Physics Qualifying Exam UCI October 4, 1978

QUANTUM MECHANICS

Qualitative

Do all the problems
One Hour
Closed Book

(Points)

(15) 1. Consider a model of liquid helium in which each atom is confined to a cube whose edge is the mean nearest neighbor distance. Taking the density of liquid helium to be 0.15 gm/cm^3, calculate the zero point energy per helium atom. Compare the result with the heat of vaporization per atom which is derivable from the heat of vaporization per mole of 0.020 kcal (1 kcal = 4.2×10^{10} erg). How do the zero point energy and heat of vaporization compare for an ordinary liquid such as water?

(10) 2. i) At what density must one begin to consider quantum effects in a (nonrelativistic) thermal plasma?

ii) Make a qualitative argument which gives the form of the ionization-recombination relation (Saha equation) between the various number densities in an equilibrium plasma.

(10) 3. Consider our sun squeezed down to the size of a neutron star, R ~ 10 km. Estimate roughly the Fermi energy of the nucleons in eV.

(15) 4. Consider two electrons in a one-dimensional box.

(a) Construct the lowest energy eigenstate, ignoring the Coulomb force.

(b) If we include the Coulomb force, the answer to (a) may not be the lowest energy state. What state is the best alternative candidate in this case?

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CLASSICAL MECHANICS

Instructions

Do a total of three problems, as follows:

(1) either #1 or #2 (30 points)

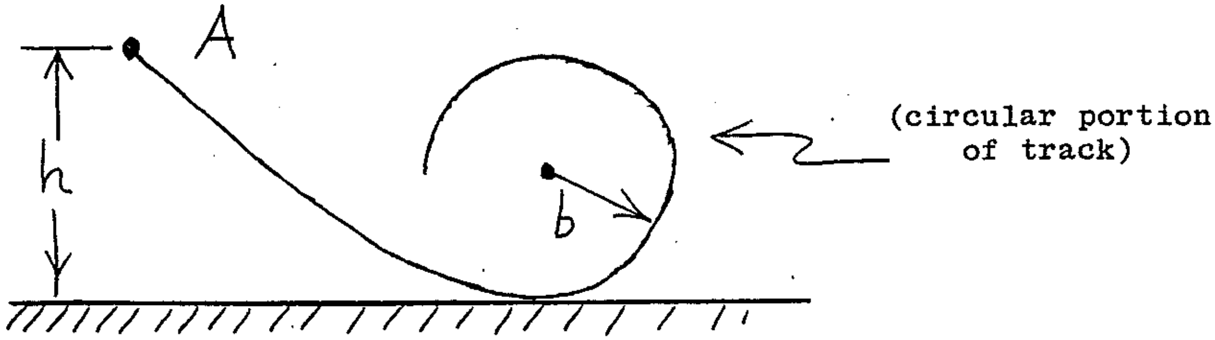
(2) either #3 or #4 (35 points)

(3) everyone do #5 (35 points)

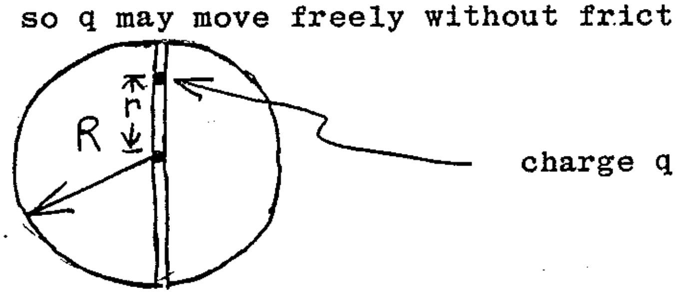
In grading, considerable attention will be given to the clarity and completeness of the solution. You can lose points if the solution appears to be random guesswork, with only a portion headed in the right direction.

Classical Mechanics

1. A small, uniform, solid sphere of radius r is initially released from rest at point A. Gravity acts downward, and the ball rolls on the track without slipping. How big must h be for the ball to roll completely around the track without falling off?



2. A positive electric charge of strength q is placed inside a uniformly charge sphere of radius R and total charge $Q>0$. The electrical charge is distributed uniformly throughout the volume of the sphere. A tiny hole is drilled along a diameter, so q may move freely without friction.

