Physics Qualifying Exam UCI October 7, 1980

QUANTUM MECHANICS

3 Hours

Do 4 Problems

Textbook and classroom notes are OK.

1. Given a particle of mass m in a one-dimensional potential $V = \alpha|x|$ where α is a constant > 0, use some appropriate approximation to estimate the energy of the ground state and the first excited state.

2. Consider a harmonic oscillator interacting with a spin-$\frac{1}{2}$ system. The Hamiltonian is

$$H = \hbar\omega\, a^{+} a + \hbar\omega\left(\frac{1+\sigma_z}{2}\right) + \hbar\lambda[a^{+}\sigma_{-} + a\sigma_{+}]$$

where as usual a, a^{+} are annihilation and creation operators for the oscillator;

$$\sigma_{\pm} \equiv \frac{\sigma_x \pm i\sigma_y}{2}$$

with $\sigma_x, \sigma_y, \sigma_z \equiv$ Pauli spin matrices.

(i) Find an operator (other than H) which corresponds to a conserved quantum number of the system.

(ii) Suppose we start at $t = 0$ in a state with $n > 0$ quanta in the oscillator and spin pointing down (i.e. along $-z$ direction). Find the probability that at some later time t the spin is pointing up.

3. A particle is bound in a finite-range central potential $V(r)$ about some fixed point. The ground state and first excited S-state wave functions are φ_0 and φ_1, respectively. The particle carries an electric charge. A second charged particle, which does not interact with V, is incident with wave vector $\vec{k}$.

3. Continued

a) Find an expression in first Born approximation for the differential cross section for scattering leading to excitation of the bound system from φ_o to φ_1.

b) Suppose the range of the potential V (and hence the extension of the wave functions φ_o and φ_1) is very much smaller than the wavelength of the incident beam. What will be the approximate angular dependence of the differential cross section in (a)?

c) What is the answer to the question in (b) if, instead of exciting the bound system, the scattering leaves it in the ground state φ_o?

4. Two spin-$\frac{1}{2}$ particles are bound together by an interaction consisting of a central potential, i.e., depending only on the inter-particle distance, and an effective spin-orbit interaction which we take to be

$$a_1 \vec{L} \cdot \vec{S}_1 + a_2 \vec{L} \cdot \vec{S}_2$$

Here $\vec{L}$ is the relative orbital angular momentum; $\vec{S}_1$ and $\vec{S}_2$ are the two spin operators; and a_1 and a_2 are constants which are almost, but not quite, equal. Find the structure of the energy levels for S and P states to zeroth order and first order in the small parameter $\epsilon \equiv a_1 - a_2$.

5. A spinless particle is scattered from a repulsive potential with $V(r) = V_o$ for $r < a$

$= 0$ for $r > a$

a) Find an expression for the S-wave scattering phase shift if the energy of the incident particle is less than V_o.

b) Use this result to find the scattering cross section at low energy.

Physics Qualifying Exam UCI October 6, 1980

Electricity and Magnetism

One Book: Jackson

15 points 1. A potential field is given by $\Phi_o(r,\theta,\varphi) = r^2 \sin 2\theta \cos\varphi$. Find the potential exterior to a grounded, conducting sphere of radius a placed in this field at the origin.

25 points 2. Write Maxwell's equations in media with $\bar{J}_{free} = 0$, $\rho_{free} = 0$.

a) Using the constitutive relations in terms of $\bar{P}$ and $\bar{M}$, reWrite Maxwell's equations with only $\bar{E}$, $\bar{B}$ terms on the left and only $\bar{P}$, $\bar{M}$ terms as bound sources on the right.

b) Write equations for the potentials $\bar{A}$, Φ in the Lorentz gauge using the $\bar{P}$, $\bar{M}$ terms as sources.

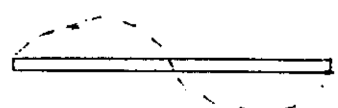
c) Now substitute for the potentials in terms of the vector fields $\bar{\pi}_e$, $\bar{\pi}_m$ defined by

$$\Phi = -\bar{\nabla}\cdot\bar{\pi}_e,\quad \bar{A} = \bar{\nabla}\times\bar{\pi}_m + \frac{1}{c}\frac{d\bar{\pi}_e}{dt}$$

d) Rearrange the equations in terms of a divergence and a curl equation and write second order equations for $\bar{\pi}_e$, $\bar{\pi}_m$ assuming the arguments of div and curl vanish.

e) Write the retarded solutions for $\bar{\pi}_e$, $\bar{\pi}_m$ in terms only of the sources $\bar{P}$, $\bar{M}$.

