Physics Qualifying Exam UCI 30 September 1981

GENERAL PHYSICS

1. A gas of free electrons is embedded in a rigid, uniform background of positive charge; the whole assembly is neutral. Derive an expression for the plasma frequency of the electrons.

2. Discuss the kinematics of pair production (photon → electron + positron). Show explicitly how momentum and energy is distributed in the final state. What is the threshold for pair production? Can pair production occur in vacuum?

3. A set of one-electron atoms occupy the sites $\ell = na_o$ of a one dimensional lattice. For large a_o the atoms are isolated and each electron satisfies an equation

$$\left[\frac{-\hbar^2}{2m} \frac{d^2}{dx^2} + v(x-\ell) \right] \varphi(x-\ell) = E_o \varphi(x-\ell) \quad .$$

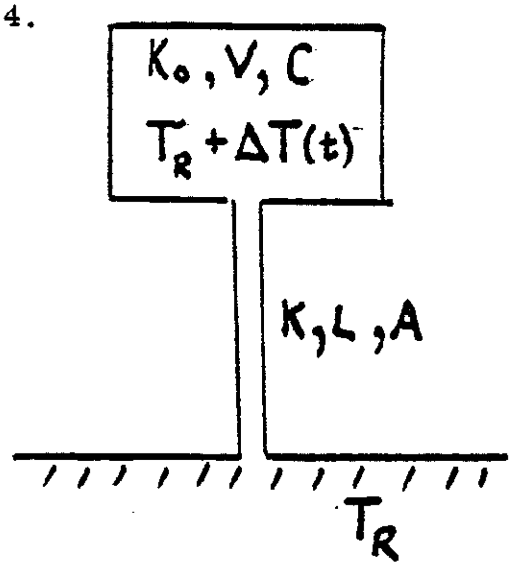
As a_o decreases, electrons begin to sense the nearest neighbor atoms and $v \to v + \delta v$. Assume small δv; choose the overall wavefunction of an electron in the lattice as

$$\varphi_k(r) = \sum_{\ell} e^{ik\ell} \varphi(r-\ell)$$

in order to satisfy Bloch's theorem. Show that

$$\mathcal{E}_k \approx E_o + \delta E \cos a_o k$$

i.e. the energy level E_o spreads into a band.

4\. 

At t = 0 a good thermal conductor of volume V, specific heat C per unit volume, thermal conductivity K_o and temperature $T_R + \Delta T$ is connected to a heat reservoir at temperature T_R through a thin cylindrical post of length L, cross sectional area A and thermal conductivity K (where $K \ll K_o$). Making reasonable idealizations find how ΔT depends on time.

5\. A rocket leaves earth Jan. 1, 1982. At each moment t sufficient propellant is fired off so that the rocket has acceleration g relative to its own rest frame at time t. This makes the astronaut feel comfortable. It arrives at its destination on Feb. 1, 1982 by its own clock. How long a time has elapsed according to those of us who elected to stay on earth?

6\. Discuss the physics behind several of the following headlines (taken from Physics Today, Search and Discovery, Sept. 1980 - Aug. 1981).

a. Period-doubling route to chaos shows universality

b. Plans for VLBI with 10 antennas

c. Tandem mirror success leads to expanded experimental facility

d. Fitch & Cronin share Nobel prize for CP violation

e. Experiments set limits and a value for neutrino mass

f. Literally two dimensional magnets.

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MATHEMATICAL PHYSICS

Closed Book

Do all problems

20 points for each problem

1. Solve the differential equation

$$2x^{3/2}\frac{dy}{dx} - x\sqrt{y} - y^{3/2} = 0$$

subject to the boundary condition $y = 0$ at $x = 1$.

2. Find an asymptotic approximation for large N(real and positive) to the function

$$y(N) = \int_0^1 dx \cos x \, e^{iN(x^2 - x)} .$$

3. A rod of square cross section ($0 \le x \le a$, $0 \le y \le a$) and length, L, is initiatlly at temperature $T = 0$ and has insulated sides except for the ends $z = 0$ and $z = L$. At time $t = 0$, the ends $z = 0$ and $z = L$ are placed in contact with a heat reservoir at $T = T_o > 0$. Find the temperature distribution for $t > 0$. Recall that

$$\nabla^2 T - \frac{c\rho}{K}\frac{\partial T}{\partial t} = 0$$

K = coeff. of heat cond.

c = specific heat

ρ = density

heat flux = $Q = -K\nabla T$ . .

The result may be expressed as a series.

