Physics Qualifying Examination

October 3, 1983
10 - 12

ELECTRICITY AND MAGNETISM

Open Book: Jackson only. Closed Notes
Show all work
Two Hours

1. An infinite cylindrical dielectric with dielectric constant ε is introduced in to a previously uniform electric field with its axis perpendicular to $\bar{E}_o$. Find the potential and electric fields both inside and ouside the dielectric.

   Hint: Investigate powers of the cylindrical coordinate ρ.

2. An infinite plane of charge at $x = 0$ is oscillated in the z direction with frequency ω and maximum current density J_o.

   a. Solve for the vector potential everywhere.

   b. What are the $\bar{E}$ and $\bar{B}$ fields?

   c. What is the time averaged Poynting rate?

3. An infinite solenoid is aligned along the z axis carrying current I through N turns/meter. It is moving in the x direction with velocity v with respect to the laboratory.

   a. Using the Lorentz transformation find the electric and magnetic fields inside the solenoid seen by the laboratory observer.

   b. Compare the result to the Lorentz force and see if the magnetic flux inside the solenoid is changed.

   c. Show that the vector potential $\bar{A} = -\frac{1}{2}\,\bar{x} \times \bar{B}$ gives the correct magnetic field in the rest frame of the solenoid.

   d. Lorentz transform the four-vector potential to the laboratory frame.

   e. Compute the laboratory $\bar{E}$ and $\bar{B}$ fields inside the solenoid from the transformed four-vector potential. Compare to part (a).

Physics Qualifying Examination — October 4, 1983

QUANTUM MECHANICS

Closed Book
One 8 ½ x 11" page of notes allowed
Show all steps.

Points

20 1. Given the matrix element

i) $\langle n|x|m\rangle = \gamma \left(\frac{n+1}{2}\right)^{1/2}$ for $m = n + 1$

$= \gamma \left(\frac{n}{2}\right)^{1/2}$ for $m = n-1$

$= 0$ otherwise.

Use i) to calculate $\langle n|x^3|r\rangle$.

20 2. Suppose a parameter α occurs in the Hamiltonian H. (It might be, e.g., the width or depth of the potential.) Show that for a stationary, normalized state that

$$\frac{\partial E}{\partial \alpha} = \frac{\partial \langle \psi|H|\psi\rangle}{\partial \alpha} = \langle \psi| \frac{\partial H}{\partial \alpha} |\psi\rangle.$$

30 3. a) The deuteron is a bound state of a proton and neutron. Construct wave functions

$$\phi_{L,S}^{J=1,M_J=1}$$

for $L = 0$, $S = 1$
$L = 2$, $S = 1$
$L = 1$, $S = 0$

using attached tables.

b. Using ψ from a) calculate the magnetic moment of the deuteron

$$\mu = \langle \psi^{J=1, M_J=1} \Big| \mu_z \Big| \psi^{J=1, M_J=1} \rangle$$

where $\mu_z = 2(\mu_n M_{s_n} + \mu_p M_{s_p}) + \frac{1}{2} M_L \mu_o$ for each of the 3 above possibilities.

$$\mu_n = -1.91\ \mu_o, \quad \mu_p = 2.79\ \mu_o$$

(where μ_o is a nuclear magneton).

c. The deuteron has $\mu = .86\ \mu_o$. What combination of the 3 states listed , consistent with the usual strong interaction conservation laws, would fit the experimental value.

30 4. a) Given the Dirac equation

$$[E - (c\underline{\alpha}\cdot\underline{p} + \beta\, mc^2 + V)]\psi = 0$$

Writing $\psi = \binom{\phi}{\chi}$ where ϕ abd χ each have 2 components express χ in terms of ϕ.

Recall $\underline{\alpha} = \begin{pmatrix} 0 & \underline{\sigma} \\ \underline{\sigma} & 0 \end{pmatrix}$ $\quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ where 1 is the 2 x 2 identity matrix.

b) Given the Dirac current

$$\underline{j} = c\ \psi^+\ \underline{\alpha}\ \psi$$

use the result of a) and find the non-relativistic reduction for $\underline{j}$. Recall $\sigma_j \sigma_i + \sigma_i \sigma_j = 2\delta_{ij}\ 1$.

CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A √ is to be understood over **every** coefficient, e.g., for −8/15 read −√(8/15).

Notation:

	J	J	...
	M	M	...
m_1 m_2			
m_1 m_2	Coefficients		
. .			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

1/2 × 1/2

m_1	m_2	J=1	J=0
+1/2	+1/2	1 (M=+1)	
+1/2	−1/2	1/2 (M=0)	1/2
−1/2	+1/2	1/2	−1/2
−1/2	−1/2	1 (M=−1)	

1 × 1/2

M	m_1	m_2	J=3/2	J=1/2
+3/2	+1	+1/2	1	
+1/2	+1	−1/2	1/3	2/3
+1/2	0	+1/2	2/3	−1/3
−1/2	0	−1/2	2/3	1/3
−1/2	−1	+1/2	1/3	−2/3
−3/2	−1	−1/2	1	

2 × 1/2

M	m_1	m_2	J=5/2	J=3/2
+5/2	+2	+1/2	1	
+3/2	+2	−1/2	1/5	4/5
+3/2	+1	+1/2	4/5	−1/5
+1/2	+1	−1/2	2/5	3/5
+1/2	0	+1/2	3/5	−2/5
−1/2	0	−1/2	3/5	2/5
−1/2	−1	+1/2	2/5	−3/5
−3/2	−1	−1/2	4/5	1/5
−3/2	−2	+1/2	1/5	−4/5
−5/2	−2	−1/2	1	

3/2 × 1/2

M	m_1	m_2	J=2	J=1
+2	+3/2	+1/2	1	
+1	+3/2	−1/2	1/4	3/4
+1	+1/2	+1/2	3/4	−1/4
0	+1/2	−1/2	1/2	1/2
0	−1/2	+1/2	1/2	−1/2
−1	−1/2	−1/2	3/4	1/4
−1	−3/2	+1/2	1/4	−3/4
−2	−3/2	−1/2	1	

1 × 1

M	m_1	m_2	J=2	J=1	J=0
+2	+1	+1	1		
+1	+1	0	1/2	1/2	
+1	0	+1	1/2	−1/2	
0	+1	−1	1/6	1/2	1/3
0	0	0	2/3	0	−1/3
0	−1	+1	1/6	−1/2	1/3
−1	0	−1	1/2	1/2	
−1	−1	0	1/2	−1/2	
−2	−1	−1	1		

2 × 1

M	m_1	m_2	J=3	J=2	J=1
+3	+2	+1	1		
+2	+2	0	1/3	2/3	
+2	+1	+1	2/3	−1/3	
+1	+2	−1	1/15	1/3	3/5
+1	+1	0	8/15	1/6	−3/10
+1	0	+1	6/15	−1/2	1/10
0	+1	−1	1/5	1/2	3/10
0	0	0	3/5	0	−2/5
0	−1	+1	1/5	−1/2	3/10
−1	0	−1	6/15	1/2	1/10
−1	−1	0	8/15	−1/6	−3/10
−1	−2	+1	1/15	−1/3	3/5
−2	−1	−1	2/3	1/3	
−2	−2	0	1/3	−2/3	
−3	−2	−1	1		

3/2 × 1

M	m_1	m_2	J=5/2	J=3/2	J=1/2
+5/2	+3/2	+1	1		
+3/2	+3/2	0	2/5	3/5	
+3/2	+1/2	+1	3/5	−2/5	
+1/2	+3/2	−1	1/10	2/5	1/2
+1/2	+1/2	0	3/5	1/15	−1/3
+1/2	−1/2	+1	3/10	−8/15	1/6
−1/2	+1/2	−1	3/10	8/15	1/6
−1/2	−1/2	0	3/5	−1/15	−1/3
−1/2	−3/2	+1	1/10	−2/5	1/2
−3/2	−1/2	−1	3/5	2/5	
−3/2	−3/2	0	2/5	−3/5	
−5/2	−3/2	−1	1		

