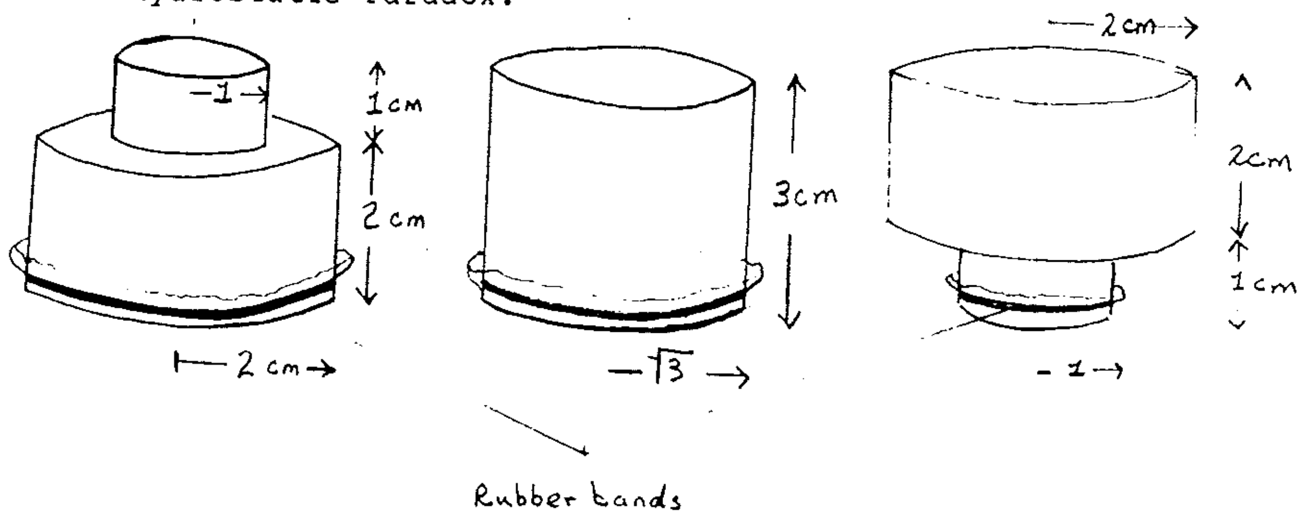
Physics Qualifying Exam

September 30, 1985
2:00 - 5:00 p.m.

CLASSICAL MECHANICS

You are allowed one sheet of notes and a calculator. As the exam probably is too long, you should not get too involved with lengthy algebra on the first pass. Each problem is equally weighted.

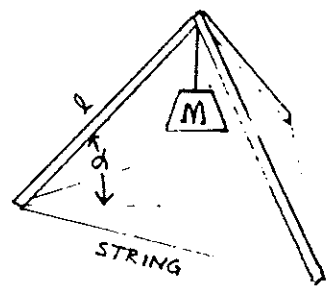
1. Hydrostatic Paradox.



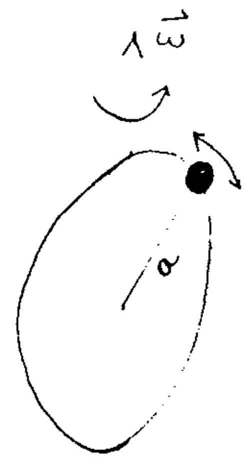
The pressure at any point in an incompressible liquid depends upon the height of liquid above that point. In the three containers shown above, the height and total volume of liquid (water at STP) is the same. The containers are sealed at the bottom by a thin mylar membrane held in place by a rubber band. The rubber band can exert a maximum vertical holding force on the mylar of 0.1 Newton.

a. If each container is picked up by grasping its sides, which will hold the water?

b. All three containers are now set on a table top and refilled if necessary. Obviously the table top must support each container by its reaction force to the liquid's weight. From part (a) you know the force across the membrane. How does this force compare to the weight of the liquid (assume the container itself is massless)? Since the forces do not balance, why do the containers not fall through the table or float above it (as the case may be)? For partial credit, explain in words what is going on. For full credit, calculate and fully explain this (apparent) paradox.



