QUANTUM MECHANICS I

3 Hours
Open Book
Answer All Questions

1. A particle in three-dimensions is bound in a potential which gives rise to a large number of bound states. Among these the S-wave ($\ell = 0$) states are found to have energy levels E_n such that E_n is proportional to $n^{2/3}$ for large n. Find and demonstrate a form for the potential which yields this spectrum. (You may use a suitable approximation to justify your result.)

2. A particle is scattered by two weak scattering centers, A & B, with scattering amplitudes $f_A(\theta)$, $f_B(\theta)$, respectively. Suppose A and B, separated by a displacement $\vec{R}$, are placed simultaneously in an incident beam, with $\vec{R}$ making a well-defined but arbitrary angle with respect to the beam direction.

   a) Find an expression for the scattering amplitude in terms of f_A, f_B in lowest non-vanishing approximation.

   b) Find the form of the differential cross section in the limits of very large and very small incident wave length.

3. Consider two spin $\frac{1}{2}$ particles, A and B. B is essentially infinitely massive and attracts A with a harmonic oscillator potential of the form

$$V(r) = \tfrac{1}{2} m_A \omega^2 r^2 \quad .$$

In addition there is a spin-orbit coupling and an interaction between the spins, $\vec{S}_A$, $\vec{S}_B$. The interaction Hamiltonian is given by

$$H_1 = V(r) + \alpha \vec{L} \cdot \vec{S}_A + \beta \vec{S}_A \cdot \vec{S}_B$$

where

$$\omega \gg \hbar\alpha \gg \hbar\beta > 0 \quad .$$

Sketch the energy level diagram for all states $< \frac{3}{2}\hbar\omega$ above the ground state. Identify the levels completely by appropriate quantum numbers and give quantitatively the splittings between levels.

4. A system with spin $\frac{1}{2}$ and given g-factor is placed in a D.C. magnetic field of magnitude H_o along the z-axis, with spin initially "up" along the +z direction. At $t = 0$ an additional D.C. magnetic field of magnitude h is applied parallel to the x-axis and is left on for all $t > 0$. Find an explicit expression for the spin wave function for times $t > 0$. Describe the motion in words.

5. Two similar harmonic oscillators in one dimension have the same frequency and equilibrium position, and interact by means of a repulsive force which is proportional to the distance between them. Write and solve the Schrödinger equation for this system; determine the energy levels and specify the quantum numbers, including parity, which correspond to the levels.

THERMODYNAMICS AND STATISTICAL MECHANICS

Two Hours. Open Book: One textbook & one math table allowed;

Do all problems

1. A system consists of three cylinders, the first containing one mole of a monatomic ideal gas at 300 K; the second, two moles of a monatomic ideal gas at 250 K; and the third, one mole of a diatomic ideal gas at 200 K. The characteristic rotational energy of the latter is 5 K, while its characteristic vibrational energy is 10,000 K.

   a) What is the maximum amount of work that can be extracted from the three bodies in a process in which all three are brought to a common final temperature?

   b) What is the final temperature, and

   c) What is the change in entropy in such a process of each subsystem? Of the total system?

2. Diatomic molecules get larger with increasing temperature for the same fundamental reasons that solids exhibit thermal expansion. Show that if the molecule is treated classically in the harmonic approximation the molecule has the same mean size at all temperatures. To obtain thermal expansion an anharmonic term is required:

$$- \epsilon r^3$$

where r is the displacement from equilibrium and ϵ is a small parameter. Find the lowest order contribution of the anharmonic term to the average size of the molecule, and to the specific heat.

