Physics Qualifying Exam UCI September 24, 1973

QUANTUM MECHANICS I

1. A one dimensional harmonic oscillator of charge e is perturbed by a weak electric field of strength E in the positive X direction.
Calculate using perturbation methods the energy shift of each level to second order.
Note: Use raising and lowering operator methods to evaluate matrix elements.

2. Angular Momentum and Spin-Statistics
   a. For a valence shell with two p electrons, write down the possible spectroscopic terms and their degeneracy.

   b. Write down an explicit, antisymmetric wave function in terms of simple particle wave functions for each state of the D term.

3. Consider the potential $V(x) = A\delta(x)$ $-a<x<a$ and

$$V(x) = +\infty, \quad |x|>a.$$

   a. Find the eigenvalue condition and wave functions.
   b. Show graphically the eigenvalue equation solutions for the case $A > 0$.
   c. What do the eigenvalues approach as $A \to \infty$? What is the physical interpretation?

4. To a first approximation, the potential that a charged particle feels from a hydrogen atom can be thought of as that due to a positive point charge at the origin plus a uniform cloud of negative charge occupying a sphere of radius a_0 about the origin.
   a. Calculate in the Born approximation the scattering amplitude for the scattering of a charged particle from this charge distribution.

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4. b. Calculate the large momentum transfer limit of the amplitude and compare it to the pure Coulomb amplitude.

   c. Show that the total elastic cross section from this potential is finite.

5. Calculate the spontaneous transition probability for dipole transitions from <u>all</u> $n = 2$ hydrogen atom levels to the ground state.

QUANTUM MECHANICS II

1. An electron beam is diffracted around an infinitely long magnetized iron whisker. Ignoring for the moment the fact that it is magnetized, explain qualitatively what kind of diffraction pattern you expect to see. Now include the fact that the whisker is magnetized. How does the pattern change? Calculate the effect.

2. Qualitative - Atomic Physics
   a. Outline the interactions, approximations and conserved quantum numbers that yield the various stages of classification of the states of an atom. That is, explain the classification scheme of atomic physics, beginning with the central field approximation and working down to the spin-orbit coupling. Give the order of magnitude of the energy splitting at each stage.
   b. Show the Hamiltonian for the interaction with an external magnetic field. What levels are split by a weak magnetic field? What are an appropriate set of state labels or quantum numbers for a strong magnetic field.

3. Obtain the electric polarizability of a hydrogen atom in its ground state by calculating the change in the ground state energy, using a perturbation theory. Note that the hydrogenic wave function for the ground state is given by:

$$\psi_o = (\pi a_o^3)^{-\frac{1}{2}} \exp(-r/a_o) \quad \text{where } a_o \text{ is the Bohr radius.}$$

To simplify the problem, assume that all the excited states lie at E = 0, although their energies are actually given by

$$E_n = -\frac{1}{n^2} \cdot \frac{e^2}{2a_o}$$

where n is the principal quantum number. Does this approximation overestimate or underestimate the polarizability? (The polarizability α is obtained from the second order perturbation energy by $W_2 = -\frac{1}{2}\alpha\mathcal{E}^2$ where $\mathcal{E}$ is the electric field strength.)

GENERAL PHYSICS

1. The attractive or repulsive nature of a force cannot be determined from differential scattering cross section measurements ($d\sigma/d\Omega$). Why? Illustrate your explanation with a sketch.

2. What are the possible (electromagnetic) multipole moments ${}^{14}_{7}N_7$? (ground state spin-parity, 1^+). Why?

3. Consider the following reactions

   a) $\gamma + p \rightarrow p + \pi^o$

   b) $\gamma + p \rightarrow p + p + \bar{p}$

   What is the threshold photon energy for each reaction in terms of pion mass and proton mass?

4. What is the energy released when a negative muon goes from a state of n = 6 to n = 5 in lead? (Z = 82)

5. What is "coherent light"? How do you measure "coherence"?

6. When you see a piece of metal, you can generally tell that it is a metal. Discuss why you can tell a metal from other forms of solids. Are there non-metals which can look like a metal?

7. Explain with short arguments and sketches:

   a. Why is the sky blue?

   b. Why is the setting sun red?

   c. Why does the sun sometimes appear green just as it sets (the green flash)?

   d. Why do things appear colorless in moonlight?

   Two of these questions rely upon both physical and physiological effects.

(Over)

8. A. Olbers' Paradox (1826) states that if (a) the universe is infinite (and Euclidean), and (b) is uniformly populated with stars with some constant average absolute luminosity, then, ignoring the screening due to stars intercepting light travelling in space, the radiant energy flux at any point in space (say on the surface of the earth) is infinite. Prove this assertion.