25 points 3. A linear antenna of length d is excited in such a way that the sinusoidal current makes a full wavelength of oscillation as shown in the figure.



a) Calculate the magnetic dipole moment.

b) Calculate the electric dipole moment.

c) Calculate the electric quadrupole moments.

d) What is the angular distribution of the power?
(Hint: Use results in the text for this part.)

35 points 4. A parallel-plate capacitor consists of two circular plates, so that the system has an axis of symmetry. The radius is a, the plate separation is ℓ, and the material filling the space between the plates has dielectric constant ϵ. The capacitor is charged by being placed in a circuit that contains a source of emf V_o and a series resistor R. If the circuit is completed at time t = 0, find the following quantities <u>within the capacitor</u> as functions of the time:

b) the electric field

b) the magnetic field

c) Poynting's vector

d) the field energy

e) the scalar potential

f) the vector potential.

Neglect edge effects and assume RC >> a/c.

Physics Qualifying Exam UCI October 8, 1980

MATHEMATICAL PHYSICS

One book plus class notes.

1. Consider the ordinary differential equation

$$x(2-x)y'' + 2(1-x)y' + y = 0$$

a) What are the singular points of this equation? (5 points)

b) Find the first three terms in the power series expansions of two independent solutions about the origin. (20 points)

2. Evaluate the following integrals

a) $$\int_0^\infty \frac{dx}{1+x^5}$$ ($12\frac{1}{2}$ points)

b) $$\int_0^\infty \frac{\cos ax}{b^2+x^2}\, dx$$ ($12\frac{1}{2}$ points)

3. Solve the integral equations

a) $$f(x) = x + \lambda \int_0^1 y(x+y)\, f(y)\, dy$$ ($12\frac{1}{2}$ points)

b) $$f(x) = \lambda \int_0^1 \sin \pi(x-y)\, f(y)\, dy$$ ($12\frac{1}{2}$ points)

In case b), what are the eigenvalues of λ?

4. A uniform string of length 5 meters hangs from two supports at the same height, 3 meters apart. By minimizing the potential energy of the string, find the equation describing the curve it forms and determine the vertical distance between the supports and the lowest point of the string. (25 points)

Physics Qualifying Exam UCI October 8, 1980

General Physics

(Closed Book; you may use a table of physical constants.)

1. Explain why the paramagnetic susceptibility of metals is independent of temperature, rather than inversely proportional to temperature, as would follow from Curie's law, and why the magnitude of the susceptibility is much smaller than expected from that law.

2. Explain the meaning and/or origin of 6 of the following terms:
   a. Landau damping.
   b. Color confinement.
   c. Reynolds' number.
   d. Microwave background radiation.
   e. Soliton.
   f. Critical opalescence.
   g. Bohm-Aharonov effect.
   h. Van der Waals forces.
   i. Heat pump.
   j. Maser.
   k. Einstein-Podolsky-Rosen paradox.

3. Give a list of the quantum numbers (spin, parity, orbital momentum, isospin) for the ground state of the deuteron.

4. What is the relation between the lifetime of a resonant state and the width of the energy level describing it.

5. List the discoveries for which the following physicist have received Nobel prizes (and give the year, if you know it).
   a. N. Basov
   b. F. Bloch
   c. L. Esaki
   d. A. Hewish
   e. J. H. D. Jensen
   f. A. Kastler
   g. P. Kusch

5. Continued

   h. W. Pauli

   i. A. Penzias

   j. B. Richter

   k. R. Schrieffer

   l. I. E. Tamm

6. Why is the light coming from a clear sky polarized? Where is the maximum polarization located relative to the sun?

7. Give a brief explanation of the following fact:
A negative muon may replace an electron in an atom of arbitrary Z and move on a "Bohr orbit" according to the same laws as the electron ($m_\mu = 207\ m_e$). The transition energies from states with $n \leq 4$ to more strongly bound states in such mesic atoms are the same for neutral atoms as for atoms from which all electrons have been removed.

8. Discuss, without necessarily deriving, the scattering of a particle from a hard sphere of radius R. Compare the total cross sections in the low and high energy limits with the geometric cross section πR^2. Explain the high energy result, i.e., discuss the concept of "shadow scattering."

