Physics Qualifying Exam UCI Sept. 27, 1971
Morning Session

ELECTRICITY AND MAGNETISM (2 hours)

Reference book: Jackson, Classical Electrodynamics

Answer any 4 problems.

1. A monochromatic plane electromagnetic wave travels in the +x direction. The electric field is

$$\vec{E} = (0,\ 4 \sin (kx - \omega t),\ E_z)$$

The wave is circularly polarized.

Find E_z. Calculate the $\vec{B}$ field. Calculate the Poynting vector for this wave. State all your assumptions, including choice of units.

2. Maxwell did not use the Lorentz gauge condition commonly used today. Rather, he assumed that the divergence of the vector potential $\vec{A}$ was zero and that the scalar potential was zero. $\vec{A}$ still obeyed a homogeneous wave equation, when no charges or currents were present. Show that these assumptions still lead to Maxwell's equations in free space.

3. An electromagnetic plane wave of frequency ω traveling in a dielectric medium characterized by ϵ_1 and μ_1 is normally incident on a conducting plane characterized by ϵ_2, μ_2 and conductivity σ. Show that in the limit of a good conductor, that is, $\frac{\sigma}{\omega\epsilon_2} \gg 1$, the reflection coefficient is approximately

$$R = \frac{|E_{reflected}|^2}{|E_{incident}|^2} \cong 1 - 2\sqrt{\frac{2\mu_2}{\mu_1} \frac{\omega\epsilon_1}{\sigma}}$$

This relation is known as the Hagen-Rubens relation and describes the infrared reflectivity of metals.

4. In an anisotropic medium, we can, by proper choice of axes, obtain linear relations between the components of $\vec{D}$ and $\vec{E}$:

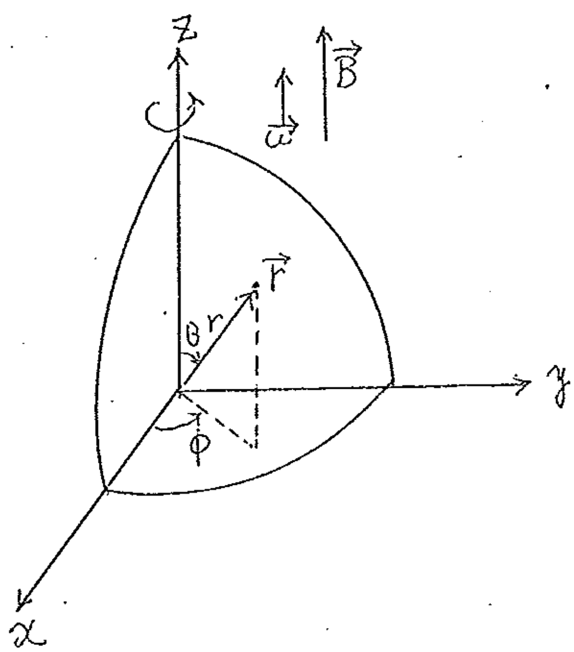
$$D_1 = \epsilon_1 E_1, \quad D_2 = \epsilon_2 E_2, \quad D_3 = \epsilon_3 E_3 \quad .$$

Assume a monochromatic plane wave, with wave vector $\vec{k} = k\vec{n}$ ($\vec{n} = (n_1, n_2, n_3)$ a unit vector in the direction of propagation) and frequency ω. Show that the allowable velocities $v = \omega/k$ must satisfy the Fresnel relation

$$\frac{n_1^2}{v^2 - v_1^2} + \frac{n_2^2}{v^2 - v_2^2} + \frac{n_3^2}{v^2 - v_3^2} = 0 \quad ,$$

where $v_i = c/\sqrt{\epsilon_i}$.

5. A perfectly conducting solid sphere of radius R with zero net charge is set into rotation with a constant angular velocity $\vec{\omega}$ in an external uniform magnetic field $\vec{B}$. $\vec{\omega}$ is parallel to $\vec{B}$.



(a) Show that the electric field at various points r, θ is given by $\vec{E} = E_r\hat{r} + E_\theta\hat{\theta}$ ($\hat{r}$, and $\hat{\theta}$ are unit vectors in direction of increasing r, θ):

$$\left.\begin{aligned} E_r &= -\omega B r \sin^2\theta \\ E_\theta &= -\omega B r \sin\theta\cos\theta \end{aligned}\right\} r < R$$

$$\left.\begin{aligned} E_r &= \frac{-\omega B R^5}{r^4}\left(1 - \frac{3}{2}\sin\theta\right) \\ E_\theta &= \frac{-\omega B R^5}{r^4}\sin\theta\cos\theta \end{aligned}\right\} r > R$$

(Continued on next page)

5. (Continued)

(Hint: At equilibrium, the total electrical force on the individual charged particle within the sphere is zero.)