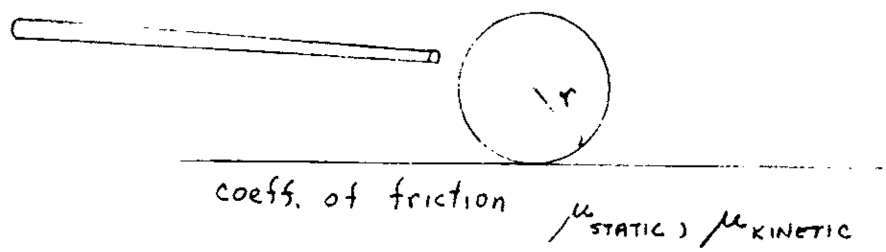
2. You are given the above triangle constructed of three rods of length l and mass m. Their tips are tied together at the vertex by the standard perfectly frictionless pivot point. A mass M is supported from the vertex by a rope. The bases of the rods are supported on the floor in an equilateral triangle configuration by an inflexible string. The angle between the floor and the rod is given by the angle α. By the Principle of Virtual Work (or any other method you may choose), calculate the tension in each string. It would be useful if you would make some sketches showing angles, forces, displacements, etc., before you start making full-blown calculations.



3. A particle of mass m is constrained to a hoop of radius "a". The hoop is rotating with constant angular velocity ω through a vertical diameter.

   a. Find the Lagrangian of this system as well as the conjugate momenta.

   b. Find a first integral of the motion. Can it be identified with any commonly known quantity? Why or why not?



4. a. At what height must the cue strike a cue ball in order that the cue ball begins to immediately roll without slipping?

   b. The cue ball now makes a head-on collision with the eightball. For partial credit, qualitatively (but correctly) describe the resulting motion. For full credit, quantitatively describe the motion assuming the original cue ball velocity was "v_o" and the eightball was at rest (masses are identical).

5. A pion (π, rest mass energy = mc^2 = 140 MeV) decays into a muon (μ, mc^2 = 106 MeV) and a neutrino (ν, mc^2 = 0) with an average lifetime of $26 \cdot 10^{-9}$ seconds.

   a. In the rest frame of the pion, what are the momenta and kinetic energies of the muon and neutrino?

   b. The same pion as seen in the laboratory is found to have a momentum of 500 MeV/c. What will its average lifetime be measured to be in the lab?

   c. What will be the maximum and minimum energies of the muon and neutrino as seen in the lab?

Physics Qualifying Exam

October 2, 1985
9:00 - 12:00 a.m.

THERMODYNAMICS/STATISTICS

Closed Book, one sheet of 8 1/2" x 11" paper

Points

(20) Problem #1 (Thermodynamics, entropy)

1. a) Two isolated bodies of constant and equal heat capacity c initially at temperature T_1 and T_2 ($T_2 > T_1$) are brought into contact so that they assume a common temperature $T_o = \frac{T_1+T_2}{2}$. What is the entropy change ΔS_1, ΔS_2 of each of them and what is the entropy change ΔS_{tot} of the entire system? Show explicitly that $\Delta S_{tot} > 0$.

b) To be specific now take $T_1 = 100K$ $T_2 = 200K$. Calculate again ΔS_1, ΔS_2, ΔS_{tot}.

c) Now body 2 ($T_2 = 200K$) is cooled down to 150K in a different way: First it is brought into contact with an equal body initially at 180K so that it is cooled down to 190K. Then by subsequent contact with bodies initially at 170K, 160K, 150K and finally 140K it reaches 150K. What are the entropy changes now?

d) How can you then cool down body 2 without any (or with minimal) entropy change of the entire system?

Points Problem #2 (Basic Statistics)

(30) 2. a) N independent particles occupy a volume V. Derive the probability P(n) of finding n particles in a given subvolume $V' = p.V$ $(p < 1)$.

b) What is the probability of finding zero, one, two particles in V'? What is the probability of finding at least three particles in V'? (Calculate for $p = \frac{1}{N}$) and take $N \to \infty$.

c) Calcualte the mean particle number $\bar{n}$ in V'.

d) Calculate $\frac{\Delta n}{\bar{n}} = \frac{\sqrt{\overline{(n-\bar{n})^2}}}{\bar{n}}$

$$[(\lim_{m\to\infty} (1-\frac{1}{m})^m = \frac{1}{e}] .$$

Points | Problem #3 (Two-level system)

(30) 3. Consider a system of N distinguishable independent particles which can occupy only two energy levels $-E_o$, $+E_o$ (e.g., a spin $\frac{1}{2}$ in a magnetic field). With N_+, N_- ($N_+ + N_- = N$) denoting the number of particles in the upper and lower energy state, respectively, the "magnetization" is $M = N_- - N_+$. Calculate:

a) the partition function Z.

b) the energy U(T) and the magnetization M(T).

c) the heat capacity C(T).

d) the free energy F(T).

e) the entropy S(T).

f) Make a qualitative sketch of U(T), M(T), S(T), C(T) (for T positive and negative).

g) What does a negative temperature mean? How can you "heat" the system to negative temperatures?