4. Consider the differential equation

$$\frac{d^2y}{dx^2} - k^2y = 1$$

subject to the boundary conditions

$$y(o) = y(1) = 0 \quad .$$

a) Find the Green's function that may be used to solve the above equation.

b) Indicate how the Green's function can be used to give a formal solution of the differential equation for $y(x)$.

5. Solve the integral equation

$$f(x) = 1 + \lambda\int_o^1 (x + 2y + xy)f(y)dy \quad .$$

Are there any values of λ for which your solution is not well behaved?

QUANTUM MECHANICS

Open Book

Do 5 Problems

1. A system has 3 degenerate states φ_1, φ_2, φ_3. An additional interaction is introduced which, in the basis given by these states, adds to the Hamiltonian a term of the form:

$$\begin{pmatrix} o & h & o \\ h & o & h' \\ o & h & o \end{pmatrix}$$

where h is a real quantity.

a) How are the energy levels shifted by this interaction?

b) If the system is initially in the state φ_1, what is the probability that after some time, t, it is in the state φ_2?

2. a) Find, in Born approximation, the scattering amplitude and differential cross-section for scattering from a square well potential of depth V_o and radius a.

   b) What is the condition for validity of this result in the low energy limit?

   c) Using the result in (a), find the low energy behavior of the S-wave scattering phase shift.

3. The wave function of a non-relativistic spin-$\frac{1}{2}$ particle can in general be given by a Pauli spinor of the form

$$\begin{pmatrix} \varphi_1(\vec{r}) \\ \varphi_2(\vec{r}) \end{pmatrix}$$

where as usual the basis states are the eigenstates of the z-component of the spin operator.

a) Does there exist at every point in space some direction along which the particle may be regarded as positively oriented? If the answer is "no", give a proof. If it is "yes", find the direction cosines of the direction along which the particle is oriented.

b) Does the spinor above provide the most general description of a non-relativistic spin-$\frac{1}{2}$ particle? Explain. Show how one may represent such a particle in a situation in which it has no preferred orientation in space. Discuss explicitly.

4. A particle bound in a harmonic oscillator potential occupies the ground state. At some instant the spring constant of the oscillator is changed very suddenly by a factor of 2. Write expressions for the probabilities P_1, P_2 that the particle is found in the first and second excited states, respectively, of the new potential. What is the ratio P_1/P_2?

5. Consider in an appropriate approximation the scattering of a high energy positron from a neutral hydrogen atom. Without necessarily evaluating integrals, find an expression for the differential cross-section. Show how the calculation differs for (i) Elastic scattering from the atom in the ground state and (ii) inelastic excitation of the atom from the 1S to 2S state. (You may ignore any atomic recoil effects.)

6. Consider the matrix element

$$\langle \beta j'm' | U_{m_1}^{(j_1)} V_{m_2}^{(j_2)} | \alpha j m \rangle$$

where U and V are irreducible tensor operators, but are otherwise unspecified, and the states are, in the usual notation, eigenstates of $\vec{J}^2$ and J_z, where $\vec{J}$ is the total angular momentum; α and β specify the remainder of a complete commuting set of observables.

a) How many such matrix elements are there? (i.e. how many different combinations of the m's?)

b) What is the number of independent constants (reduced matrix elements) in terms of which the above can be expressed?

c) Write this matrix element explicitly in terms of these constants. Identify all quantities explicitly.

d) What are the restrictions on j' and m' in order for the above matrix element to be non-vanishing?

e) Suppose U and V are components of the angular momentum operator itself; does this lead to further restrictions in (d)?

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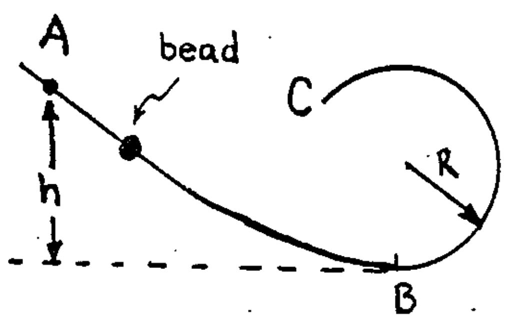
CLASSICAL MECHANICS

Open Book

Do all 5 Problems

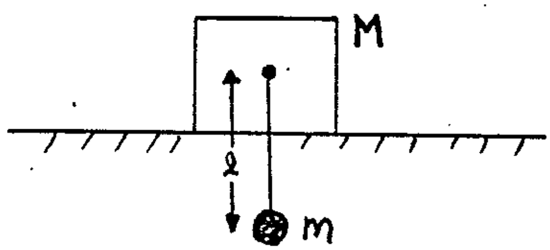
1. A massive bead is released from rest at point A on a frictionless, inclined track.
   a) What is the minimum height, h, for which the bead will eventually leave the track?
   b) For what shape of segment A → B is the travel time least?