(b) Show that there is an induced charge characterized by a distribution within the volume of the sphere of

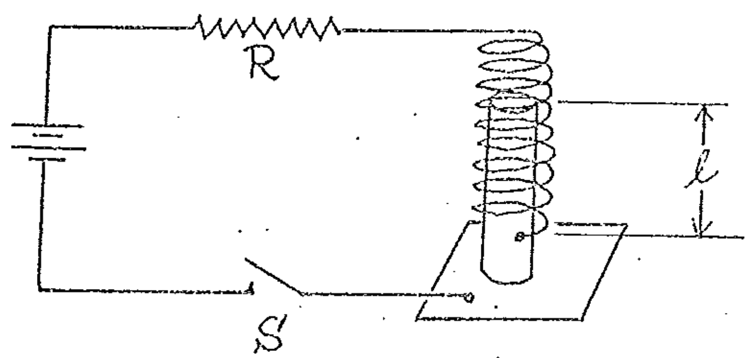
$$\rho = -2\omega B$$

(c) Show that there is a charge distribution on the surface of the sphere of

$$\sigma = \omega BR\left(\frac{5}{2}\sin^2\theta - 1\right)$$

6. Derive the expression for the refractive index for right and left circularly polarized radiation of a medium consisting of N electronic oscillators per unit volume. The oscillators are characterized by elastic force $= -k\vec{r}$, where $\vec{r}$ is the radius vector from the center of the orbit to the instantaneous position of the electron. A static magnetic field H_o is oriented along the direction of propagation $\hat{z}$. Neglect damping. What happens if linearly polarized light is sent along the direction of the static magnetic field? This phenomenon is called Faraday rotation. Note that the polarization of this system is $\vec{P} = -Ne\vec{r}$.

7. Consider the circuit shown below. The rigid solenoid has length L, N turns, cross-sectional area A. The ferromagnetic rod of negligible resistance has length L, cross section a, permeability $5000\mu_o$, and mass M. The lower end of the solenoid is in electrical contact with this rod and the rod stands on a horizontal conducting plate, thus completing the circuit. Switch S is closed at time t = 0. Find the current in the circuit as a function of time. Consider all possible values of R.



PHYSICS QUALIFYING EXAM - U.C.I.

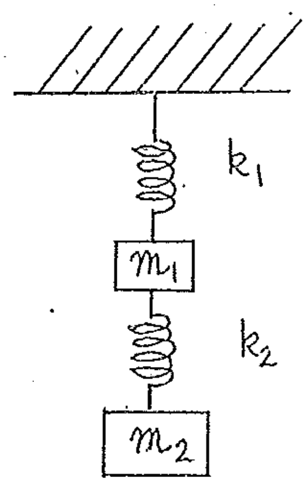
September 27, 1971
Morning Session

Classical Mechanics (2 hours)

Reference Book: Goldstein, Classical Mechanics

Answer any 4 problems

1. Consider the system illustrated below. Mass 1 is held fixed and mass 2 is displaced 2 units of length downward and then both are released from rest. Find the subsequent motion of the system.



The masses move only up and down. Assume $m_1=2$, $m_2=3$, $k_1=10$, and $k_2=20$.

2. Contrary to what you might expect, a satellite orbiting the earth increases its velocity because of the resistance of the earth's atmosphere. Investigate the problem.

3. An early form of electrodynamics used the following velocity dependent potential for a 2-charge system:

$$V = \frac{1}{r} - \frac{(\dot{r})^2}{r}$$

(velocity of light=one)

A) Find the force

B) Find the generalized momentum

C) Write the Lagrangian

D) Find the Hamiltonian

E) Discuss how you would go about determining how a particle moves under such a potential. No solution is required, only a discussion of methods which might lead to a solution.

4. A particle moves under the action of a central force which varies inversely as the cube of the distance from the force center.

A) What conservation laws hold for this motion?

B) Investigate how the particle moves.

5. At one point Dirac was led to consider Lagrangians which were homogeneous linear in the velocities. Investigate the problems associated with deriving a Hamiltonian in such a case.

6. Consider a particle of mass m moving in a central force field described by the potential function $V(\vec{r})$.

a) Show that the following relation must be satisfied if a stable circular orbit is to exist at $r = \rho$.

$$\frac{V''(\rho)}{V'(\rho)} + \frac{3}{\rho} > o$$

b) Consider the stability of circular orbits in a screened Coulomb potential central field where

$$V(r) = \frac{-k}{r} e^{-r/a}$$

and $k > o$ and $a > o$. For what values of the radius of the orbit is the orbit stable?

Physics Qualifying Exam | UCI | September 27, 1971 Afternoon Session

Quantum Mechanics (3 hours)

Closed Book.

Answer any 4 problems

1. Derive the quantum mechanical version of the Virial theorem for a particle of mass m moving in a potential $V(\vec{r})$. Compare your result with the corresponding theorem in classical mechanics as to the physical content of the quantities involved.
   Recall that classically the theorem for this case is:

$$\langle T \rangle = -\tfrac{1}{2} \langle \vec{F} \cdot \vec{r} \rangle$$

   where T is the kinetic energy, F the force and $\langle \rangle$ denotes time average.