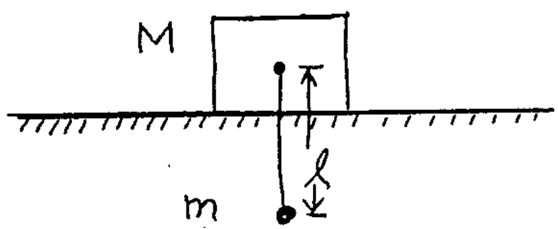


(15 points) (a) If q is given the velocity V_0 at $t = 0$, find an expression for its position as a function of time, for the portion of its trajectory when $0 < r < R$.

(15 points) (b) Say whatever you can that is simple about its motion when $r > R$, and in particular find its velocity at $r = \infty$ in terms of q, Q, R, and V_0.

3. An elementary result of mechanics is that a ball tossed vertically into the air strikes the ground again at the time $t_o = 2V_o/g$, where V_o is the ball's initial velocity and g is the acceleration of gravity.
Consider a ball tossed vertically into an atmosphere that opposes its motion with the frictional force $\vec{F} = -\eta\vec{v}$. How much longer does the ball stay in the air, if η is small? (More precisely, if t_o is the time the ball strikes the ground, what is the correction to the elementary formula first order in η?)

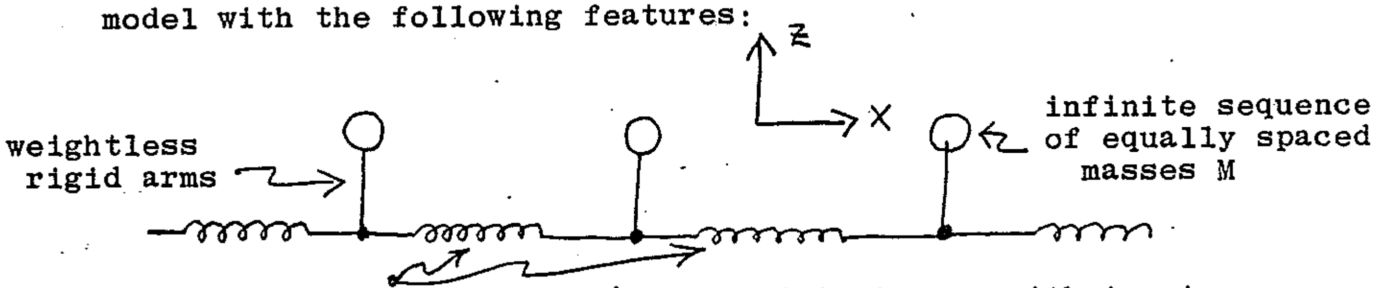
4. A mass M slides on a frictionless shelf, while a second mass m dangles below on a string of length ℓ. Gravity acts downward,



(15 points) (a) Set up a Lagrangian for the system, and find the equations of motion for the relevant dynamical variables.

(20 points) (b) The mass M is given a sharp, very brief impulsive blow at $t = 0$, so at $t = 0+$ it has velocity V_o. Assume the mass on the string never moves very far from the vertical, and find an expression for the position of M as a function of time. What is the <u>average</u> velocity with which M moves?

5. At a Physics Department colloquium last year, the speaker discussed in detail a number of properties of a physical model with the following features:



identical torsion fibers. Each resists torque with torsion constant k. Assume torque is linear with angle of twist.

Gravity acts downward.

(15 points) (a) Find the equation of motion obeyed by θ_i, the angle made by the i^{th} weightless arm with the vertical. THE MASSES MOVE ONLY IN THE PLANE PERPENDICULAR TO THE X AXIS.

(15 points) (b) Find the frequency of small amplitude waves in θ_i, as a function of wave vector, for the case where each mass points upward as shown. Comment on any unusual feature of the result.

(5 points) (c) Describe, very briefly, the subject of the colloquium where this example arose.

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GENERAL PHYSICS

One Hour

Do all the problems

Open Book

(Points)

(5) 1. If a cup of tea is stirred with a rotary motion, the tea leaves are found to collect in the center of the bottom of the cup. Explain.

