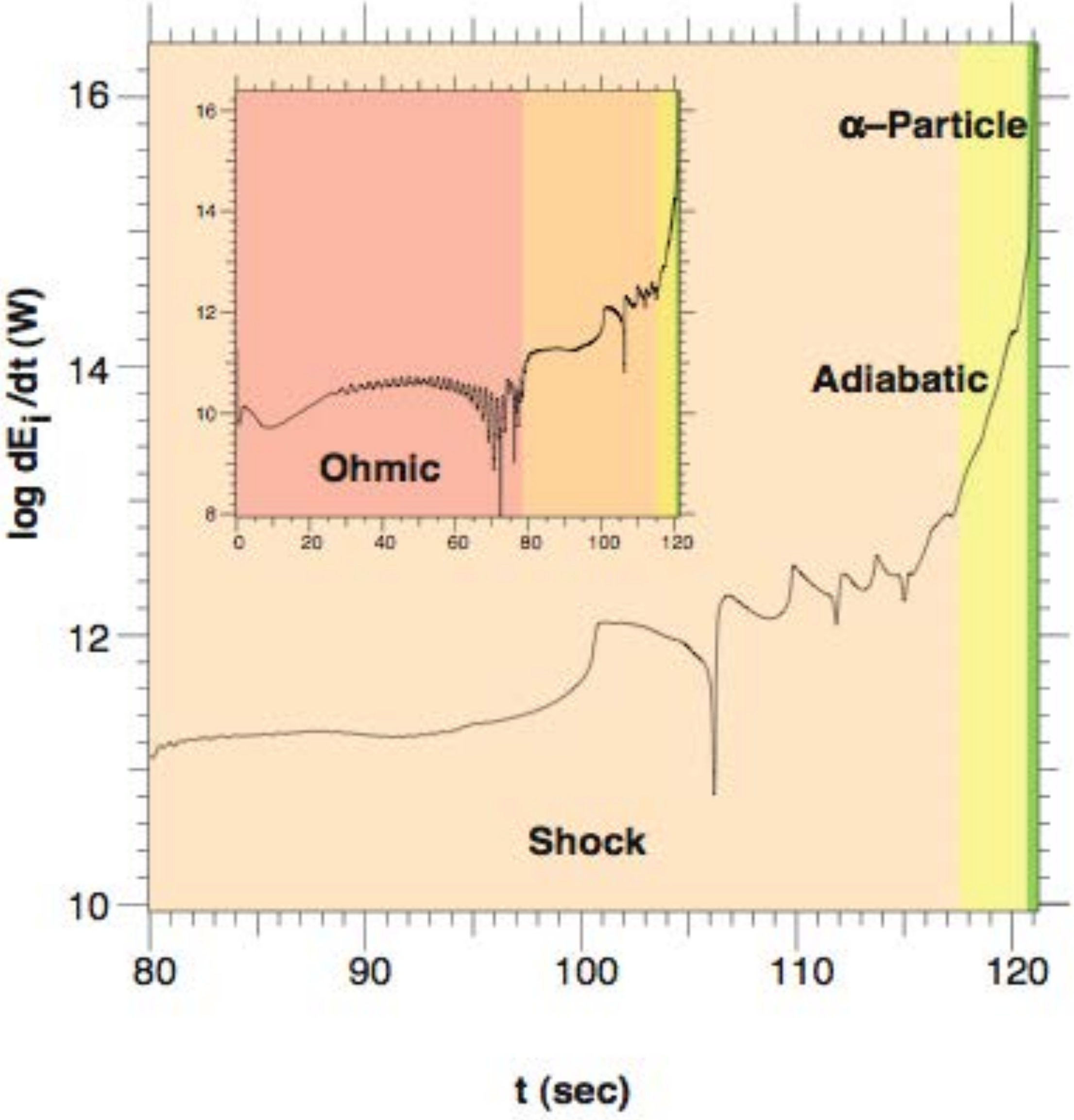
- Believe we have solved the “pinch stability” problem
  - Staged Z-pinch
  - Experiments, theory, simulations
- Energy coupled to the target at the fastest possible rate
  - Stable, shock front internal to liner, compresses target
  - Flux compression of both B_Θ and B_z
  - Inductive currents formed at target surface
  - Target compressed magneto-inertially
- To be fielded on the Univ. Nevada, Reno, Zebra Facility
  - Funded by the ARPA-E Alpha Program
  - 1 MA, 120 ns, 200 kJ
  - Ag liner and DT target near Lawson Criterion, less with D2
- SNL Z Facility
  - Currently investigating MagLIF
  - Predictions for ignition with a Staged Z-pinch
  - Energy gain >20X