3. In a <u>one</u> dimensional ferromagnetic solid the dispersion relation for spin waves is

$$\hbar\omega = C(1 - \cos k a)$$

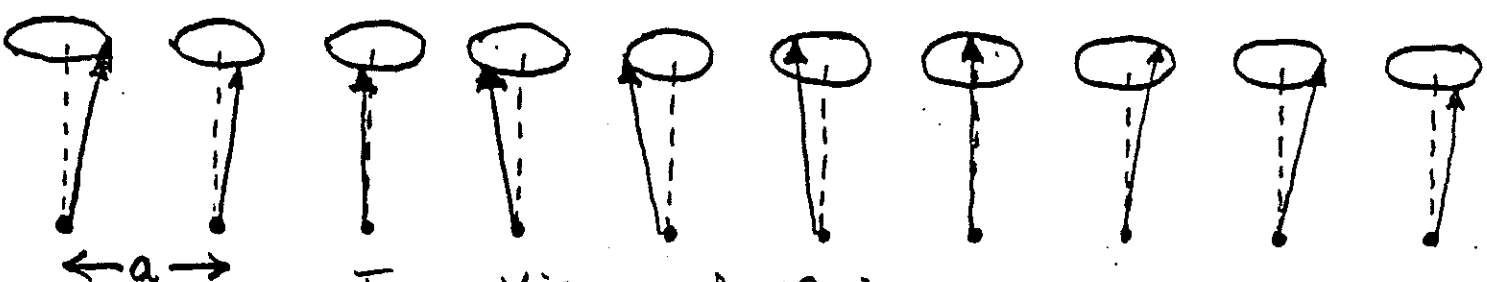
where C is a constant, k is the wave number, and a is the lattice constant. Such a wave is pictured below; note that the wavelength can't be smaller than the lattice constant. Assume that arbitrarily many spin waves in a given mode can be excited.

a) At low temperatures only the low energy modes are excited. Approximate the dispersion relation by its low energy form and find the appropriate cutoff frequency.

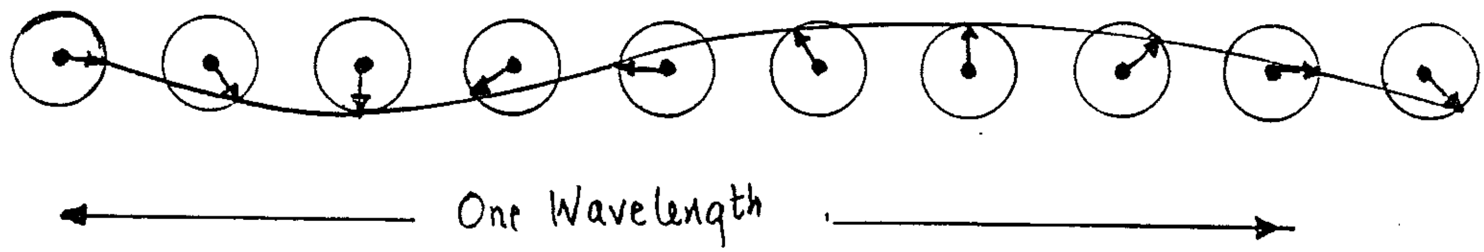
b) Assuming the dispersion relation and cutoff of a) give an exact expression for the specific heat. (What is the chemical potential, and why?)

c) Find the limiting form of the temperature dependence of the specific heat at low temperatures.

Spin Wave Viewed in Perspective



Top View of Spinwave



MATHEMATICAL PHYSICS

Do all problems.

1. a. The function $f(z) = \dfrac{1}{(e^z + 1)^2}$ is expanded in a series of the form

$$f(z) = \sum_{n=0}^{\infty} a_n (z - 1)^n \quad .$$

Discuss the convergence of the series.

b. Evaluate the integral

$$\oint_c \frac{dz(3z^3 + 2)}{(z-1)(z^2+9)}$$

where c is the circle $|z - 2| = 2$.

c. Through use of an appropriate contour, evaluate

$$\int_0^{\infty} \frac{dx \, \ln(x)}{1 + x^2} \quad .$$

2. Given the differential operator

$$L = \frac{d^2}{dx^2} - K^2$$

which operates on the interval $0 \le x \le 3$. We want to solve

$$L u = f$$

subject to the boundary conditions $u(o) = u(3) = 0$.

a. Construct the Green's function that may be used to generate the solution to the above equation.

b. For the choice $f(x) = 1$, obtain the explicit form of $u(x)$ through use of the Green's function.

3. Solve the integral equation

$$f(x) - \lambda \int_0^1 dy \sinh(x-y) f(y) = 1.$$

What values of λ are eigenvalues of the equation?