2. Consider a system of N identical atoms each of which has two energy states $|0\rangle$ and $|1\rangle$ separated in energy by $\hbar\Omega$. The life time of the upper state $|1\rangle$ is τ. The matrix elements of the electric dipole operators are given by

$$\langle i|\vec{\mu}|j\rangle = \delta_{ij}\vec{\mu} \; ; \; i,j = 0,1$$

   Assume that $\hbar\Omega \gg k_B T$ where k_B is the Boltzmann's constant and T is the temperature of the system.

   i) Assuming that the atoms are all initially in the ground state and that the atoms couple to the EM radiation by a dipole interaction, calculate the rate of energy absorption from a "weak" electromagnetic radiation of frequency $\omega = \Omega$. By "weak" we mean that the relaxation time, τ, times the transition rate is much smaller than unity.

   ii) Now derive a rate equation which gives the number of atoms n in the excited state when the EM radiation is not "weak." At approximately what intensity of the radiation does the energy absorption begin to saturate?

iii) If the volume of this system is V and the atoms are uniformly distributed in this volume, what is the absorption constant of the system for a "weak" EM radiation (i.e., far below saturation)?

3. A hydrogen atom is placed in a crystal at a point where the crystalline electric fields lead to an added potential of the form:

$$V_c(x,y,z) = A\ (x^2 + y^2 - 2z^2)$$

acting on the hydrogenic electron. Here, A is a constant.

a) Working in the representation in which L^2 and L_z are diagonal, find the splitting in the energy levels of the 2p-states as a function of A.

b) Is there any degeneracy for $A \neq 0$? If so, explain the physical origin of the degeneracy. If the potential $V_c' = B(x^2 + 2y^2 - 3z^2)$ is applied in addition to V_c, is any residual degeneracy lifted? Why?

c) What are the eigenfunctions for $A \neq 0$?

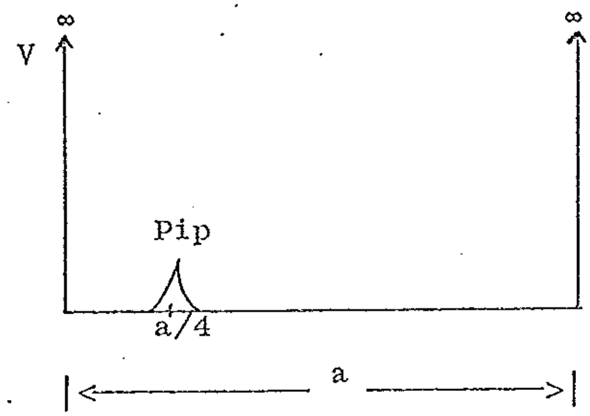
Note: In working out this problem the following functions may be useful:

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

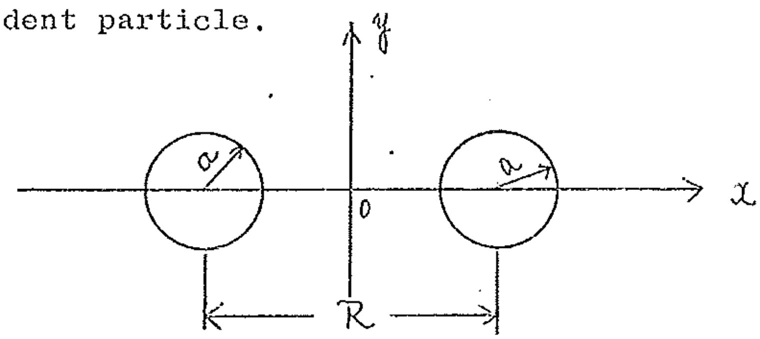
$$Y_1^{\pm 1} = \mp\sqrt{\frac{3}{8\pi}}\ e^{\pm i\varphi}\sin\theta$$

4. A particle of mass M is in a one dimensional box of length a with infinitely high walls. The box has a very small pip on the floor at point a/4 as shown. The integrated area of the pip is W. What is the probability of finding the particle

in the left hand half of the box in its ground state to the lowest non-vanishing order in the effect of the pip. What expansion parameter must be small to justify this approximation?



5. A particle of mass M is incident on a potential $V(\vec{r})$ that consists of two identical spherical wells. The attractive potential is a constant, V_o, inside the well of radius a. They are separated by a distance R > 2a, as shown in the sketch below. Assume that a is very small compared to the de Broglie wavelength of the incident particle.



A particle propagating initially in the y direction scatters from the potential.

a) Calculate the differential scattering cross section per unit solid angle in the first Born approximation. Note that the differential cross section in the first Born approximation is given by:

$$\frac{d\sigma}{d\Omega} = \left| f_{\vec{k}}(\hat{k}') \right|^2$$

where

$$f_{\vec{k}}(\hat{k}') \approx -\frac{m}{2\pi\hbar^2}\int e^{i\vec{q}\cdot\vec{r}'}\, V(\vec{r}')\,d^3r'$$

$\vec{q} = \vec{k} - \vec{k}'$

$\hbar\vec{k}$ = momentum of the incident particle

$\hbar\vec{k}'$ = momentum of the particle after scattering

$\hat{k}'$ = unit vector in the direction of $\vec{k}'$.

b) Form the ratio of the cross section calculated in part a) to that appropriate for a single isolated well of the same depth and radius. Sketch the behavior of this ratio as a function of the scattering angle for these limiting cases:

i) The de Broglie wavelength of the incident particle is very large compared to R.

ii) The de Broglie wavelength is comparable to R.

iii) The de Broglie wavelength is very small compared to R.

c) State the physical reasons for the behavior you found in part i) and iii) of (b).