3/2 × 3/2

M	m_1	m_2	J=3	J=2	J=1	J=0
+3	+3/2	+3/2	1			
+2	+3/2	+1/2	1/2	1/2		
+2	+1/2	+3/2	1/2	−1/2		
+1	+3/2	−1/2	1/5	1/2	3/10	
+1	+1/2	+1/2	3/5	0	−2/5	
+1	−1/2	+3/2	1/5	−1/2	3/10	
0	+3/2	−3/2	1/20	1/4	9/20	1/4
0	+1/2	−1/2	9/20	1/4	−1/20	−1/4
0	−1/2	+1/2	9/20	−1/4	−1/20	1/4
0	−3/2	+3/2	1/20	−1/4	9/20	−1/4
−1	+1/2	−3/2	1/5	1/2	3/10	
−1	−1/2	−1/2	3/5	0	−2/5	
−1	−3/2	+1/2	1/5	−1/2	3/10	
−2	−1/2	−3/2	1/2	1/2		
−2	−3/2	−1/2	1/2	−1/2		
−3	−3/2	−3/2	1			

2 × 3/2

M	m_1	m_2	J=7/2	J=5/2	J=3/2	J=1/2
+7/2	+2	+3/2	1			
+5/2	+2	+1/2	3/7	4/7		
+5/2	+1	+3/2	4/7	−3/7		
+3/2	+2	−1/2	1/7	16/35	2/5	
+3/2	+1	+1/2	4/7	1/35	−2/5	
+3/2	0	+3/2	2/7	−18/35	1/5	
+1/2	+2	−3/2	1/35	6/35	2/5	2/5
+1/2	+1	−1/2	12/35	5/14	0	−3/10
+1/2	0	+1/2	18/35	−3/35	−1/5	1/5
+1/2	−1	+3/2	4/35	−27/70	2/5	−1/10
−1/2	+1	−3/2	4/35	27/70	2/5	1/10
−1/2	0	−1/2	18/35	3/35	−1/5	−1/5
−1/2	−1	+1/2	12/35	−5/14	0	3/10
−1/2	−2	+3/2	1/35	−6/35	2/5	−2/5
−3/2	0	−3/2	2/7	18/35	1/5	
−3/2	−1	−1/2	4/7	−1/35	−2/5	
−3/2	−2	+1/2	1/7	−16/35	2/5	
−5/2	−1	−3/2	4/7	3/7		
−5/2	−2	−1/2	3/7	−4/7		
−7/2	−2	−3/2	1			

2 × 2

M	m_1	m_2	J=4	J=3	J=2	J=1	J=0
+4	+2	+2	1				
+3	+2	+1	1/2	1/2			
+3	+1	+2	1/2	−1/2			
+2	+2	0	3/14	1/2	2/7		
+2	+1	+1	4/7	0	−3/7		
+2	0	+2	3/14	−1/2	2/7		
+1	+2	−1	1/14	3/10	3/7	1/5	
+1	+1	0	3/7	1/5	−1/14	−3/10	
+1	0	+1	3/7	−1/5	−1/14	3/10	
+1	−1	+2	1/14	−3/10	3/7	−1/5	
0	+2	−2	1/70	1/10	2/7	2/5	1/5
0	+1	−1	8/35	2/5	1/14	−1/10	−1/5
0	0	0	18/35	0	−2/7	0	1/5
0	−1	+1	8/35	−2/5	1/14	1/10	−1/5
0	−2	+2	1/70	−1/10	2/7	−2/5	1/5
−1	+1	−2	1/14	3/10	3/7	1/5	
−1	0	−1	3/7	1/5	−1/14	−3/10	
−1	−1	0	3/7	−1/5	−1/14	3/10	
−1	−2	+1	1/14	−3/10	3/7	−1/5	
−2	0	−2	3/14	1/2	2/7		
−2	−1	−1	4/7	0	−3/7		
−2	−2	0	3/14	−1/2	2/7		
−3	−1	−2	1/2	1/2			
−3	−2	−1	1/2	−1/2			
−4	−2	−2	1				