   Note: Track shape is circular from B to C, slope can be discontinuous at B.

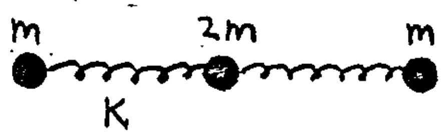


2. A mass M slides on a frictionless shelf, while a second mass, m, hangs below on a massless rod of length, ℓ.

Set up a Lagrangian for the system and find the equations of motion. (You may treat this as a two-dimensional problem; i.e. ignore motion perpendicular to the plane of the drawing.)



3. Three masses connected by two identical springs are constrained to move in a straight line



Find:

a) the vibrational modes of the system,

b) the mode frequencies.

4. The Lagrangian for a particle is

$$L = \frac{1}{2} m \dot{\vec{r}}^{\,2} + \vec{r} \cdot (\dot{\vec{r}} \times \vec{c})$$

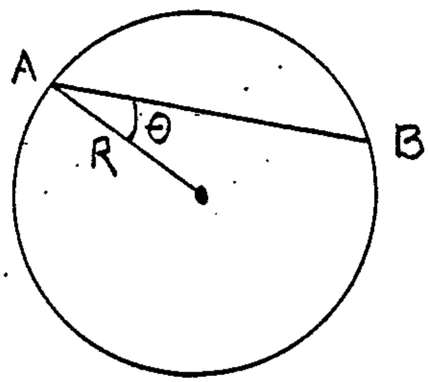
where $\vec{c}$ is a fixed vector.

a) Find the equations of motion.

b) Find the Hamiltonian of this system.

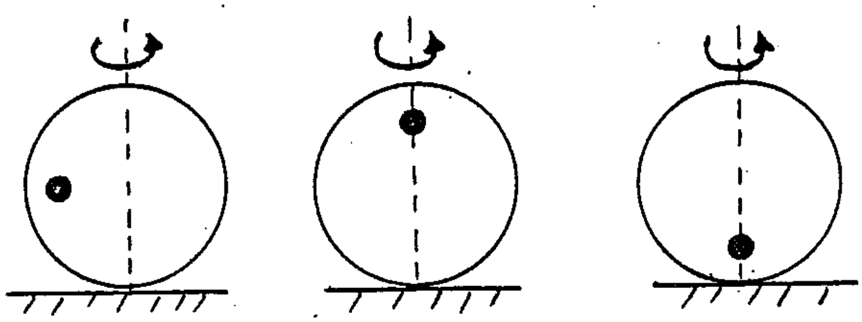
c) Can you translate this system into masses, springs, etc?

5. a)



A straight line, frictionless track runs in a tunnel from point A to B through earth of constant density. What is the travel time as a function of θ for a train propelled by gravity?

b) A coin with an off-center hole is spun on a table. Where will the hole migrate, e.g., top, bottom, side? Justify your answer.



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ELECTRICITY AND MAGNETISM

CLOSED BOOK: Math tables will be provided upon request during exam.

Do all problems; relative weighting of the problem is provided at the start of each section of a problem.

Use the supplied bluebooks for all work.

Identify the bluebooks with your number only.

Use a separate bluebook for scratch paper.

Time: 9 a.m. - 11:30 a.m.

1. A pair of coaxial, perfectly conducting cylinders carry equal currents in opposite directions. The medium between the cylinders has dielectric constant, ϵ, and permeability, $\mu = \mu_o$. The radius of the inner cylinder is a, the radius of the outer cylinders is b, and b>>a.

(10) a. Calculate the capacitance, C, per unit length of the system.

(10) b. Calculate the inductance, L, per unit length of the system.

(5) c. What is the relation among the speed of light, c, in the system and L and C?

2. A planar metallic surface at $x = d$ (at potential, φ_d) is parallel to a planar metallic surface at $x = 0$ (at potential $\varphi_o = 0$). The $x = 0$ surface may free charges to the system in unlimited quantity. Charges of only one species (charge q, mass m) are released, with speed equal to zero when released at $x = 0$. Assume motion of the charges is slow enough that electrostatics formulas apply.