(15) 2. Estimate the total <u>classical</u> free energy of the present Sun ($M_\odot \sim 10^{30.3}$kg, $R_\odot \sim 10^{8.8}$ m, $\langle T_\odot \rangle \sim 10^{7.1}$ °K), and thence its lifetime if there are no other energy sources.

(5) 3. The bulbs connected as shown are common 110 volt bulbs. When the circuit is closed, one bulb does not emit visible light. Which is it?

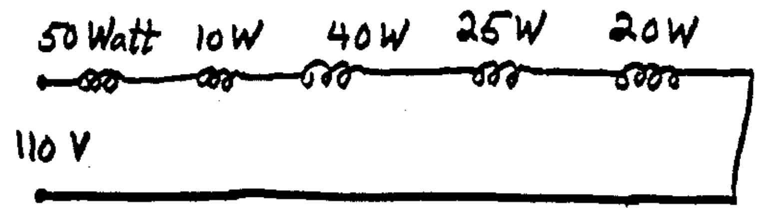
(a) 10 watt

(b) 20 watt

(3) 25 watt

(4) 40 watt

(5) 50 watt



(5) 4. An experimenter notices that the mercury column height of barometer 1, kept in the basement stands consistently <u>lower</u> than that of barometer 2, kept on the second floor. He discovers that the mercury of barometer 1 is contaminated with a little water. The best explanation of the difference in mercury heights is that:

a) The atmospheric pressure decreases with increasing altitude.

b) The weight of a given mass of mercury increases as the mercury is brought nearer to the earth.

c) Water vapor exerts a pressure.

d) Mercury evaporates less easily in the presence of water vapor.

e) The evaporation of water cools the mercury and causes it to contract.

(Points)

5. A scientist, while conducting an experiment, burns his finger. He finds it more comfortable to stick his finger in water rather than to wave it in the air. This indicates that:

(5)

a) Water is a better conductor than air, and the heat loss is mainly by conduction.
b) Water sets up stronger convection currents than air, and the loss is mainly by convection.
c) Water has a higher specific heat than air, and hence can absorb more heat.
d) A body radiates faster when surrounded by water than air, and the heat loss is mainly by radiation.

6. Which of the following would be MOST apt to undergo the LEAST temperature change over a period of a few hours?

(5)

a) A glass of water containing melting ice.
b) A pan of water covered with a tight-fitting lid and placed on a heating unit.
c) An open container of gasoline with air of constant temperature bubbling through it.
d) A melting ice cube placed in the freezing unit of a fast-freeze food locker.
e) The water at the bottom of a small pond while the surface is freezing over.

7. A bar magnet is supported by a delicate coiled spring attached to one end. The magnet is set into harmonic motion in a vertical line. Its period is T. Now there is brought up from below the lower end of the magnet a closed coil of wire so that the motion of the magnet is into and out of the coil. The following facts are all true, except:

(10)

a) The period of the motion will now be longer
b) The frequency of the motion will not be less
c) The motion of the magnet will not be altered by the presence of the coil
d) Lenz's Law gives rise to damping forces
e) an e.m.f. is induced in the coil and the coil acquires polarity

(Points)

8. A parallel plate condenser has two vertical plates 12 mm apart and a plate of glass 10 mm thick between the plates. The condenser (capacitor) is charged with +Q on one plate, -Q on the other, and then the charging connectors are removed. If now the glass plate is withdrawn:

(5)

- a) mechanical work must be expended upon it
- b) mechanical work will be done by it
- c) the quantity of electricity on the positive plate will be greater than +Q
- d) there will be no change in the energy stored in the capacitor
- e) the quantity of electricity on the positive plate will be less than +Q

9. Standard resistance coils are usually wound double, that is, the coil is wound back on itself. This is done to

(5)

- a) increase the resistance
- b) decrease the resistance of the coil
- c) minimize inductive effects in the coil
- d) increase the IR drop in the coil
- e) protect the meter with which the coil may be used

10. Very little sound comes from a violin string alone. More is produced when the string is fixed to a sounding box. The additional energy now appearing in the sound wave is accounted for by which of the following?

(5)

- a) Thermal energy
- b) The PE stored in the curved body of the sounding box whose walls are under tension.
- c) Energy drawn from the surrounding atmosphere.
- d) A mechanical advantage governed by the string length and the bridge height.
- e) The increased energy supplied by the bow.