d functions

$$d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2}$$

$$d^1_{0,0} = \cos\theta \qquad d^1_{1,1} = \frac{1+\cos\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}} \qquad d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2}$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta-1}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta+1}{2}\sin\frac{\theta}{2}$$

$$d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2 \qquad d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta \qquad d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta$$

$$d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta \qquad d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2$$

$$d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta-1) \qquad d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta \qquad d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta+1)$$

$$d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Sign convention is that of Wigner (Group Theory, Academic Press, New York, 1959), also used by Condon and Shortley (The Theory of Atomic Spectra, Cambridge Univ. Press, New York, 1953), Rose (Elementary Theory of Angular Momentum, Wiley, New York, 1957), and Cohen (Tables of the Clebsch-Gordan Coefficients, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The signs and numbers in the current tables have been calculated by computer programs written independently by Cohen and at LBL. Table extended April 1974.

Physics Qualifying Exam

October 3, 1983
2 - 5 p.m.

CLASSICAL MECHANICS

Closed book and notes
Do all problems

1. In a closed system of particles, when is:

   a) $\vec{P}$ conserved?

   b) $\vec{L}$ conserved?

   c) $\vec{E}$ conserved?

2. What is the velocity of a 3 MeV electron?

3. What is the nominal supply voltage to a TTL integrated circuit?

4. A Solid sphere of mass M and radius R is released at the top of a frictionless incline (height=h). The level part of the track has coefficients of static and kinetic friction μ_s and μ_k respectively. If the sphere is purely rolling by point A, how high up the second frictionless incline will it go?

5. Given a particle of mass M in an attractive force field

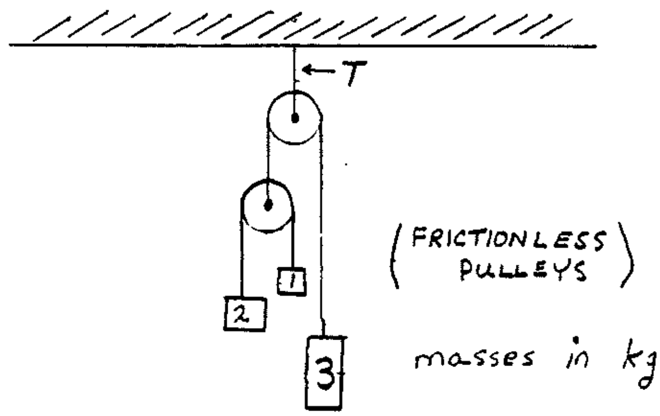
$$\vec{F}(r) = -\frac{k}{r^3}\hat{r}$$

a) If it is in a circular orbit of radius R, and is then "nudged" a small amount, what will be the frequency of the small oscillations about R? (Give the answer in terms of the constants of motion.)

b) For any arbitrary initial conditions, what will be the exact equation of the orbit (i.e., r(θ)). Hint: you may find the relations

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} \quad \text{and} \quad u = \frac{1}{r}$$

useful.



6. What is the tension in the string holding the above system to the ceiling?

Physics Qualifying Exam October 5, 1983
10 - 12

THERMODYNAMICS AND STATISTICAL MECHANICS

Allowed material: P. K. Pathria, Statistical Mechanics

Solve all 4 problems

Points

20 1. The Helmholtz potential F is sometimes referred to as the available work at constant temperature, or the Helmhlotz free energy. Using a concrete example, show that F indeed has this meaning.

25 2. Assume that there is only one acoustic phonon branch with the dispersion relation given by

$$\omega = v_s k$$

where v_s is the sound velocity. Develop the Debye theory of heat capacity on this model in two dimensions. How does the heat capacity C_V depend on temperature for very low T? Discuss the difference between this model and the three dimensional case.

25 3. A certain ion has the ground state with S = 0. The lowest excited state is an S = 1 state that lies an energy Δ above the ground state. Calculate the paramagnetic susceptibility of N such ions imbedded in the lattice of a non-magnetic host crystal. Assume that the ions do not interact with each other. Also assume that $kT \gg g\mu_B H$ and $\Delta \gg g\mu_B H$.