6. The Hamiltonian for the Zeeman effect in the presence of spin-orbit coupling is given approximately by

$$H = \beta(\vec{L} + 2\vec{S})\cdot\vec{B} + 2A\ \vec{L}\cdot\vec{S}$$

where β is the Bohr magneton; $\vec{L}$ and $\vec{S}$ are orbital and spin angular momentum operators, respectively; and A is the spin-orbit coupling constant. Note that $[H,L^2] = [H,S^2] = 0$, so that no mixing occurs between states of different (ℓ,s). Now consider one electron in a p-state ($\ell = 1$, $s = \frac{1}{2}$).

a) Describe in words how you would find the energy levels of this electron in the weak field limit and the strong field limit, respectively. What basis functions would you use for each limit? Sketch the energy levels and label them with appropriate quantum numers.

b) Obtain exact energy eigenvalues of this Hamiltonian for (ℓ = 1, s = $\frac{1}{2}$) by solving secular equations. Work in the representation which diagonalizes L_z and S_z, i.e., $|M_L, M_S>$. [Hint: $[H,J_z] = 0$ where $J_z = L_z + S_z$.]

c) Express the eigenvectors in terms of $|M_L, M_S>$.

The following properties of the operators L_+ and L_- may prove useful:

$$L_+|\ell,m> = \hbar\sqrt{(\ell - m)(\ell + m + 1)}\,|\ell,m + 1>$$

$$L_-|\ell,m> = \hbar\sqrt{(\ell + m)(\ell - m + 1)}\,|\ell,m - 1>$$

where
$$L^2|\ell,m> = \hbar\ell(\ell + 1)|\ell,m>$$
$$L_z|\ell,m> = \hbar m|\ell,m> .$$

PHYSICS QUALIFYING EXAM - U.C.I.

September 28, 1971

Morning Session

Thermodynamics and Statistical Mechanics (2 hours)

Answer any 4 problems

Reference book: K. Huang, Statistical Mechanics

1. Consider a system of N independent classical one-dimensional anharmonic oscillators with a potential energy

$$\varphi(x) = ax^2 - bx^3 - cx^4 .$$

Obtain an expression for the specific heat which contains contributions of lowest nonvanishing order in b and c. When will this expression be valid?

2. Two boxes containing molecules of spin $\frac{1}{2}$ and magnetic moment μ are joined by a small tube. The respective volumes of the boxes are V_1 and V_2. The molecules are free to move from one box to the other and the total number of molecules is N. There is a constant magnetic field of strength B in box 2. Calculate the ratio of the number of molecules in the two boxes in equilibrium at a temperature T. [Stirling's approximation: $\ln(n!) \approx n\ln(n) - n$]

3. A gas of N particles is contained in a cylinder of cross-sectional area A, but of infinite height. The "gravitational potential" is mgz. Each molecule is a rigid rotator, with energy levels $K(K+1)\hbar^2/2I$ each $(2K+1)$ fold degenerate. (I is the moment of inertia.) Each molecule has a nondegenerate electronic ground state, and a 3-fold degenerate excited state at energy ϵ_o (all higher states can be neglected). What is the entropy of this system assuming that kT is large compared to the splitting of translational and rotational states?

4. As is well known, a superconductor can be made to undergo a transition from the superconducting state to the normal state by applying a sufficiently strong magnetic field H. This transition from the superconducting into the normal state is a reversible phase transition in the H-T plane across a threshold curve quite like any other phase transition, for instance, evaporation in the p-T plane. Just as the evaporation phase transition is described by the Gibbs free energy G(p,T), the superconducting transition is described by an analogous thermodynamic free energy g(H,T). It can be shown that

$$g(H,T) = \begin{cases} \Phi_o - \frac{1}{8\pi} H_c^2(T) & |H| \le H_c \\ \Phi_o - \frac{1}{8\pi} H^2 & |H| \ge H_c \end{cases}$$

where H_c is the magnetic field at which the transition occurs and Φ_o is a function only of the temperature.

If $H_c(T) = H_o \left[1 - \left(\frac{T}{T_c}\right)^2\right]$ where H_o and T_c are constants, find (a) the difference between the entropy of the normal and superconducting states as a function of T; (b) the heat released or absorbed per mole at the transition, and (c) the molar specific heat difference between the normal and superconducting states.

5. In a temperature range near absolute temperature T, the tension force F of a stretched plastic rod is related to its length L by the expression

$$F = aT^2 (L - L_o)$$

where a and L_o are positive constants, L_o being the unstretched length of the rod. When $L = L_o$, the heat capacity C_L of the rod (measured at constant length) is given by the relation $C_L = bT$ where b is a constant.

a) Express the change in entropy dS in terms of dE and dL where E is the internal energy of the rod.

b) Compute $\left(\frac{\partial S}{\partial L}\right)_T$ using the appropriate Maxwell relation.

c) Knowing $S(T_o,L_o)$, find S(T,L) at any other temperature T and length L.

d) If one starts at $T = T_i$ and $L = L_i$ and stretches the thermally insulated rod quasi-statically until it attains the length L_f, what is the final temperature T_f? Is T_f larger or smaller than T_i?