B. If we take into account the screening effect ignored in A, then one can show that the energy flux (per unit area per unit time) at any point in space is the same as the average energy flux at the surface of a star. Prove this assertion.

9. Consider the elastic scattering process

$$\bar{\nu}_e + e^- \rightarrow \bar{\nu}_e + e^-$$

for an incident anti neutrino on an electron. Derive the equation relating initial and final neutrino energies and the scattering angle.

10. Are negative temperatures realizable? If not, why? If so, describe such a physical system.

11. What is the "Island of stability" in nuclear physics? Why might it exist?

12. A. In a glass of water at 0^oC, there floats a cube of ice. Enough heat is added to melt the ice at 0^oC. Does the water level rise, fall or remain constant? Why? The water temperature is raised to 2^oC, Does the water level rise, fall or remain constant?

B. Do winds in the northern hemisphere blow clockwise or counterclockwise around a center of low pressure? Why?

13. Compare the ground state energy of a particle in a one dimensional infinite square well with that of a finite square well. Discuss the differences, if any.

14. Plasmas exist in metals and as hot, ionized gases. Describe briefly how these two types of plasmas differ, how the relevant parameters vary in magnitude and what sort of phenomena might be seen in one case, but not the other.

MATHEMATICAL PHYSICS

1. Solve by Fourier transformation

$$(\nabla^2 - \frac{1}{L^2})\psi(\vec{x}) = A\delta(\vec{x})$$

Subject to the boundary conditions

$$\psi(\vec{x}),\ \nabla\psi(\vec{x}) \text{ vanish as } |\vec{x}| \to \infty .$$

2. a. Discuss the analytic properties of the function

$$F(z) = \int_{-\infty}^{\infty} \frac{e^{-x^2}dx}{x-z}$$

where the integration is along the real x-axis.

b. Consider a different function (G(z) = F(z) for Re $z > 0$. For Re $z < 0$, G(z) is defined by analytic continuation. How is this accomplished and how do the analytic properties of G(z) differ from F(z)?

3. In a spherical sample of uranium 235 the neutron density $n(\underset{\sim}{x}, t)$ obeys the differential equation

$$\nabla^2 n + n = \frac{\partial n}{\partial t} ,$$

with the condition $n = 0$ at the surface. Assuming that n is independent of the polar and azimuthal angles θ and φ, what is the smallest value of the radius a of the sphere for which n will erupt exponentially with time?

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4. The electronic polarizability of a medium at frequency ω is

$$\alpha(\omega) = \alpha'(\omega) - i\alpha''(\omega)$$

Given that

$$\alpha''(\omega) = \frac{e^2}{m} \sum_{i=1}^{N} \frac{\Gamma_i \omega}{(\omega^2-\omega_i^2)^2+(\Gamma_i\omega)^2}$$

where ω_i, Γ_i, and e^2/m are constants. Also $\omega_i \gg \Gamma_i$. Find $\alpha'(\omega)$ and discuss the physical and mathematical basis of the derivation.

5. Assume that A is a Hermitian Matrix.
   a. Show that $(A+iI)^{-1}(A-iI)$ is unitary where I is the unit matrix.
   b. If A has the eigenvalues $(\lambda_1\lambda_2\lambda_3\cdots\lambda_n)$ what are the eigenvalues of $(A+iI)^{-1}(A-iI)$?

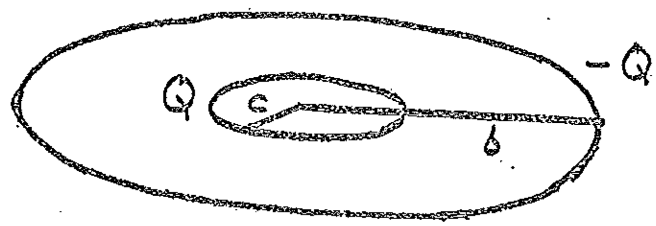
6. Consider the irreducible representations of the rotation group $D^{(j)}$.
   a. What are the characters of these representations?
   b. Show how to reduce the product representation $D^{(3)} \times D^{(2)}$ by the method of characters.