(Points)

11. A solid cylinder rolls down an inclined plane in time <u>t</u>. A hole is then bored at its geometric axis. The time now to roll down the plane is, compared to <u>t</u>:

(5)

a) greater
b) less
c) the same
d) governed by the size of the hole
e) indeterminate

12. A cylinder of radius R has a string wrapped around it. The free end of the string is held fixed and the cylinder allowed to "unroll" in a vertical line. The <u>linear</u> acceleration of the cylinder is:

(5)

a) g
b) gR
c) mg
d) 2/3g
e) 1/2g

13. A body is <u>slightly</u> denser than water and <u>very</u> compressible. When submerged and let go it will:

(5)

a) sink a ways and stop
b) rise to the surface
c) sink fast for a while, then slow up
d) sink faster and faster
e) stay where it is first submerged

14. In an astronomical telescope if the mirror diameter were doubled without altering the radius of curvature, all of the following occur <u>except</u>:

(10)

a) the image intensity would be 4 times as great
b) the overall magnification would be doubled
c) the f-number would be halved
d) spherical aberration would increase
e) the same photographic action could be obtained in 1/4 the previous exposure time

(Points)

15. The amount of heat passing through the bottom of a coffee pot on a gas stove is INDEPENDENT of:

(5)
a) the temperature gradient in the bottom of the pot.
b) the thickness of the material in the bottom of the pot.
c) the kind of material of which the pot is made.
d) the area of the bottom of the pot.
e) the amount of coffee in the pot.

16. Two men stand on opposite sides of a camp fire. They find it difficult to make each other heard. The reason for this comes most appropriately under the title of:

(5)
a) refraction
b) diffraction
c) interference
d) polarization
e) reverberation

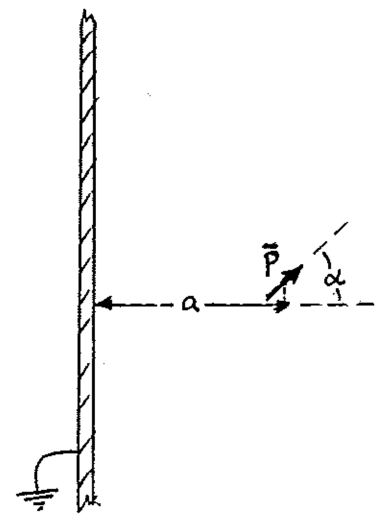
ELECTRICITY AND MAGNETISM

Open Book

(35) 1. For the dipole and infinite conducting plane shown, calculate:

(i) the surface charge on the conductor; and

ii) the resultant torque on the dipole.



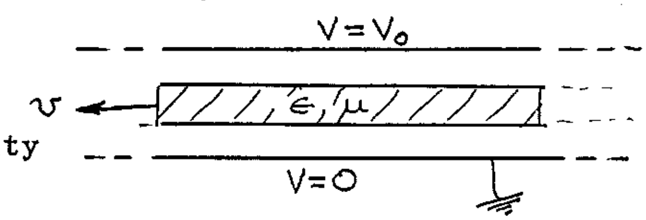
(20) 2. i) Use the Lorentz transformation of electric field and magnetic induction, and the result

$$\bar{P}'_{\perp} = \gamma(\bar{P}_{\perp} - \bar{v} \times \bar{M}_{\perp}/c^2)$$

$$\bar{M}'_{\perp} = \gamma(\bar{M}_{\perp} - \bar{v} \times \bar{P}_{\perp})$$

to derive the (perpendicular components of the) constitutive relations in the primed frame, given those in a medium in the unprimed frame.

ii) In the figure at the right, a slab of material with permeability $\underline{\mu}$ and permittivity $\underline{\epsilon}$ is moving with velocity $\underline{v}$ as shown. Give an expression, to first order in $\underline{v}$, for the current density $\underset{\sim}{J}$ (the source term in Ampere's law) in the lab frame. Do not carry out the details of the evaluation.



Sketch where the current would flow.

3\. Given a uniform slab of plasma, occupying the volume $|z| \le a$. Take as a fact that its dielectric constant can be expressed as

(45)

$$\epsilon = \epsilon_o \left(1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2} \right) .$$

where ω is an applied frequency and $\omega_{p\alpha}$ is the plasma frequency of species α.