6. The equation of state of a Van der Waals gas is

$$(p + \frac{a}{v^2})(v - b) = RT.$$

a) show that the molar energy is

$$E(T,v) = C_v T - \frac{a}{v} + \text{constant}.$$

[Hint: the Maxwell relations may prove useful.]

b) Use this result to calculate the change in temperature in the case of free expansion of one mole of a Van der Waals gas from volume v_1 to volume v_2. (Neglect any temperature dependence of C_v in this calculation.)

PHYSICS QUALIFYING EXAM - U.C.I.

September 28, 1971

Morning Session

Quantum Mechanics (1 hour)

Closed Book.

Answer any 2 problems from (7) through (10).

Answer any 3 problems from (11) through (16).

7. We are given a system described by a Hamiltonian operator H_o. It has N nondegenerate energy levels whose energies are given by $E_1^{(o)}$, $E_2^{(o)}$, ..., $E_N^{(o)}$. (These N levles are the complete spectrum of H_o). We now perturb this system by adding a term V to the Hamiltonian H_o yielding a new Hamiltonian $H = H_o + V$. The energies of the eigenstates of H will be denoted by E_1, E_2, ..., E_N. We are interested in the sum of the energy levels of the perturbed system, which is given by

$$Q = \sum_{S=1}^{N} E_S \quad .$$

One could obviously compute an approximate value Q by calculating each perturbed energy eigenvalue E_s by perturbation theory, starting with the energy $E_s^{(o)}$ of the Sth eigenstate of H_o, and summing the resulting expansion over S from 1 to N.

You are asked to prove that the exact value of Q is obtained if E_s is calculated only to first order in the perturbation V.

8. Using the $1s^2\ 2s^2\ 2p^2$ electronic configuration of Carbon as an example, mathematically justify the Hund's rule which states that the ground state of the configuration has the maximum value of the total spin S consistent with the Pauli principle. You may assume that the exchange integral is positive definite.

9. Consider any L-S level of an atom characterized by a definite J value. Using a group theoretical argument, show that the degeneracy of this L-S level is completely lifted if the atom is placed in a uniform magnetic field.

10. Describe quantitatively two experiments that demonstrate the particle-like behavior of electromagnetic radiation.

ANSWER 3 PROBLEMS FROM THE FOLLOWING:

11. The groups SU_3 and SU_6 have caused much excitement in elementary particle physics over the past few years. How does SU_6 differ from SU_3 (as far as elementary particle physics is concerned).

12. Give the quantum numbers of the initial and final states for the atomic transitions giving rise to the spectral series:

a) Sharp
b) Principal
c) Diffuse
d) Fine

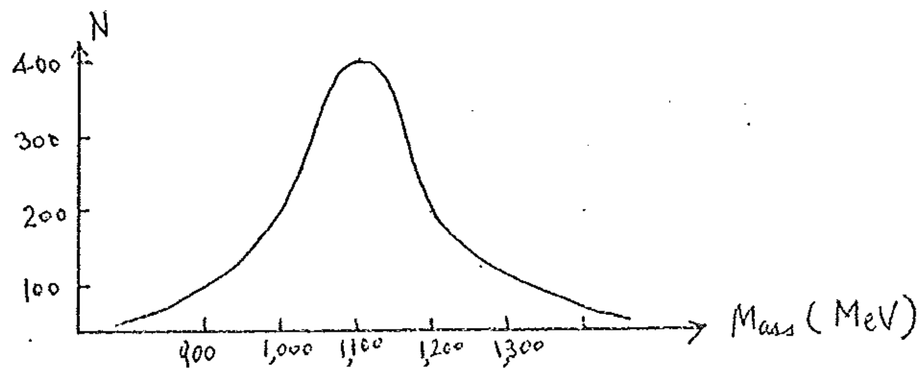
13. Estimate the cross section for the reaction

$$\bar{\nu}\, p \to n e^{+}$$

where the $\bar{\nu}$ comes from the decay

$$\mu^{+} \to e^{+}\, \nu\, \bar{\nu} \qquad \text{(at rest).}$$

14. In a particular experiment a very large number of A-mesons is produced. When the masses of all these mesons are histogramed we find:



What is the lifetime of the A-meson (in seconds)? What is the mean free path for decay of an A-meson at 2 GeV/c?

15. It is noted that the linewidth of spectral lines produced by an electrical discharge through a gas becomes broader as the temperature is increased. What mechanisms are responsible for this broadening?

16. The quantum mechanical phase difference between any two points r_1 and r_2 within the bulk of a superconducting material is given by

$$\varphi(\vec{r}_1) - \varphi(\vec{r}_2) = \int_{r_1}^{r_2} \frac{2e\vec{A}\cdot d\vec{\ell}}{\hbar}$$

where $\vec{A}$ is the vector potential and 2e is the charge of the Cooper pair. Show that the magnetic flux contained within a superconducting ring must be an integral multiple of $\frac{h}{2e}$.