9. "Heathcliff gazed intently at the spectacular sunset, his attention absorbed by the beautiful rainbow surrounding the fiery globe."

   Aside from the wretched quality of the prose, what is wrong with the preceding paragraph?

Physics Qualifying Exam UCI October 7, 1980

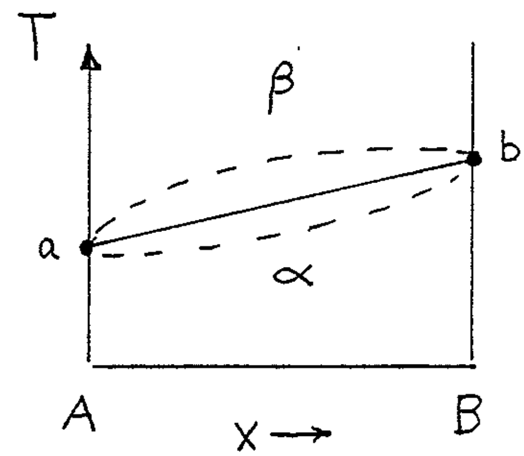
STATISTICAL MECHANICS

Open Book: two texts and one math tables book

Three Hours

Work either problem 1 or 2 and both problems 3 & 4.

1. Consider a low temperature phase transition in a metallic alloy $A_{1-x}B_x$, as pictured here in the $P = 0$ plane, between a high temperature (β) phase and a low temperature (α) phase. The dotted loop shows what the coexistence curve would look like if the two phases were allowed to adjust to equality of chemical potential. However, diffusion in solids requires thermal activation; hence at low temperatures the atoms are inhibited from moving around, and x is constrained to remain equal throughout the solid. Thus, the two phases coexist along a single, solid line ab. Find a formula analogous to the Clausius-Clapeyron equation for the slope of the solid line, $dx/dT|_{ab}$.



2. A model for salad dressing

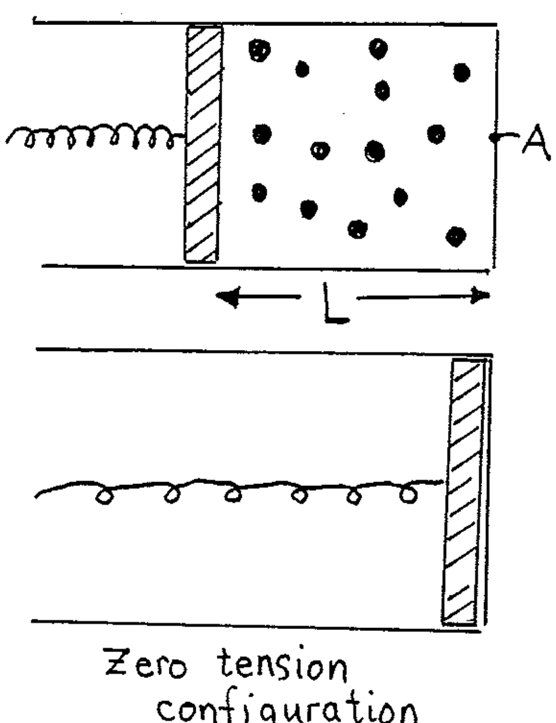
A certain oil has Gibbs free energy $G_o(T)$ equal to that of vinegar, i.e., $G_o(T) = G_v(T)$ at all temperatures. When the two are mixed there is a repulsion between oil and vinegar atoms such that the total free energy per atom for a mixture whose oil concentration is constrained to the value x is

$$G = x\, G_o + (1-x)G_v + E_{ov}\, x\,(1-x) - RT\, S_{mix}$$

where the interaction E_{ov} is positive. What is the entropy of mixing S_{mix}? Using the appropriate minimization

2. (Continued)
principle and graphical analysis of the resulting equations show that there exists a critical temperature T_c below which there are two equilibirium choices of x. Estimate T_c and draw the xT phase diagram exhibiting the two phase region ("Miscibility gap".) Physically what happens to the mixture when our goodly gourmet graduate student chef prepares his salad dressing in the hot, hot desert, but then carries the jar up to the cold, cold snowcapped peaks?