7. By contour integration evaluate the integral

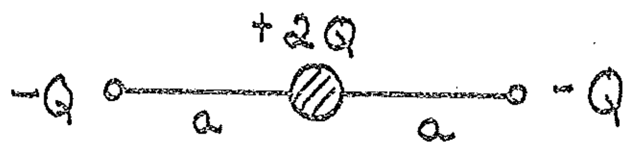
$$I(\alpha) = \text{P.V.} \int_0^\infty \frac{x^\alpha dx}{x^2-1} \quad \text{where } 0 < \alpha < 1.$$

ELECTRICITY AND MAGNETISM

1. We begin with a ring charge −Q of radius b, concentric and coplanar with a smaller ring charge Q of radius c.



We want to model this approximately at large distances as a linear quadrapole which looks like:



What should we take a to be in terms of b and c?

2. A certain plasma consists of a gas of free electrons, neutralized by a uniform positive background charge. The plasma is in a DC magnetic field, B, which is in the z-direction. Consider the propagation of low-frequency electromagnetic waves parallel to B in the plasma. (Hint: such waves are circularly polarized and purely transverse.) Show that for one sign of circular polarization the waves propagate, and for the other sign are attenuated. The propagating waves are called whistlers (in ionospheric physics) or helicons (in solid state physics). What is their dispersion relation? What is their group velocity?

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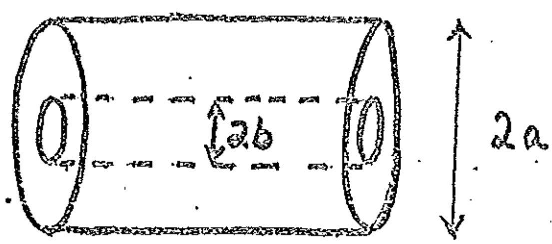
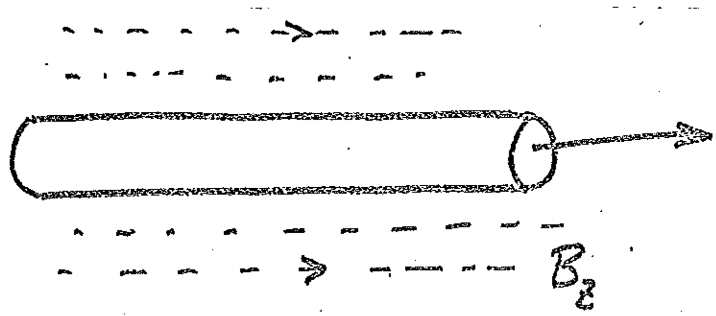
3. Derive an approximate formula for the energy loss of a fast charged particle traveling through matter, so as to display explicitly the principal dependence of the energy loss on velocity, charge, and mass of the particle.

4. A coaxial cavity with perfectly conducting walls and end plates is excited in its lowest mode. In terms of W, the energy stored in the electromagnetic field, determine the time-averaged pressure exerted by the field on the <u>side</u> walls and <u>end</u> walls.



5. An infinitely long cyclindrical current distribution, uniform to radius a, produces its own self-field B_θ as it propagates along an imposed homogeneous field B_z.



The electrons have a net charge density per unit length Q. The electrons have some perpendicular motion, $v_\perp << v_z$. Also, $B_\theta << B_z$. To first order in $v_\perp/v_z$ and B_θ/B_z, find the frequencies of the electron motion. What sort of orbits do those frequencies represent?

6. A collisionless electron gas in a magnetic field is neutralized by a fixed array of ions. We wish to propagate electrostatic waves through the electrons with $\omega \gg kv_{thermal}$. Take the magnetic field effectively infinite, so the electrons move only in straight lines along the field (cyclotron radius is zero). Using Newton's laws (or the collisionless Boltzmann (Vlasov) equation) and Poisson's equation, calculate the dispersion relation of such waves, and comment. How would you include weak collisions with neutral atoms (impurities)?

Physics Qualifying Exam UCI September 25, 1973

THERMODYNAMICS AND STATISTICAL MECHANICS

1. The potential energy V or a linear chain of (N + 1) atoms is given by

$$V = \sum_{n=1}^{N} \{\tfrac{1}{2}A(U_n - U_{n-1})^2 + B(U_n - U_{n-1})^3 + C(U_n - U_{n-1})^4\}$$

where U_n is the displacement of the n^{th} atom from its equilibrium.

a. Show that the cubic anharmonic term causes thermal expansion of the chain, and obtain the thermal expansion coefficient. Assume B and C are small compared with A.

b. What is the heat capacity of the chain at temperature T.

2. A classical gas consists of diatomic molecules whose atoms interact through an harmonic force. Neglect interactions between atoms in different molecules. Let m be the mass of the atoms and ω the frequency of vibration of the molecule.

a. Using the microcanonical ensemble, find the pressure and entropy as a function of temperature and volume and number of molecules.

b. Do the same using the canonical ensemble.