PHYSICS QUALIFYING EXAM - U.C.I.

September 28, 1971

Afternoon Session

Mathematical Physics (2 Hours)

Answer any 4 problems

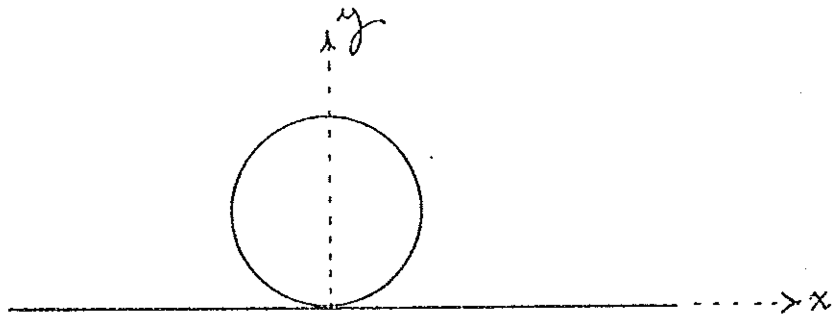
Reference Book: Any one of the following books-

1) Boas, Mathematical Methods in Physical Sciences
2) Arfken, Mathematical Methods for Physicists
3) Mathews & Walker, Mathematical Methods for Physics
4) Irving & Mullineux, Mathematics in Physics and Engineering

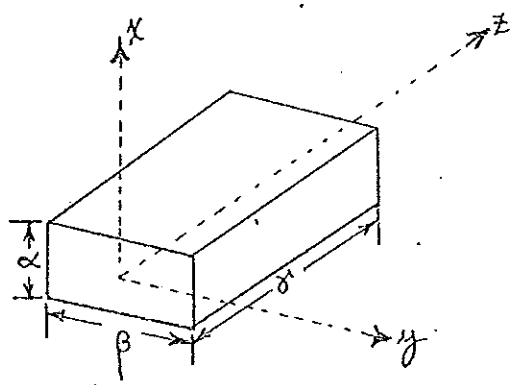
1. Evaluate:

$$\int_{-\infty}^{\infty} \frac{\sin x\, dx}{x^2+4x+5}$$

2. A cylinder (of unit diameter) rests on an infinite plane (with a thin piece of electrical insulation separating them). The cylinder is kept at 100 volts and the plane is kept at 0 volts. Find the electrical potential in the region outside the cylinder. Sketch some of the equipotentials in this region. (Hint: conformal mapping may be useful).. - SEE FIGURE ON NEXT PAGE



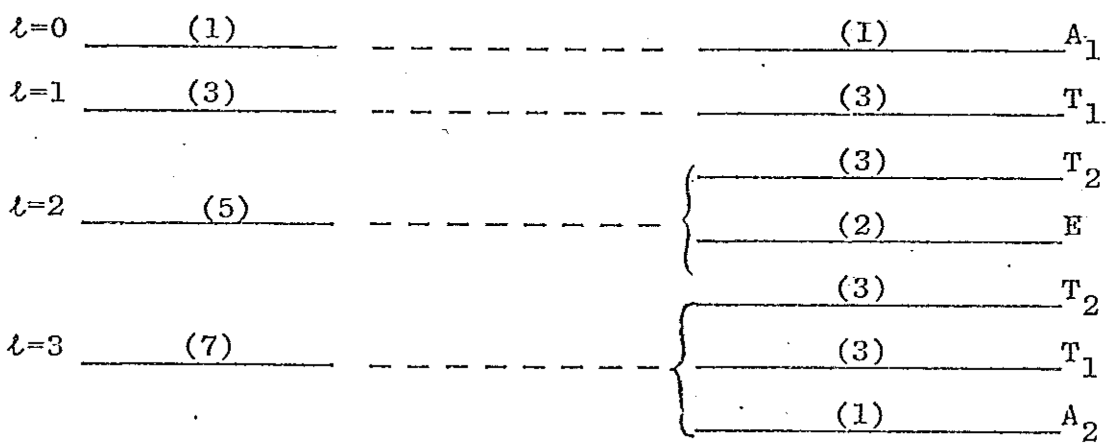
3. A rectangular bar is initially at constant temperature, T_i. At t=0 it is immersed into a cooling bath ($T=0^{o}$) and the end at z=0 is uniformly heated at a constant rate (q joules/sec). Find the temperature distribution in the rod T(x,y,z) as a function of time. Assume the thermal conductivity, k, is equal to unity.



4. A unit point mass (m=1) is suspended by a spring (of spring constant k). The mass is in a fluid which exerts a frictional force on the mass which is proportional to the velocity of the mass (the constant of proportionality is c). At t=0 a sharp blow (parallel to the spring's axis) is delivered to the mass (which was originally at rest at the equilibrium position). Assume the impulsive force has the explicit form $F(t)=a\ \delta(t)$. If $c < k/m$, find x(t), the position of the mass as a function of time. Assume that the motion is one-dimensional.

5. Neglecting spin, the electrons of an atom are each specified by the quantum numbers n, ℓ, and m. In free space, the energy of an electron depends only on n and ℓ (assuming each electron moves in a spherically symmetric potential).