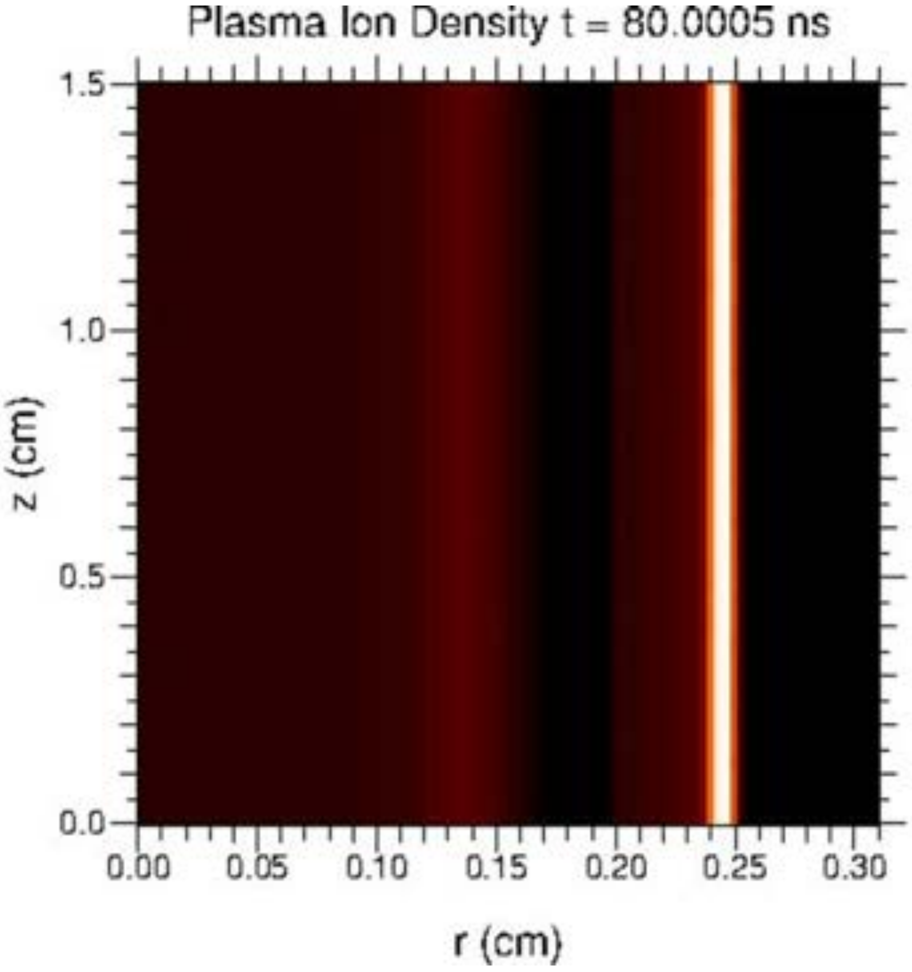
Movies of Ion density and Ion Temperature