Derive and plot the $(\omega - k)$ dispersion relation(s) for symmetric, slow $(\omega \ll kc)$, small-amplitude waves, propagating as $\exp[i\omega t - ikx]$.

Discuss the z-profiles of the allowed excitations.

Note: Show that you can derive the wave electric field from a potential.

QUANTUM MECHANICS II

Open Book

Do all problems

Relative Weight

10 1. Consider a particle of mass m moving in the earth's gravitational field so that the potential energy is zero at the earth's surface z = 0. Draw V(z). Calculate the (one dimensional) energy eigenvalues in the WKB approximation.

8 2. A particle of mass m is scattered off of a potential $V = \beta\delta(\underline{r})$. Calculate the phase shifts $\delta_\ell(E)$ in the Born approximation. Plot the cross section σ versus energy E. From applying conservation of probability, at what energy must this approximation for σ be wrong.

6 3. An $\ell = 2$ operator $Q^{\ell=2}$ has a matrix element squared

$$|\langle\psi_{j_2=2,m_2=1}|Q^{\ell=2}_{m=0}|\psi_{j_1=1,m_1=1}\rangle|^2 = 11 \quad .$$

Using the Wigner-Eckart Theorem and the attached Table of Clebsch-Gordan Coefficients, calculate

a) $|\langle\psi_{j_2=2,m_2=2}|Q^{\ell=2}_{m=1}|\psi_{j_1=1,m_1=1}\rangle|^2$

b) $|\langle\psi_{j_2=2,m_2=0}|Q^{\ell=2}_{m=0}|\psi_{j_1=1,m_1=0}\rangle|^2$.

Relative Weight

10 4. Consider an spin $\frac{1}{2}$ particle (mass m) with a magnetic moment $\underline{\mu} = \gamma\underline{S} = \frac{1}{2}\gamma\hbar\underline{\sigma}$ which moves through a uniform magnetic field (perpendicular to the motion) $\underline{B} = B_o\hat{z}$ for a distance L.

a) Calculate the phase change $\beta = (\Delta k)L$ for the particle going through this field. Δk is the change in $k(=p/\hbar)$ in the weak B field.

b) Calculate the precessional angle $\alpha = \omega t$ for the particle's spin after passing through the B field.

c) For a precessional angle of $\alpha = 2\pi$, what is β? Discuss <u>briefly</u> the significance of this result with regard to the spin $\frac{1}{2}$ nature of the particle.

CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A $\sqrt{\ }$ is to be understood over every coefficient; e.g., for -8/15 read $-\sqrt{8/15}$.

Notation:

	J	J	...
	M	M	...
m_1 m_2			
m_1 m_2	Coefficients		
. .			

1/2 × 1/2

m_1 m_2	1, +1		
+1/2 +1/2	1		

m_1 m_2	1, 0	0, 0
+1/2 -1/2	1/2	1/2
-1/2 +1/2	1/2	-1/2

m_1 m_2	1, -1
-1/2 -1/2	1

1 × 1/2

m_1 m_2	3/2, +3/2
+1 +1/2	1

m_1 m_2	3/2, +1/2	1/2, +1/2
+1 -1/2	1/3	2/3
0 +1/2	2/3	-1/3

m_1 m_2	3/2, -1/2	1/2, -1/2
0 -1/2	2/3	1/3
-1 +1/2	1/3	-2/3

m_1 m_2	3/2, -3/2
-1 -1/2	1

2 × 1

m_1 m_2	3, +3
+2 +1	1

m_1 m_2	3, +2	2, +2
+2 0	1/3	2/3
+1 +1	2/3	-1/3

m_1 m_2	3, +1	2, +1	1, +1
+2 -1	1/15	1/3	3/5
+1 0	8/15	1/6	-3/10
0 +1	6/15	-1/2	1/10

m_1 m_2	3, 0	2, 0	1, 0
+1 -1	1/5	1/2	3/10
0 0	3/5	0	-2/5
-1 +1	1/5	-1/2	3/10

m_1 m_2	3, -1	2, -1	1, -1
0 -1	6/15	1/2	1/10
-1 0	8/15	-1/6	-3/10
-2 +1	1/15	-1/3	3/5