If this atom is put in a crystal of octahedral symmetry some of these levels split:



(The numbers in parentheses are the degeneracies of the levels).

In atomic physics you learned the selection rules for electric dipole transitions from one ℓ level to another:

$$\Delta \ell = \pm 1.$$

The problem here is to find the selection rules for electric dipole transitions in an atom when it is in a octahedral crystal (i.e. is $A_1 \leftrightarrow T_1$ allowed?, etc.) The character table for the octahedral group may be found on the last page of this exam.

6. Find the equation of the path of a light ray traveling from (x_1, y_1) to (x_2, y_2) in a medium which has an index of refraction proportional to 1/x, i.e. $n(x,y) \propto 1/x$. What simple geometric path is this?

7. Let φ be a solution of the differential equation

$$\frac{d^2\varphi}{dx^2} + K^2 \varphi = 0 \qquad 0<x<\ell$$

which satisfies the boundary conditions

$$\varphi(o) = 0 \qquad \text{and} \qquad \varphi(\ell) = \ell\varphi'(\ell).$$

a) What is the equation that the eigenvalues $\{K_n\}$ satisfy?

b) Find the eigenvalues.

c) Find the eigenfunctions $\{\varphi_n(x)\}$ of this differential operator, and normalize them to unity.

d) Find the value of

$$I_{mn} = \int_o^\ell \varphi_m(x)\ \varphi_n(x)\ dx\ .$$

	$y = f(t),\ t > 0$ $(y = f(t) = 0,\ t < 0)$	$Y = L(y) = F(p) = \int_0^\infty e^{-pt} f(t)\, dt$
L27	$\delta(t - a), \quad a \geq 0$ (See Section 7.)	e^{-pa}
L28	$f(t) = \begin{cases} g(t-a), & t > a > 0 \\ 0, & t < a \end{cases}$ $= g(t-a)u(t-a)$	$e^{-pa}G(p)$ [$G(p)$ means $L(g)$.]
L29	$e^{-at}g(t)$	$G(p + a)$
L30	$g(at), \quad a > 0$	$\frac{1}{a} G\left(\frac{p}{a}\right)$
L31	$\frac{g(t)}{t}$ (if integrable)	$\int_p^\infty G(u)\, du$
L32	$t^n g(t)$	$(-1)^n \frac{d^n G(p)}{dp^n}$
L33	$\int_0^t g(\tau)\, d\tau$	$\frac{1}{p} G(p)$
L34	$\int_0^t g(t - \tau)h(\tau)\, d\tau = \int_0^t g(\tau)h(t - \tau)\, d\tau$ (convolution of g and h, often written as $g * h$; see Section 8)	$G(p)H(p)$

L35 Transforms of derivatives of y (see Section 3):

$L(y') = pY - y_0$

$L(y'') = p^2 Y - py_0 - y_0'$

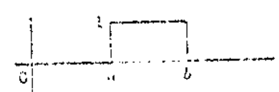
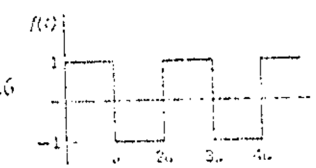
$L(y''') = p^3 Y - p^2 y_0 - py_0' - y_0''$, etc.

$L(y^{(n)}) = p^n Y - p^{n-1} y_0 - p^{n-2} y_0' - \cdots - y_0^{(n-1)}$

The character table for the cubic group is:

O (432)		E	$8C_3$	$3C_2 = 3C_4^2$	$6C_2$	$6C_4$
	A_1	1	1	1	1	1
	A_2	1	1	1	−1	−1
$(x^2 - y^2, 3z^2 - r^2)$	E	2	−1	2	0	0
(R_x, R_y, R_z) (x, y, z)	T_1	3	0	−1	−1	1
(xy, yz, zx)	T_2	3	0	−1	1	−1