4. A fundamental principle of geometrical optics states that a light ray describes a trajectory which makes its transit time through matter an extremum. Mathematically, if $n(x,y)$ is the index of refraction at point (x,y), the path is such that

$$\int ds\, n(x,y)$$

is an extremum.

Consider the case where $n(x,y)$ depends on only x, $\lim_{x\to+\infty} n(x) = +1$, $\lim_{x\to-\infty} n(x) = n_o > 1$, and $n(x)$ is a monotonically decreasing function x.

A light ray moves through the medium, and as $x \to -\infty$ its trajectory is a straight line that makes an angle θ with the x axis. If $n_o \sin\theta < 1$, show the equation of its trajectory $y(x)$ is found from

$$y(x) = n_o \sin\theta \int^x \frac{dx'}{[n^2(x') - n_o^2 \sin^2\theta]^{\frac{1}{2}}} .$$

What happens to the ray when $n_o \sin\theta > 1$?

ELECTRICITY AND MAGNETISM

Do all problems.

1. A charge Q is distributed uniformly along the z axis from z = -a to z = + a. Find the potential at a point (r,θ) where r > a as an expansion in solutions to Laplace's equation.

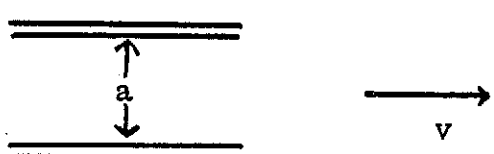
2. A "hard" ferromagnet is in the shape of a right circular cylinder of length L and radius a. The cylinder has a permanent magnetization M_o, uniform throughout its volume and parallel to its axis. The cylinder is oriented along the z axis from z = -L/2 to z = L/2, with magnetization along $\hat{z}$

   a) Determine the magnetic field $\bar{H}$ on the cylinder's axis above the cylinder.

   b) Expand for z >> L, a and find the magnetic dipole moment.

   c) What is the effective magnetic current density and where is it located?

3. Two thin, parallel, infinitely long, nonconducting rods, a distance 'a' apart, with identical constant charge density λ per unit length in their rest frame, move with a relativistic velocity v, with respect to a stationary reference frame.



   a) Calculate the $\bar{E}$ and $\bar{B}$ fields and the force per unit length between the rods in a frame of reference moving with the rods.

b) Calculate charge and current densities in the stationary frame of reference.

c) Calculate the $\bar{E}$ and $\bar{B}$ fields in the stationary reference frame from the charge and current densities.

d) Show that these $\bar{E}$ and $\bar{B}$ in the stationary frame agree with those obtained by a Lorentz transformation of the fields from the comoving frame.

e) Calculate the force per unit length in the stationary frame and compare with that in the comoving frame.

GENERAL PHYSICS

Logically discuss any 6 problems. Credit will be awarded only for the logical development steps toward a solution. In most cases, you need to identify the problem and then discuss.

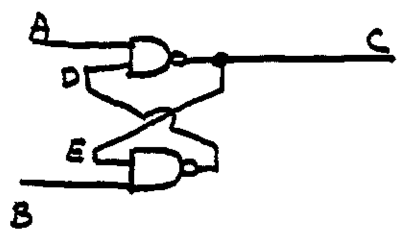
1. $\Sigma^- \rightarrow n + \pi^-$

$m_n = 939.6$ MeV

$m_{\Sigma^-} = 1197.3$ MeV

$m_{\pi^-} = 139.6$ MeV

$\tau = 1.5 \times 10^{-10}$ sec.

2. 

3. The index of refraction for hard x-rays in crystals is less than 1!

4. The words - 0, the words!

LCD	CRT
RAM	BCD
SQUID	LED
EPR	LASER
FWHM	
FET	

5. A 75 watt incandescent light has a coated glass bulb approximately 6 cm in diameter while the same type 150 watt bulb is approximately 9 cm in diameter.