m_1 m_2	3, -2	2, -2
-1 -1	2/3	1/3
-2 0	1/3	-2/3

m_1 m_2	3, -3
-2 -1	1

1 × 1

m_1 m_2	2, +2
+1 +1	1

m_1 m_2	2, +1	1, +1
+1 0	1/2	1/2
0 +1	1/2	-1/2

m_1 m_2	2, 0	1, 0	0, 0
+1 -1	1/6	1/2	1/3
0 0	2/3	0	-1/3
-1 +1	1/6	-1/2	1/3

m_1 m_2	2, -1	1, -1
0 -1	1/2	1/2
-1 0	1/2	-1/2

m_1 m_2	2, -2
-1 -1	1

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_l^{-m} = (-1)^m Y_l^{m*}$$

$$d^l_{m,0} = \sqrt{\frac{4\pi}{2l+1}}\, Y_l^m e^{-im\phi}$$

2 × 1/2

m_1 m_2	5/2, +5/2
+2 1/2	1

m_1 m_2	5/2, +3/2	3/2, +3/2
+2 -1/2	1/5	4/5
+1 +1/2	4/5	-1/5

m_1 m_2	5/2, +1/2	3/2, +1/2
+1 -1/2	2/5	3/5
0 +1/2	3/5	-2/5

m_1 m_2	5/2, -1/2	3/2, -1/2
0 -1/2	3/5	2/5
-1 +1/2	2/5	-3/5

m_1 m_2	5/2, -3/2	3/2, -3/2
-1 -1/2	4/5	1/5
-2 +1/2	1/5	-4/5

m_1 m_2	5/2, -5/2
-2 -1/2	1

3/2 × 1/2

m_1 m_2	2, +2
+3/2 +1/2	1

m_1 m_2	2, +1	1, +1
+3/2 -1/2	1/4	3/4
+1/2 +1/2	3/4	-1/4

m_1 m_2	2, 0	1, 0
+1/2 -1/2	1/2	1/2
-1/2 +1/2	1/2	-1/2

m_1 m_2	2, -1	1, -1
-1/2 -1/2	3/4	1/4
-3/2 +1/2	1/4	-3/4

m_1 m_2	2, -2
-3/2 -1/2	1

3/2 × 1

m_1 m_2	5/2, +5/2
+3/2 +1	1

m_1 m_2	5/2, +3/2	3/2, +3/2
+3/2 0	2/5	3/5
+1/2 +1	3/5	-2/5

m_1 m_2	5/2, +1/2	3/2, +1/2	1/2, +1/2
+3/2 -1	1/10	2/5	1/2
+1/2 0	3/5	1/15	-1/3
-1/2 +1	3/10	-8/15	1/6

m_1 m_2	5/2, -1/2	3/2, -1/2	1/2, -1/2
+1/2 -1	3/10	8/15	1/6
-1/2 0	3/5	-1/15	-1/3
-3/2 +1	1/10	-2/5	1/2

m_1 m_2	5/2, -3/2	3/2, -3/2
-1/2 -1	3/5	2/5
-3/2 0	2/5	-3/5

m_1 m_2	5/2, -5/2
-3/2 -1	1

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

2 × 3/2

m_1 m_2	7/2, +7/2
+2 +3/2	1

m_1 m_2	7/2, +5/2	5/2, +5/2
+2 +1/2	3/7	4/7
+1 +3/2	4/7	-3/7

m_1 m_2	7/2, +3/2	5/2, +3/2	3/2, +3/2
+2 -1/2	1/7	16/35	2/5
+1 +1/2	4/7	1/35	-2/5
0 +3/2	2/7	-18/35	1/5

m_1 m_2	7/2, +1/2	5/2, +1/2	3/2, +1/2	1/2, +1/2
+2 -3/2	1/35	6/35	2/5	2/5
+1 -1/2	12/35	5/14	0	-3/10
0 +1/2	18/35	-3/35	-1/5	1/5
-1 +3/2	4/35	-27/70	2/5	-1/10

m_1 m_2	7/2, -1/2	5/2, -1/2	3/2, -1/2	1/2, -1/2
+1 -3/2	4/35	27/70	2/5	1/10
0 -1/2	18/35	3/35	-1/5	-1/5
-1 +1/2	12/35	-5/14	0	3/10
-2 +3/2	1/35	-6/35	2/5	-2/5