(5) a. Write Poisson's equation for the system and a simple expression for the current density. How is a particle's kinetic energy related to the voltage at any given point between the plates?

(5) b. What is the steady-state boundary condition on the electric field at $x = 0$? Give a brief qualitative explanation supporting your answer.

(5) c. Eliminate ρ, charge density, and v, particle speed, and obtain a differential equation for the potential in terms of the current density.

(20) d. Obtain a solution for the current density in terms of q, d, m, and the potential difference between the plates. Comment on the difference of the effect on electrons and ions.

(10) e. Now consider the emitter as a wire, radius a, surrounded by a coaxial circular cylinder collector of inner radius b at φ_b. Write an expression for the current per unit length, I, as a function of radius. What is Poisson's equation in cylindrical coordinates with the charge density expressed in terms of I and the local potential φ?

(20) f. Assume a solution of the form $-I = \frac{8\pi\epsilon_o}{9}\left(\frac{2q}{m}\right)^{\frac{1}{2}} \frac{\varphi^{3/2}}{r\beta^2}$.,

Here β is a function of the variable $\gamma = \ln(\frac{r}{a})$ which can be determined by transforming Poisson's equation to an equation for β in terms of γ. Obtain the first two terms of a power series for $\beta = \beta(\gamma)$.

3. Consider a transverse wave ($\underset{\sim}{E}$, $\underset{\sim}{B}$, $\underset{\sim}{k}$ all perpendicular to one another) propagating in a conducting medium with permeability μ, dielectric constant ϵ, and conductivity σ (i.e. $\underset{\sim}{J} = \sigma \underset{\sim}{E}$).

(5) a. i. Write Maxwell's equations.

ii. Assume that the fields vary as $\exp(i\, \underset{\sim}{k} \cdot \underset{\sim}{x} - i\omega t)$.

Do the usual manipulations to get rid of ∇ and $\frac{\partial}{\partial t}$ and rewrite the equations in terms of $\underset{\sim}{k}$ and ω.

(10) b. i. Eliminate $\underset{\sim}{H}$ from the equations and state the resulting wave equation for $\underset{\sim}{E}$.

ii. What is the dispersion relation (that is, $k^2 = k^2(\omega)$)?

(20) c. Find k_i and k_r ($k = k_r + ik_i$). Some useful thoughts: check answer by considering $\sigma \to 0$ case; $\sin 2\alpha = 2 \sin\alpha \cos\alpha$; $\sin \frac{\alpha}{2} = \left[\frac{1 - \cos\alpha}{2} \right]^{\frac{1}{2}}$.

(10) d. In a good conductor σ is very large. Find an approximate expression for the evanescent distance of a transverse wave penetrating a good conductor. Give an order of magnitude estimate for this distance in copper [$\sigma \approx 5 \times 10^{17} \text{sec}^{-1}$ Gaussian Units)] at $f = 10$ MHz.

(20) e. Going back to the equations you wrote in a. i., consider longitudinal waves, i.e. $\underset{\sim}{k} \parallel \underset{\sim}{E}$, and show that longitudinal fields cannot exist for much time in a good conductor without an applied current. What is meant by "much time" in quantitative terms?

4. Consider a system of charges varying in time as

$$\rho(\underset{\sim}{x},t) = \rho(\underset{\sim}{x})e^{-i\omega t} \qquad \underset{\sim}{J}(\underset{\sim}{x},t) = \underset{\sim}{J}(\underset{\sim}{x})e^{-i\omega t}$$

(remember, the real part is the physical quantity). The vector potential in the Lorentz gauge is

$$\underset{\sim}{A}(\underset{\sim}{x},t) = \frac{1}{c}\int d^3x' \int dt' \, \frac{\underset{\sim}{J}(\underset{\sim}{x}',t')}{|\underset{\sim}{x}-\underset{\sim}{x}'|}\,\delta\!\left(t' + \frac{|\underset{\sim}{x}-\underset{\sim}{x}'|}{c} - t\right)$$

if there are no boundary surfaces present. Let $k \equiv \omega/c$. Consider a thin, center-fed, linear antenna of length d such that $\underset{\sim}{J}(\underset{\sim}{x}) = I\sin\left(\frac{kd}{2} - k|z|\right)\delta(x)\delta(y)\hat{z}$ for $|z| \le d/2$. Further limit the discussion to the radiation zone $d << \frac{2\pi}{k} << r$, where in this zone $|\underset{\sim}{x} - \underset{\sim}{x}'| = r - \hat{n}\cdot\underset{\sim}{x}'$, with $\hat{n} \equiv \underset{\sim}{x}/|\underset{\sim}{x}|$.