Points Problem #4 (Specific heat of solids)

(20) 4. A crude model of the lattice specific heat of solids (Einstein model) treats the N atoms as independent harmonic oscillators of frequency ω_o.

a) Derive the specific heat for this model.

b) What are the low and high temperature limits of C(T)?

c) Why does the Einstein model give a wrong low temperature behavior?

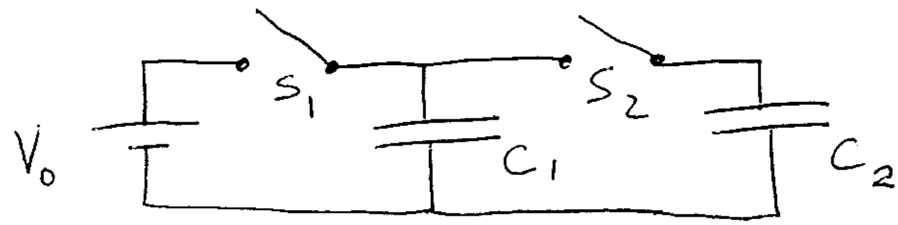
Physics Qualifying Examination

30 September 1985
9:00 - 11:00 a.m.

ELECTRICITY AND MAGNETISM

The exam period is two hours and is to be taken without the use of books or notes. All three problems carry the same point possibility. In the event you cannot complete a problem, outline in as much detail as possible how you would go about solving the problem if more time were available.

May you and the exam have a good impedance match.



1. The capacitors in the figure are uncharged initially. Switch S_1 is closed so that C_1 is charged to a voltage V_0. Switch S_1 is opened and finally switch S_2 is closed so that capacitor C_2 is charged by capacitor C_1.

   a. Find the final voltage across the capacitors.

   b. Determine the total stored energy when only C_1 is charged to V_0. Also determine the total stored energy of the final configuration

   c. Explain why the stored energy was not conserved.

2. Starting from Maxwell's Equations, estimate the skin depth of copper for an incident radio frequency wave at $\omega = 6 \times 10^7 \text{sec}^{-1}$. Use the conductivity $\sigma = 10^{18} \text{sec}^{-1}$. The skin depth, δ, is the distance from the metal surface at which the $\underset{\sim}{E}$ or $\underset{\sim}{H}$ has dropped to 1/e of the magnitude of the field incident on the surface.

3. Consider a star at a distance of 3×10^{21} cm. from the earth. Interstellar neutral atoms contribute to scattering of light. Suppose (for this calculation) the neutral atom density is about $10^{0}\,cm^{-3}$. What fraction of light at $\lambda = 5500$ Å is scattered out of an observer's direct line of sight by the interstellar neutrals? Qualitatively, what significance would dust particles at $5 \times 10^{-13}\,cm^{-3}$ have on scattering of visible light?

Notes to help you:

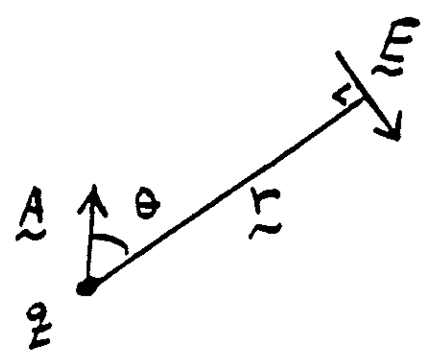
1. Several justifiable approximations and estimates must be made to get the final numbers.

2. The natural frequencies, ω_o, of electrons in neutral atoms are such that $\omega_o \gg \omega$ of starlight. Thus, the scattering cross section is modified from the Thompson cross section.

3. Far field electric field (in MKS) of a charge accelerated as shown in the figure is approximately

$$E(t) = \frac{-qA_{RET}\sin\theta}{4\pi\varepsilon_o c^2 r} \quad \text{MKS}$$

$$= \frac{-qA_{RET}\sin\theta}{c^2 r} \quad \text{CGS}$$



where A_{RET} is the acceleration at the retarded time $t - r/c$.