m_1 m_2	7/2, -3/2	5/2, -3/2	3/2, -3/2
0 -3/2	2/7	18/35	1/5
-1 -1/2	4/7	-1/35	-2/5
-2 +1/2	1/7	-16/35	2/5

m_1 m_2	7/2, -5/2	5/2, -5/2
-1 -3/2	4/7	3/7
-2 -1/2	3/7	-4/7

m_1 m_2	7/2, -7/2
-2 -3/2	1

3/2 × 3/2

m_1 m_2	3, +3
+3/2 +3/2	1

m_1 m_2	3, +2	2, +2
+3/2 +1/2	1/2	1/2
+1/2 +3/2	1/2	-1/2

m_1 m_2	3, +1	2, +1	1, +1
+3/2 -1/2	1/5	1/2	3/10
+1/2 +1/2	3/5	0	-2/5
-1/2 +3/2	1/5	-1/2	3/10

m_1 m_2	3, 0	2, 0	1, 0	0, 0
+3/2 -3/2	1/20	1/4	9/20	1/4
+1/2 -1/2	9/20	1/4	-1/20	-1/4
-1/2 +1/2	9/20	-1/4	-1/20	1/4
-3/2 +3/2	1/20	-1/4	9/20	-1/4

m_1 m_2	3, -1	2, -1	1, -1
+1/2 -3/2	1/5	1/2	3/10
-1/2 -1/2	3/5	0	-2/5
-3/2 +1/2	1/5	-1/2	3/10

m_1 m_2	3, -2	2, -2
-1/2 -3/2	1/2	1/2
-3/2 -1/2	1/2	-1/2

m_1 m_2	3, -3
-3/2 -3/2	1

$$d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^1_{1,1} = \frac{1+\cos\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

$$d^1_{0,0} = \cos\theta$$

2 × 2

m_1 m_2	4, +4
+2 +2	1

m_1 m_2	4, +3	3, +3
+2 +1	1/2	1/2
+1 +2	1/2	-1/2

m_1 m_2	4, +2	3, +2	2, +2
+2 0	3/14	1/2	2/7
+1 +1	4/7	0	-3/7
0 +2	3/14	-1/2	2/7

m_1 m_2	4, +1	3, +1	2, +1	1, +1
+2 -1	1/14	3/10	3/7	1/5
+1 0	3/7	1/5	-1/14	-3/10
0 +1	3/7	-1/5	-1/14	3/10
-1 +2	1/14	-3/10	3/7	-1/5

m_1 m_2	4, 0	3, 0	2, 0	1, 0	0, 0
+2 -2	1/70	1/10	2/7	2/5	1/5
+1 -1	8/35	2/5	1/14	-1/10	-1/5
0 0	18/35	0	-2/7	0	1/5
-1 +1	8/35	-2/5	1/14	1/10	-1/5
-2 +2	1/70	-1/10	2/7	-2/5	1/5

m_1 m_2	4, -1	3, -1	2, -1	1, -1
+1 -2	1/14	3/10	3/7	1/5
0 -1	3/7	1/5	-1/14	-3/10
-1 0	3/7	-1/5	-1/14	3/10
-2 +1	1/14	-3/10	3/7	-1/5

m_1 m_2	4, -2	3, -2	2, -2
0 -2	3/14	1/2	2/7
-1 -1	4/7	0	-3/7
-2 0	3/14	-1/2	2/7

m_1 m_2	4, -3	3, -3
-1 -2	1/2	1/2
-2 -1	1/2	-1/2

m_1 m_2	4, -4
-2 -2	1

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta$$

$$d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2$$

$$d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1)$$

$$d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Sign convention is that of Wigner (Group Theory, Academic Press, New York, 1959), also used by Condon and Shortley (The Theory of Atomic Spectra, Cambridge Univ. Press, New York, 1953), Rose (Elementary Theory of Angular Momentum, Wiley, New York, 1957), and Cohen (Tables of the Clebsch-Gordan Coefficients, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The signs and numbers in the current tables have been calculated by computer programs written independently by Cohen and at LBL. (Table extended April 1974.)