6. Fusion of hydrogen in the sun produces helium plus ? .

7. A string with metal beads is held upright from the floor and then released.

(see illustration right side of page)

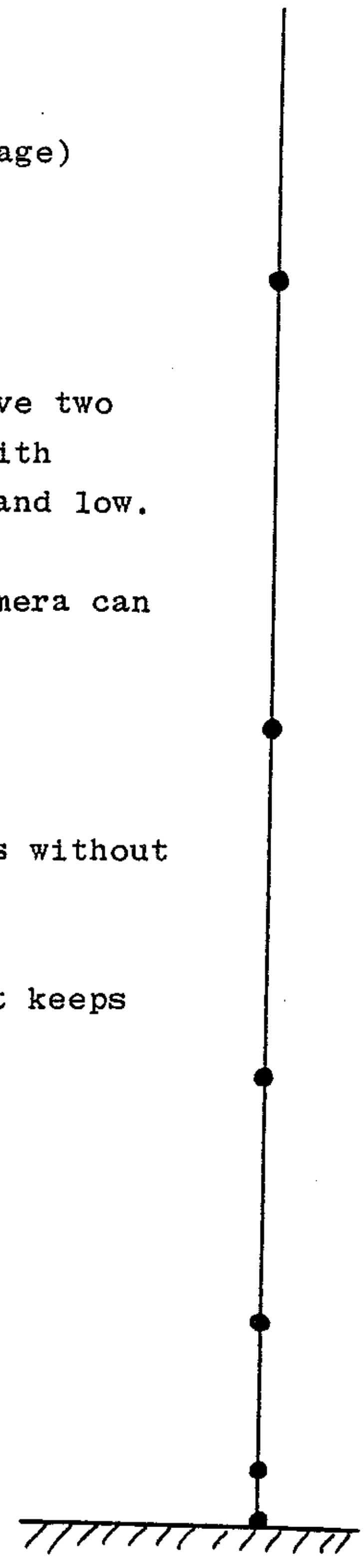
8. Many places along the Atlantic coast of the USA have two equal high tides and two equal low tides "daily" with approximately 6 hours and 13 minutes between high and low.

9. With the same ambient lighting, the f stop on a camera can be 1.4 or 8 to take a good picture.

10. "Summer of the gluons: Jets of evidence."

11. An audio amplifier accepts up to 1 mV input signals without distortion. A 2 mV signal has a distorted output.

12. A clock is thrown as a projectile upward so that it keeps maximum proper time!



Physics Qualifying Exam Fall, 1979

QUANTUM MECHANICS II

1 hour
Closed Book
Answer 5 Questions

1. Describe several experiments which demonstrate the "wave-particle duality" of radiation and matter.

2. Describe briefly (giving specific examples) the key components of a laser and the basic principles underlying its operation.

3. a) Give the following properties and quantum numbers which correspond to the deuteron:
   (i) spin, (ii) parity, (iii) relative orbital angular momentum, (iv) isotopic spin, (v) magnetic moment (approximate).

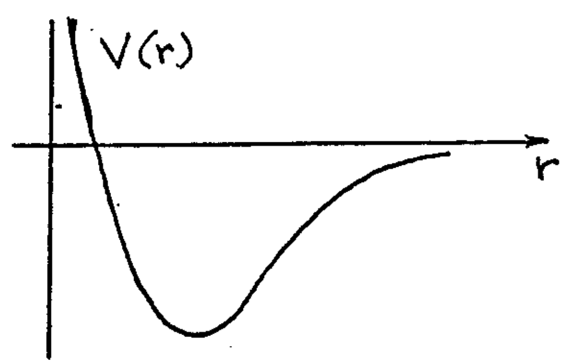
   b) Give a relation between the binding energy of the deuteron and the quantity which characterizes the "radius" of the deuteron.

4. a) What is meant by the "natural line width" of an atomic spectroscopic line?

   b) What is meant by "Doppler broadening" of such a line?

   c) Give an estimate of Doppler broadening in a gaseous discharge at temperature T. Compare the magnitudes of the natural and Doppler widths expected for a typical optical line at a reasonable temperature.