Table 3

Conversion table for symbols and formulas

The symbols for mass, length, time, force, and other not specifically electromagnetic quantities are unchanged. To convert any equation in Gaussian variables to the corresponding equation in mks quantities, on both sides of the equation replace the relevant symbols listed below under "Gaussian" by the corresponding "mks" symbols listed on the right. The reverse transformation is also allowed. Since the length and time symbols are unchanged, quantities which differ dimensionally from one another only by powers of length and/or time are grouped together where possible.

Quantity	Gaussian	mks
Velocity of light	c	$(\mu_0\epsilon_0)^{-1/2}$
Electric field (potential, voltage)	$\mathbf{E}(\Phi, V)$	$\sqrt{4\pi\epsilon_0}\,\mathbf{E}(\Phi, V)$
Displacement	$\mathbf{D}$	$\sqrt{\frac{4\pi}{\epsilon_0}}\,\mathbf{D}$
Charge density (charge, current density, current, polarization)	$\rho(q, \mathbf{J}, I, \mathbf{P})$	$\frac{1}{\sqrt{4\pi\epsilon_0}}\rho(q, \mathbf{J}, I, \mathbf{P})$
Magnetic induction	$\mathbf{B}$	$\sqrt{\frac{4\pi}{\mu_0}}\,\mathbf{B}$
Magnetic field	$\mathbf{H}$	$\sqrt{4\pi\mu_0}\,\mathbf{H}$
Magnetization	$\mathbf{M}$	$\sqrt{\frac{\mu_0}{4\pi}}\,\mathbf{M}$
Conductivity	σ	$\frac{\sigma}{4\pi\epsilon_0}$
Dielectric constant	ϵ	$\frac{\epsilon}{\epsilon_0}$
Permeability	μ	$\frac{\mu}{\mu_0}$
Resistance (impedance)	$R(Z)$	$4\pi\epsilon_0 R(Z)$
Inductance	L	$4\pi\epsilon_0 L$
Capacitance	C	$\frac{1}{4\pi\epsilon_0}C$

Table 4

Conversion table for given amounts of a physical quantity

The table is arranged so that a given amount of some physical quantity, expressed as so many mks or Gaussian units of that quantity, can be expressed as an equivalent number of units in the other system. Thus the entries in each row stand for the same amount, expressed in different units. All factors of 3 (apart from exponents) should, for accurate work, be replaced by (2.997930 ± 0.000003), arising from the numerical value of the velocity of light. For example, in the row for displacement (D), the entry ($12\pi \times 10^5$) is actually ($2.99793 \times 4\pi \times 10^5$). Where a name for a unit has been agreed on or is in common usage, that name is given. Otherwise, one merely reads so many Gaussian units, or mks units.

Physical Quantity	Symbol	Rationalized mks	Gaussian	
Length	l	1 meter (m)	10^2	centimeters (cm)
Mass	m	1 kilogram (kg)	10^3	grams (gm)
Time	t	1 second (sec)	1	second (sec)
Force	F	1 newton	10^5	dynes
Work	W	1 joule	10^7	ergs
Energy	U	1 joule	10^7	ergs
Power	P	1 watt	10^7	ergs sec^{-1}
Charge	q	1 coulomb (coul)	3×10^9	statcoulombs
Charge density	ρ	1 coul m^{-3}	3×10^3	statcoul cm^{-3}
Current	I	1 ampere (coul sec^{-1})	3×10^9	statamperes
Current density	J	1 amp m^{-2}	3×10^5	statamp cm^{-2}
Electric field	E	1 volt m^{-1}	$\frac{1}{3} \times 10^{-4}$	statvolt cm^{-1}
Potential	Φ, V	1 volt	$\frac{1}{300}$	statvolt
Polarization	P	1 coul m^{-2}	3×10^5	dipole moment cm^{-3}
Displacement	D	1 coul m^{-2}	$12\pi \times 10^5$	statvolt cm^{-1} (statcoul cm^{-2})
Conductivity	σ	1 mho m^{-1}	9×10^9	sec^{-1}
Resistance	R	1 ohm	$\frac{1}{9} \times 10^{-11}$	sec cm^{-1}
Capacitance	C	1 farad	9×10^{11}	cm
Magnetic flux	ϕ, F	1 weber	10^8	gauss cm^2 or maxwells
Magnetic induction	B	1 weber m^{-2}	10^4	gauss
Magnetic field	H	1 ampere-turn m^{-1}	$4\pi \times 10^{-3}$	oersted
Magnetization	M	1 ampere m^{-1}	10^{-3}	magnetic moment cm^{-3}
*Inductance	L	1 henry	$\frac{1}{9} \times 10^{-11}$	

Physics Qualifying Examination

October 1, 1985
2:00 - 5:00 p.m.