3. A plane wave advances with velocity v into a stationary fluid whose pressure P and density d are everywhere constant. Behind the wave, the pressure and density are P' and d' and the fluid moves with velocity u; these quantities are everywhere constant.

a. Show that the conservation of matter is expressed by the equation

$$\frac{d}{d'} + \frac{u}{v} = 1$$

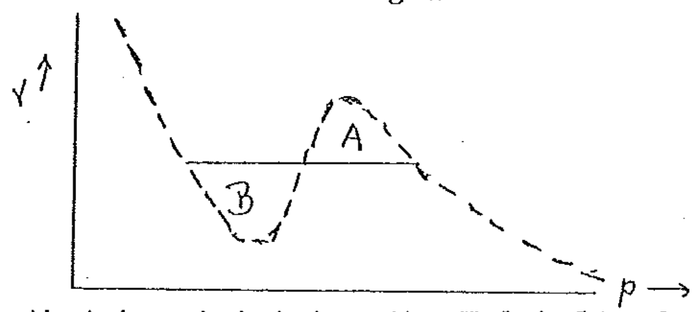
b. Show that conservation of momentum is given by

$$P' = P + duv$$

c. Now assume the fluid is an ideal gas, $P = RdT$, and has a specific heat $C_v = \frac{R}{\gamma - 1}$. Show that conservation of energy is expressed by

$$\frac{RT'}{\gamma - 1} = \frac{u^2}{2} + RT\left(\frac{1}{\gamma - 1} + \frac{u}{v}\right)$$

4. The isotherm for a van der Waals gas has a shape as given by the dashed line in the diagram



Show that by minimizing the Helmholtz free energy we obtain the Maxwell construction, i.e., the true isotherm is the one given by the straight line with the areas of A an B equal.

5. A cylinder of radius R and length L rotates about its axis with constant angular velocity ω. It contains N particles. Evaluate the density distribution of an ideal gas enclosed in the cylinder. Ignore gravity. Do a classical calculation, assuming the thermal equilibrium is established at temperature T. (Hint: You can either try to guess the distribution function and carry on from there, or you can proceed through the usual statistical mechanical machinery. Note that the Hamiltonian describing the motion is $H^* = H - \omega L$, where H is the Hamiltonian in the coordinate system at rest and L is the angular momentum.)

6. Use the barometric formula

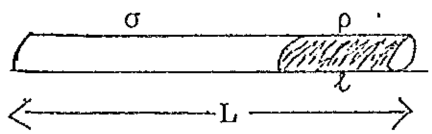
$$\rho = \rho_o \, e^{-\frac{Mg\,z}{RT}} \qquad (M = \text{molecular weight}),$$

to show that the heat capacity of an unbounded gas column under gravitational field equals C_p. Express the answer in terms of the mass of the air column.

CLASSICAL MECHANICS

1. A particle moves on a smooth circular wire under the action of a force inversely proportional to the n^{th} power of the distance from, and directed toward, a point on the wire. Find the value of n such that the force exerted by the wire on the particle at the point of contact is constant in time.

2. A stick is made so that part of its total length L is more dense. Thus the section ℓ has



density ρ and the rest of the rod has density σ. Both ρ and σ are in units in which water has a density of one. Now we throw this stick in a bathtub. Show that in order for the stick to float <u>perfectly vertically</u> we must have

(a) $X \equiv \sigma(L-\ell) + (\rho-1)\ell \leq L - \ell$

(b) $X^2 \geq \sigma(L-\ell)^2 - (\rho-1)\ell^2$

3. Consider a mass m connected to a spring with force constant k, under conditions where gravitational forces may be neglected. The mass undergoes harmonic oscillations with amplitude A. Suppose the force constant of the spring actually varies in time <u>very</u> slowly, i.e., the relative change in k in one cycle is << 1. How does the amplitude of oscillation behave?

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4. A particle of mass m is suspended at rest from a point A by an elastic string of negligible mass and force constant k, whose initial unstretched length is a. At time $t = 0$ the point A begins to oscillate up and down so that its upward displacement at time t is $x_o \sin\omega t$. Find the length of the string at time t.

5. A smooth tube of mass M is bent into a circular helix of radius a and pitch $2\pi b$. The helix is mounted so as to rotate freely about its axis, which is vertical. A particle of mass m is dropped into the tube at the top when the tube is at rest and slides down the tube. Find the angular velocity of the helix and the vertical distance through which the particle falls as functions of t.

6. (a) Find the ratio of times required to traverse the same path for two particles having different masses but the same potential energy.

   (b) Find the ratio of times required to traverse the same path for two particles of the same mass but potential energies differing by a constant factor.