(70) a. Find $dP/d\Omega$, the time averaged power radiated per unit solid angle.

(10) b. What is the radiation resistance?

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THERMODYNAMICS AND STATISTICAL MECHANICS

Closed Book

Solve 4 problems out of the 5

3 hours

Points

(20) 1. An ideal monoatomic gas is characterized by two equations of state:

$$PV = NRT \quad \text{and } U = \frac{3}{2} NRT \quad .$$

Find the expression for the entropy S(U,V and N) as a function of U, V, and N. Then verify that the two equations of state given above can be retrieved from your result. What is the third equation of state?

(30) 2. Consider the process of adiabatic demagnetization along the following steps. In solving this problem, ignore the difference between the local field H and the external field H_e; i.e., ignore the demagnetization effect, and assume that there is no volume change.

i) A paramagnetic salt is magnetized isothermally at $T = T_i$ by increasing the magnetic field from $H = 0$ to H_i.

The isothermal susceptibility of this salt is given approximately by:

$$\chi_T = \frac{a}{T} \qquad \text{where a is a positive constant.}$$

a) What is the change in the entropy of the salt?

(Hint: Use the appropriate Maxwell relation.)

b) Knowing the physical picture of spin alignment in a magnetic field and also knowing the statistical mechanical nature of entropy, can you explain why the entropy changes in the direction you found? (increase or decrease)

c) What is the change in the internal energy of the salt?

ii) Now the paramagnetic salt is thermally isolated from the surrounding, and the magnetic field is decreased from $H = H_i$ to $H = H_f$ (adiabatic demagnetization).

d) Show that the temperature change per unit change in the magnetic field, $\left(\frac{\partial T}{\partial H}\right)_S$ is given by

$$\left(\frac{\partial T}{\partial H}\right)_S = -\frac{\mu_o VTH}{C_H}\left(\frac{\partial \chi_T}{\partial T}\right)_H$$

where C_H is the heat capacity at constant H given by

$$C_H(T,H) = T\left(\frac{\partial S}{\partial T}\right)_H ,$$

and μ_o is the permittivity of free space.

e) Now assume that the functional form of $C_H(T,H)$ is given by an equation of state:

$$C_H(T,H) = \frac{V}{T^2}(b + aH^2) \text{ where b is a constant.}$$

Find the final temperature as a function of T_i, H_i and H_f.

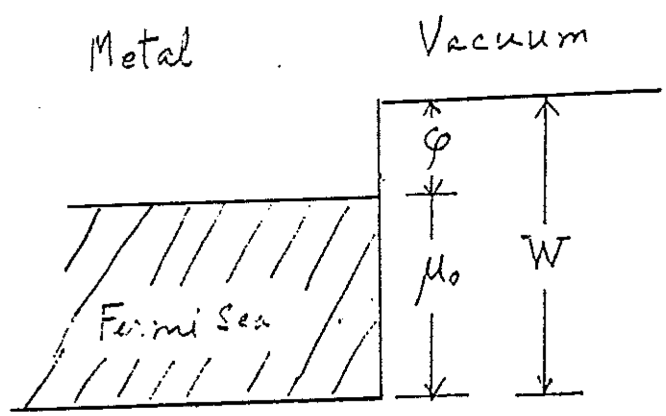
f) Sketch a graph of S vs T for this salt at different H values, and explain the two processes above on the graph.

(20) 3. Show that if X is a coordinate of an entirely classical system in contact with a heat bath at temperature T and $\Delta X = X - \bar{X}$, then

$$\overline{\Delta X^2} = kT\left(\frac{\partial \bar{X}}{\partial F}\right)_T$$

where F is the generalized force on X due to external forces. Apply this result to the mean square fluctuation of the charge $\overline{\Delta q^2}$ on a capacitor plate at temperature T and find the expression for $\overline{\Delta q^2}$. Estimate the noise voltage appearing across a capacitance of 1 μF at room temperature (300 K).

(20) 4. Consider a metal as a gas of free electrons confined in a box and obtain the thermionic emission current density J at temperature T. Assume that the potential energy of an electron is constant at -W inside the metal and zero in vacuum outside. Also, assume that the work function φ is much greater than kT. Assume that all electrons with z-component of kinetic energy greater than W will escape the metal.



(20) 5. Describe Gibbs' paradox using a concrete example. What is the nature of this paradox? In answering this question, try to demonstrate the extent of your knowledge of statistical mechanics.