QUANTUM MECHANICS

Do as many problems as you can. Credit is indicated.
100 is perfect.

1. a) List all of the absolute symmetries (conservation laws) that you know; list some approximate ones. (5)

   b) Explain why the Schrodinger equation is first order in time rather than second order as in classical wave equations. (5)

   c) Does the density matrix $\rho = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ describe a pure state or a mixed state - explain. (5)

   d) Consider a spin 1 system in a state represented by the column vector $\psi = \frac{1}{\sqrt{26}}\begin{bmatrix} 1 \\ 4 \\ -3 \end{bmatrix}$. What is the probability that the measurement of angular momentum $S_x = \frac{\hbar}{2}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ yields the value zero. (10)

   e) Why is the Zeeman effect usually anomalous? (5)

   f) Why are the triplet states in Helium lower than the corresponding singlet states? (10)

2. An atom with no permanent magnetic moment is diamagnetic. Neglecting the spin of the electron, calculate the induced diamagnetic moment for a hydrogen atom in its ground state when a weak magnetic field B is applied. (20)

3. A static uniform electric field E is applied to a vacuum (strong and weak interactions are neglected.) What is the probability per unit time of producing pairs? Explain why the results of perturbation theory are incorrect. (20)

4. Neutrons can be scattered in the Coulomb field of a nucleus because of their magnetic moment. Write down the Hamiltonian of the interaction and calculate the spin averaged differential cross section in the Born approximation. (20)

5. Show that the source free Maxwell equations may be put into Dirac form through the introduction of Kramer's vector $\underset{\sim}{F} = \underset{\sim}{E} + i \underset{\sim}{H}$ and the definition of suitable matrices S. ie

$$P_\beta \, S_\beta \, \underset{\sim}{F} = \frac{i}{c} \frac{\partial}{\partial t} \underset{\sim}{F}$$

where P_β is a momentum operator. Use this representation to show that the photon has spin 1. (20)

Physics Qualifying Examination

October 1, 1985
9:00 - 11:00 a.m.

MATHEMATICAL PHYSICS

2 Hours. One textbook allowed; no notes.
Do all problems. All problems count equally.

1, Consider the integral equation

$$f(x) = A + \lambda \int_0^1 (x + y) f(y)\,dy$$

where A = a constant.

(a) If $A \neq 0$, find the solution of the equation.

(b) For what value (or values) of λ does part (a) have no solution?

(c) If $A = 0$, does the equation have any nontrivial solutions? If so, state the conditions for such solution(s) to exist, and give the solution(s) explicitly.

2. Evaluate the integral

$$I = \int_0^1 \frac{\ln\left(\frac{1}{x}\right)}{1 - x^2}\,dx$$

Show all work.

3. Find the general solution y(x) of the differential equation

$$y'' + 3y' + 2y = \exp[e^x] \quad .$$

4. Given a function $\phi(\vec{r})$ in 3-dimensions defined inside a sphere of radius a. ϕ satisfies the eigenvalue equation

$$\nabla^2\phi + \lambda r^2\phi = 0$$

subject to the boundary condition $\phi = 0$ on the surface of the sphere. Obtain an upper limit on the lowest eigenvalue λ_o.

5. An infinite slab of material fills the region $-\frac{L}{2} < x < \frac{L}{2}$. The temperature distribution $T(x,t)$ is governed by the diffusion equation

$$\nabla^2 T = \frac{1}{K}\frac{\partial T}{\partial t} .$$

The entire slab is originally at temperature $T = 0$. At time $t = 0$ a pulse of energy is emitted along the central plane ($x = 0$) which gives rise to an initial temperature distribution corresponding to a delta function: $T(x,o) = \beta\delta(x)$. If the boundary planes $x = \pm\frac{L}{2}$ are maintained at $T = 0$, find an expression for $T(x,t)$ throughout the slab for all $t > 0$.

6. Two independent random variables x_1, x_2 are defined over the range $(-\infty, +\infty)$ and have probability distributions given by

$$P(x_i) = \frac{\gamma_i/2\pi}{(x_i - a_i)^2 + \frac{\gamma_i^2}{4}} \qquad i = 1,2 .$$

Let $x \equiv x_1 + x_2$ define another random variable. What is the probability distribution of x?