	$y = f(t),\ t > 0$ $(y = f(t) = 0,\ t < 0)$	$Y = L(y) = F(p) = \int_0^\infty e^{-pt} f(t)\,dt$	
L1	1	$\frac{1}{p}$	$\text{Re}\, p > 0$
L2	e^{-at}	$\frac{1}{p+a}$	$\text{Re}\,(p+a) > 0$
L3	$\sin at$	$\frac{a}{p^2+a^2}$	$\text{Re}\, p > \lvert\text{Im}\, a\rvert$
L4	$\cos at$	$\frac{p}{p^2+a^2}$	$\text{Re}\, p > \lvert\text{Im}\, a\rvert$
L5	$t^k,\quad k > -1$	$\frac{k!}{p^{k+1}}$ or $\frac{\Gamma(k+1)}{p^{k+1}}$	$\text{Re}\, p > 0$
L6	$t^k e^{-at},\quad k > -1$	$\frac{k!}{(p+a)^{k+1}}$ or $\frac{\Gamma(k+1)}{(p+a)^{k+1}}$	$\text{Re}\,(p+a) > 0$
L7	$\frac{e^{-at} - e^{-bt}}{b-a}$	$\frac{1}{(p+a)(p+b)}$	$\text{Re}\,(p+a) > 0$ and
L8	$\frac{ae^{-at} - be^{-bt}}{a-b}$	$\frac{p}{(p+a)(p+b)}$	$\text{Re}\,(p+b) > 0$
L9	$\sinh at$	$\frac{a}{p^2-a^2}$	$\text{Re}\, p > \lvert\text{Re}\, a\rvert$
L10	$\cosh at$	$\frac{p}{p^2-a^2}$	$\text{Re}\, p > \lvert\text{Re}\, a\rvert$
L11	$t \sin at$	$\frac{2ap}{(p^2+a^2)^2}$	$\text{Re}\, p > \lvert\text{Im}\, a\rvert$
L12	$t \cos at$	$\frac{p^2-a^2}{(p^2+a^2)^2}$	$\text{Re}\, p > \lvert\text{Im}\, a\rvert$
L13	$e^{-at} \sin bt$	$\frac{b}{(p+a)^2+b^2}$	$\text{Re}\,(p+a) > \lvert\text{Im}\, b\rvert$
L14	$e^{-at} \cos bt$	$\frac{p+a}{(p+a)^2+b^2}$	$\text{Re}\,(p+a) > \lvert\text{Im}\, b\rvert$
L15	$1 - \cos at$	$\frac{a^2}{p(p^2+a^2)}$	$\text{Re}\, p > \lvert\text{Im}\, a\rvert$

	$y = f(t),\ t > 0$ $(y = f(t) = 0,\ t < 0)$	$Y = L(y) = F(p) = \int_0^\infty e^{-pt} f(t)\,dt$	
L16	$at - \sin at$	$\frac{a^3}{p^2(p^2+a^2)}$	$\text{Re}\, p > \lvert\text{Im}\, a\rvert$
L17	$\sin at - at \cos at$	$\frac{2a^3}{(p^2+a^2)^2}$	$\text{Re}\, p > \lvert\text{Im}\, a\rvert$
L18	$e^{-at}(1 - at)$	$\frac{p}{(p+a)^2}$	$\text{Re}\,(p+a) > 0$
L19	$\frac{\sin at}{t}$	$\arctan \frac{a}{p}$	$\text{Re}\, p > \lvert\text{Im}\, a\rvert$
L20	$\frac{1}{t} \sin at \cos bt,$ $a > 0,\ b > 0$	$\frac{1}{2}\left(\arctan \frac{a+b}{p} + \arctan \frac{a-b}{p}\right)$	$\text{Re}\, p > 0$
L21	$\frac{e^{-at} - e^{-bt}}{t}$	$\ln \frac{p+b}{p+a}$	$\text{Re}\,(p+a) > 0$ and $\text{Re}\,(p+b) > 0$
L22	$1 - \text{erf}\left(\frac{a}{2\sqrt{t}}\right),\quad a > 0$	$\frac{1}{p} e^{-a\sqrt{p}}$	$\text{Re}\, p > 0$
L23	$J_0(at)$	$(p^2+a^2)^{-1/2}$	$\text{Re}\, p > \lvert\text{Im}\, a\rvert$
L24	$f(t) = \begin{cases} 1, & t > a > 0 \\ 0, & t < a \end{cases}$ [unit step, often written $f(t) = u(t-a)$]	$\frac{1}{p} e^{-pa}$	$\text{Re}\, p > 0$
L25	$f(t) = u(t-b) - u(t-a)$ 	$\frac{e^{-ap} - e^{-bp}}{p}$	All p
L26		$\frac{1}{p} \tanh \frac{1}{2} ap$	$\text{Re}\, p > 0$

Physics Qualifying Exam | UCI | September 28, 1971
Afternoon Session

General Topics (2 hours)

Answer any 10 questions. **CLOSED BOOK**

Be *brief* and to the point.

1. List the fundamental interactions known to us at the present time and describe the nature of each. Be specific about the ranges and the strengths. For each of these interactions give an example of a physical phenomenon which is dominated by it.

2. Suppose magnetic monopoles were found to exist. How could electromagnetic theory be adapted to cover this possibility?

3. Discuss the relation between symmetry or invariance principles and conservation laws in classical or quantum mechanics. Discuss more than three such examples.

4. Speculate as to who may win the next Nobel Prize in physics. Support your choice with comments about his contribution to modern physics. If you are unfamiliar with names of individuals, discuss areas of physics.

5. You are given n points in space, with a one ohm resistor connecting each pair of points. Find the resistance between any two points.

6. What are the properties of black holes? (In cosmology, that is!) How would you detect them?

7. What are tachyons? Describe some of their "unusual" properties.

8. Describe the results of the Dickie-Roll-Etvös experiment.

9. Nuclear powered electrical generating stations are constrained to operate at lower temperatures than conventional power stations because of the metallurgy of reactor interiors. For this reason, nuclear power stations generate more thermal pollution than conventional power plans. Justify the latter statement.

10. Describe briefly how the general theory of relativity treats gravity.

11. Why is there essentially no atmosphere on the moon?

12. Why is the harmonic oscillator so important a physical system?

13. Explain, in one sentence, the mechanism in quantum electrodynamics that accounts for the force between charged particles.

14. In what sense is a physical theory "true" or "false"?