5. A molecule consists of two atoms each of mass M. The potential energy as a function of interatomic distance is given roughly by the figure:
Discuss the origin of the form of the potential, and give the low lying spectrum of the molecule.



6. Describe the Lamb Shift and the ground state hyperfine structure of hydrogen.

Useful constants:

$\hbar = 1.06 \times 10^{-27}$ erg-sec

$k_{Boltzmann} = 1.38 \times 10^{-16}$ erg ${}^{o}K^{-1}$

CLASSICAL MECHANICS

Open Book

Do 3 of the 4 Problems

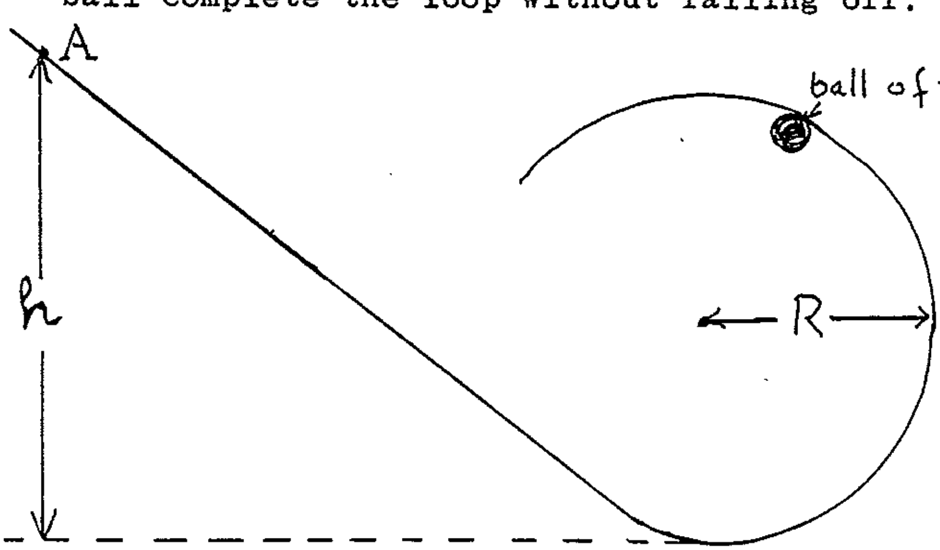
1. A relatively good model for the vibrations of a triatomic molecule of the type SO_2, NO_2, H_2O is given by three masses $m \neq M$ connected by elastic "springs" capable of longitudinal and transverse motions (with different spring constants, say k, K, respectively):

Assuming that the motion is two-dimensional and that there is no coupling between the longitudinal (x-direction) and transverse (y-direction) modes find:

a) the number of distinct vibration modes and their qualitative aspect;

b) the frequencies of the modes, assuming $M = 2m$, $K > k$.

2. A small uniform ball of radius r is released from rest at the point A of an inclined track. Assuming that the ball rolls without slipping determine the height h required to make the ball complete the loop without falling off.



(Moment of inertia of a ball of radius r is $I = \frac{2}{5} mr^2$.)

3. Use the Hamilton-Jacobi method to determine the motion of a two-dimensional harmonic oscillator:

   a) Write down the Lagrangian and the Hamiltonian for the problem.

   b) Write down the time-dependent Hamilton-Jacobi equation and find a complete integral.

   c) Use Jacobi's theorem to determine the trajectories and the motion along them.

4. A rigid body has principal moments of inertia $I_1 < I_2 < I_3$ corresponding to the three principal axes with unit vectors $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$.

   If one adds a small mass ε at the point $\vec{q} = x_1\hat{e}_1 + x_2\hat{e}_2 + x_3\hat{e}_3$ show that the change in the value of I_1 is

$$I_1(\varepsilon) = I_1 + \varepsilon(x_1^2 + x_2^2)$$

   and that the axis $\hat{e}_1$ changes to

$$\hat{e}_1(\varepsilon) = \hat{e}_1 + \varepsilon[x_1x_2\hat{e}_2/(I_2 - I_1) + x_1x_3\hat{e}_3/(I_3 - I_1)] \quad .$$

   (Hint: Expand the kinetic energy to order ε.)