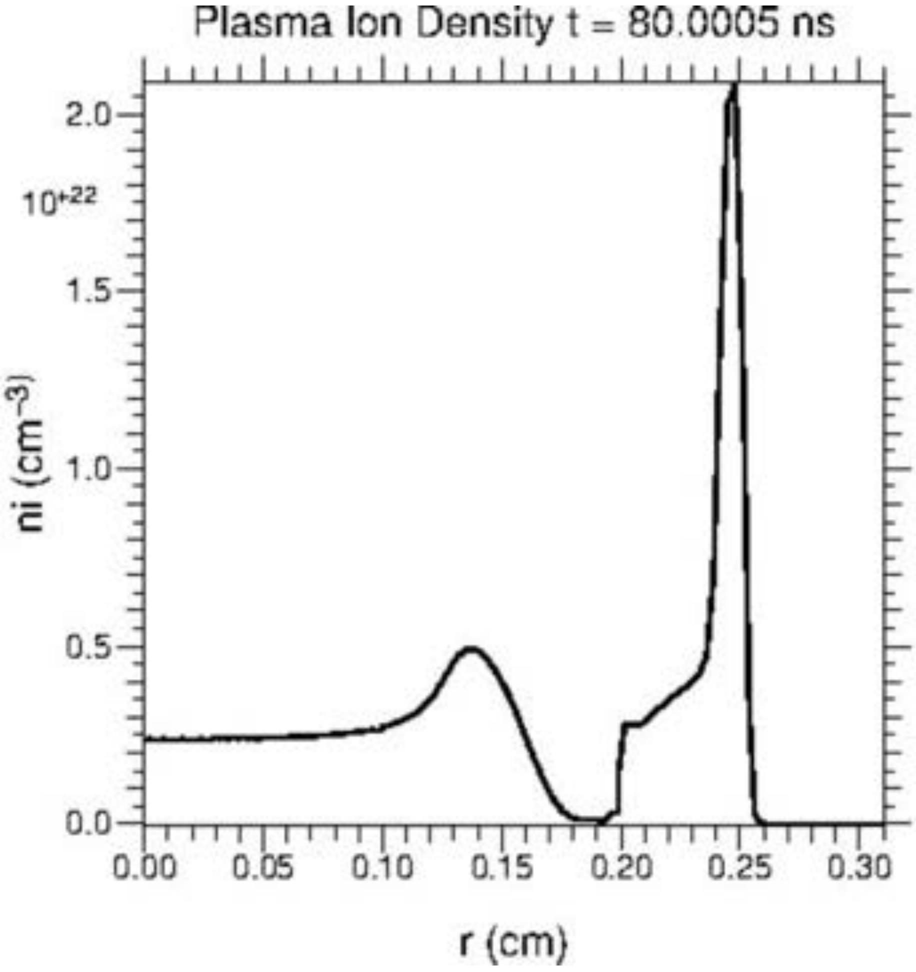


Heating Mechanisms





Plasma Ion Density t = 80.0005 ns



Plasma Ion Density t = 80.0005 ns

ABSTRACT

MACH2

Continuity Equation:

$$\frac{\partial\rho}{\partial t} = -\nabla\cdot(\rho\vec{u})$$

Momentum Equation:

$$\rho\frac{\partial v^i}{\partial t} = -\rho v^j\nabla_j v^i + \nabla_j[-(P + Q + \tfrac{1}{3}u_R)\delta^{ji} + \tfrac{1}{\mu_0}(B^jB^i - \tfrac{1}{2}B^2\delta^{ji}) + \sigma^d_{ji}]$$

Electron Specific Energy Equation:

$$\rho\frac{\partial\epsilon_e}{\partial t} = -\rho\vec{v}\cdot\nabla\epsilon_e - P_e\delta^{ji}\nabla_i v_j + \eta J^2 - \vec{J}\cdot(\frac{\nabla P_e}{en_e}) + \nabla\cdot(\kappa_e\nabla T_e) - ac\rho\chi_{planck}(T_e^4 - T_R^4) - \rho c_{v_e}\frac{(T_e - T_i)}{\tau_{ei}}$$

Ion Specific Energy Equation:

$$\rho\frac{\partial\epsilon_i}{\partial t} = -\rho\vec{v}\cdot\nabla\epsilon_i + [-(P_i + Q)\delta^{ji} + \sigma^d_{ji}]\nabla_i v_j + \nabla\cdot(\kappa_i\nabla T_i) + \rho c_{v_e}\frac{(T_e - T_i)}{\tau_{ei}}$$

Radiation Energy Density:

$$\frac{\partial u_R}{\partial t} = -\rho\vec{v}\cdot\nabla u_R - \tfrac{4}{3}u_R\nabla\cdot\vec{v} + \nabla\cdot(\rho\chi_{ros}\nabla u_R) + ac\rho\chi_{planck}(T_e^4 - T_R^4)$$

Magnetic Induction:

$$\frac{\partial\vec{B}}{\partial t} = \nabla\times(\vec{v}\times\vec{B}) - \nabla\times(\eta\vec{J}) - \nabla\times(\frac{\vec{J}\times\vec{B}}{en_e}) + \nabla\times(\frac{\nabla P_e}{en_e})$$