THERMODYNAMICS AND STATISTICAL MECHANICS

Do all 4 problems
Problems count equally
CLOSED BOOK

1. Consider an ideal gas with a specific heat ratio $\gamma = C_P/C_V$ which is constant. For a quasistatic adiabatic process, prove:

   a. That PV^γ = constant, where P is the pressure and V is the volume of the gas.

   b. That the work done W is given by

   $$W = \frac{P_i V_i - P_f V_f}{\gamma - 1} ,$$

   where the subscripts i and f refer to the initial and final states, respectively.

   c. That the initial and final temperatures, T_i and T_f, are related by

   $$\frac{T_i}{T_f} = \left(\frac{V_f}{V_i} \right)^{\gamma-1}$$

2. a. Derive the thermodynamic relation

   $$\left(\frac{\partial U}{\partial V} \right)_T = T\left(\frac{\partial P}{\partial T} \right)_V - P$$

   where U is the internal energy.

   b. Apply this relation to black body radiation whose pressure P is one-third of the energy density u and derive the Stefan-Boltzmann law $u = bT^4$ where b is a constant.

3. An aqueous solution at temperature T contains a small concentration of magnetic ions with net spin $\frac{1}{2}$ and is placed in an inhomogeneous magnetic field pointing in the z-direction and having a magnitude specified by

$$H(z) = H(o) + \Delta z$$

where Δ is a constant.

a. Let $n_+(z)dz$ be the mean number of ions whose spin quantum number $s = +\frac{1}{2}$ and which are located between z and z + dz. What is the ratio $n_+(z_1)/n_+(o)$?

b. Let $n(z)dz$ be the total mean number of ions (of both directions of spin orientation) located between z and z +dz. What is the ratio $n(z_1)/n(o)$? If $\Delta > 0$ and $z_1 > 0$, is this ratio less than, equal to, or greater than unity?

4. The potential energy V of a linear chain of N atoms is given by

$$V = \sum_{n=1}^{N} \{\tfrac{1}{2}A(u_n - u_{n-1})^2 + B(u_n - u_{n-1})^3 + C(u_n - u_{n-1})^4\}$$

where u_n is the displacement of the nth atom from its equilibrium position. Assuming that B and C are small quantities, calculate the thermal expansion coefficient of the chain to first order in B and C. The following integrals may be useful:

$$\int_{-\infty}^{\infty} dx\, e^{-x^2} = \sqrt{\pi}\ , \qquad \int_{-\infty}^{\infty} dx\, x^2 e^{-x^2} = \frac{\sqrt{\pi}}{2}$$

$$\int_{-\infty}^{\infty} dx\, x^4 e^{-x^2} = \frac{3\sqrt{\pi}}{4}$$

MATHEMATICAL PHYSICS
Open Book
Do all 3 problems

1. An infinite cylinder of radius a is heated. the temperature obeys

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = \frac{1}{k}\frac{dT}{dt} .$$

The surface temperature is prescribed,

$$T(a,t) = f(t). \qquad t \geq 0$$

and initially, $T(r,o) = 0$.

(a) Find an expression for the temperature at all later times.

(b) If $f(t) = T_o \frac{t}{\tau}$, find $T(t)$.
For long times, what is the t-dependence?

2. A differential equation

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + 2 = 0$$

comes from minimizing some integral, I, over an area (x,y).

(a) What is the integrand of I.

(b) Now suppose we wish to find a function ψ which satisfies the condition that I be a minimum in the region and ψ be zero on the elliptical boundary

$$g = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0 .$$

Try a choice $\psi = d_1 g$ and find d_1.

3. The random variable x has the probability distribution

$$f(x) = \frac{1}{\sqrt{2}} e^{-x^2/2} \qquad (-\infty < x < \infty)$$

(a) Find $\langle x \rangle$ and $\langle x^2 \rangle$.

(b) n independent measurements x_i are made. Let $\bar{x} = \frac{1}{n}(x_1 + x_2 + \ldots + x_n)$. Find $\varphi(\bar{x})$, the probability distribution of $\bar{x}$. Find $\langle \bar{x} \rangle$.

(c) Let $s = \frac{1}{n}(x_1^2 + x_2^2 + \ldots + x_n^2)$. Find $\psi(s)$, the probability distribution of s. Find $\langle s \rangle$.