Points

30 4. Consider a system that consists of N distinguishable noninteracting particles, and suppose that each particle has only two eigenstates with energies ε_o and $-\varepsilon_o$. ($\varepsilon_o > o$)

a) If there is no restriction on the total energy of the system, how many microstates are there?

b) If the total energy of the system is fixed at E, how many microstates are there? Get the answer in terms of N, E, and ε_o.

c) Find the temperature of the system as a function of E using the concepts of microcanonical ensemble.

d) Find the range of E for which a thermodynamic equilibrium is possible. Discuss the physical significance of your answer.

e) Now use the concepts of canonical ensemble, and find the entropy and the internal energy of the system.

f) Find the specific heat of the system.

Physics Qualifying Examination

October 5, 1983
2:00 - 5:00

MATHEMATICAL PHYSICS

One textbook plus class notes permitted

Do all problems

20 points

1. Evaluate the line integral and state any restrictions on the parameters a, b:

$$\int_0^\infty x^a (x + e^{ib})^{-1} dx$$

2. Consider the function defined for $-\infty < x < \infty$:

$$f(x) = \begin{cases} \frac{1}{2} x^2 & \text{for } x > 0, \\ -\frac{1}{2} x^2 & \text{for } x < 0. \end{cases}$$

10 points

a. In the sense of distribution theory, calculate $f'(x)$, $f''(x)$ and $f'''(x)$ (state which test function space you are using).

10 points

b. What are the Fourier transforms of $f(x)$ and $f'''(x)$ (in the sense of distribution theory, of course) and how are they related?

5 points

c. Can you find an ordinary differential equation for which $f(x)$ is a Green's function (fundamental solution)?

15 points

3. Solve the integral equation (and state what kind it is, etc.)

$$y(s) = \lambda \int_0^1 \sin \pi(t - s)\, y(t)\, dt.$$

4. This problem consists of several parts, which can be solved separately. You may start with any part you wish, but must describe the exact assumptions you make or ready-made results (e.g., from notes) you use. Credit will be given for educated guesses, if time is running out to do all the calculations.

10 points

a. Using Hamilton's principle, show that the equation governing the small transverse vibration of a heavy string of constant linear density, around its vertical equilibrium position is:

$$\frac{\partial^2 u}{\partial t^2} - g\frac{\partial}{\partial x}\left(x \frac{\partial u}{\partial x}\right) = 0 \quad , \tag{1}$$

where g is the acceleration of gravity, and the coordinate x is measured upward from the lower end of the string of length L. From the variational principle (or by physical reasoning) find the boundary conditions $u(0,t) = P$, $u(L,t) = P$.

20 points

b. Use the method of separation of variables to solve for Eq. (1) the following boundary-value problem

$$u(0,t) = \text{finite},$$
$$u(L,t) = 0. \tag{2}$$

(Use a substitution $z = a\lambda^{\alpha}x^{\beta}$ to reduce the Sturm-Liouville problem to one for the Bessel function $J_0(z)$, and denote the successive zeroes of this function by $\mu_{on} = \mu_n$, the first three roots are 2.4, 5.6, and 8.7; the normalization integral you may need is

$$\int_0^{\infty} [J_0(\mu_n \frac{z}{a})]^2 \, dz = (a^2/2)[J_1(\mu_n)]^2 \quad .$$

10 points c. In terms of the eigenfunctions X_n found in point b) write down the fundamental solution (Green's function) for the Cauchy problem for Eq. (1).

15 points d. Either using the result obtained in c), or directly, solve Eq. (1) with boundary conditions (2), and the following initial conditions:

$$u(x,0) = 0,$$

$$u_t(x,0) = VJ_0\left(2\mu_3 \frac{\sqrt{x}}{\sqrt{L}}\right) \qquad (3)$$

35 points 5. Describe the symmetry group of a molecule consisting of one atom of mass M and three identical atoms of mass m, situated at the vertices of a triangular pyramid the three identical atoms forming an equilateral triangle. What does the symmetry tell you about the possible normal modes of such a molecule?