3. A system of N monatomic, noninteracting atoms is contained in a cylinder of area A, enclosed by a weightless piston which is attached to the bottom of the cylinder by a spring. The spring is under zero tension when the volume is zero.
a) Write down the two equations of state for U(T,V,N) and P(T V N) in terms of the equilibrium volume.
b) Find the equilibrium volume by two methods:
   i) using a thermodynamic minimization principle, and
   ii) by statistical evaluation, using the L dependent partition function.
c) Evaluate the fluctuation in the volume at equilibrium



4. A large evacuated chamber is filled with noninteracting, massless spin $\frac{1}{2}$ fermions; these are enclosed by walls through which the fermions cannot penetrate, but which act as a source and sink where the fermions are created and annihilated in such a way as to bring the fermions in the cavity to thermal equilibrium with the walls at temperature T. To what well-known problem is the situation analogous and what is the difference? What is the fermion chemical potential, and why? What is the energy per unit volume in the cavity? What is the pressure?

Physics Qualifying Exam UCI October 6, 1980

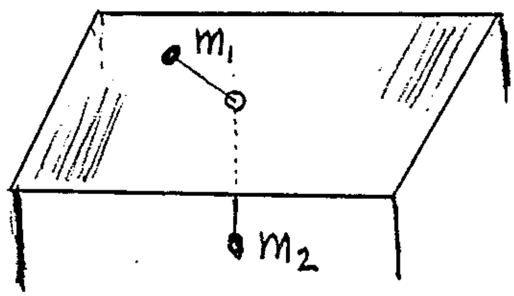
Classical Mechanics

Open Book

Do All Four Problems
Note: Problems have different point values.

30 Points

1. Two masses m_1 and m_2 are connected by a weightless string of length ℓ passing through a small hole in a horizontal table with m_2 hanging below the table; m_2 can move only in the vertical direction.



a) Choose an appropriate set of coordinates to describe this system and write the Lagrangian in terms of these.

b) Determine the equations of motion.

c) Under what conditions will m_2 be stationary?

d) If m_2 is slightly displaced from its stationary condition, small oscillations will ensue in the vertical direction. What will be the frequency of these oscillations?

20 Points

2. The Lagrangian for a particle is

$$L = \tfrac{1}{2} m \dot{\vec{r}}^2 + \dot{\vec{r}} \cdot (\vec{r} \times \vec{c})$$

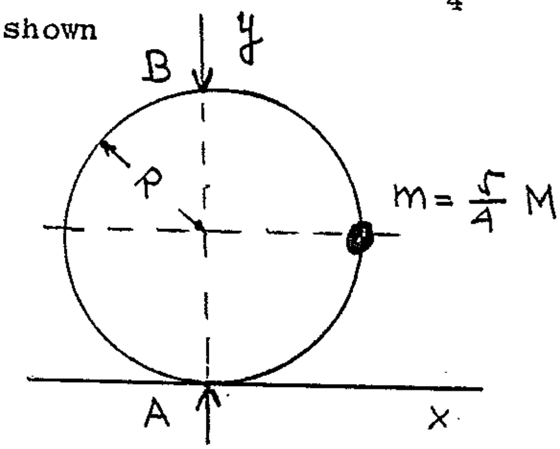
$\vec{c}$ is a given fixed vector

a) Find the equations of motion.

b) Find the Hamiltonian of this system.

30 Points

3. A thin disk of radius R and mass M lies in the x-y plane and and has a point mass $m = \frac{5}{4} M$ attached on its edge as shown



The moment of inertia tensor of the disk (without the extra mass point) about its center of mass is (z axis out of the paper)

$$I = \frac{MR^2}{4} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

a) Find the moment of inertia tensor of the disk about point A.

b) Find the moment of inertia tensor of the combination, mass point and disk, about A.

c) The disk is constrained to rotate about the y axis with angular velocity ω by pivots at A and B. Describe the angular momentum vector about A as a function of time and find the torque at A.

20 Points 4. Let $\Phi(q_i, P_j)$ be a generating function for a canonical transformation

$$Q_j = \frac{\partial \Phi}{\partial P_j}$$

Use the above to eliminate P_j in favor of Q_j and define

$$X(q_i, Q_j) = \sum_i Q_i P_i - \Phi$$

a) Find:

$$\frac{\partial X}{\partial q_i} = ?$$

$$\frac{\partial X}{\partial Q_j} = ?$$

b) Show that

$$\frac{\partial p_i}{\partial Q_j} = - \frac{\partial P_j}{\partial q_i}$$