Physics Qualifying Exam UCI May 15, 1972
9:00 - 11:00 a.m.

MECHANICS

Do 4 of the 6 Problems (2 hours)

1. Consider a simple pendulum consisting of a rigid massless rod of length ℓ hanging from a fixed point. At the end of the rod is a point mass M. Write the Hamiltonian and Hamilton's equations of motion. Solve in the limit of small oscillation.

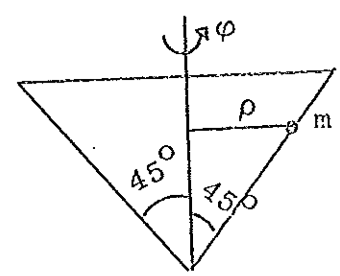
   Now suppose the point of support, which has mass M', oscillates horizontally with frequency ω_o. Write the Hamiltonian.
   Is it constant in time? Why?
   Is it equal to the total energy of the system? Why?

2. A point mass m moves on the inner surface of a right circular cone under the influence of gravity. The cone has a half-angle of 45^o.

   a) Using general coordinates ρ (radial distance from axis) and θ (angular position), derive the equations of motion.

   b) For what value of ρ will uniform circular motion be possible?

   c) Let ρ equal the value ρ_o for which circular motion is possible. Displace ρ slightly, so that $\rho = \rho_o(1+\Delta)$, where $\Delta \ll 1$. Find the frequency of oscillation of ρ about ρ_o.



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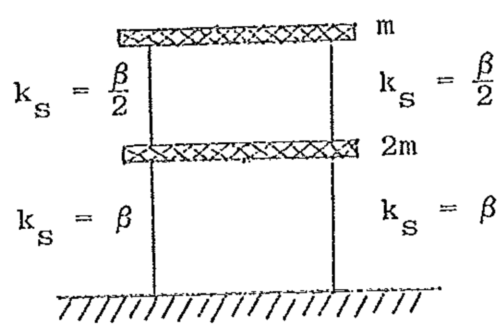
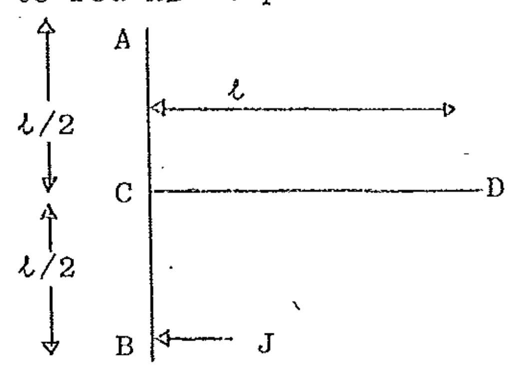


An approximate dynamical model of a 2-story steel-frame building can be constructed from two concentrated masses and two pairs of massless "shear springs" as shown in the figure. During lateral vibration of the building due to an earthquake, it is assumed that the floors move only parallel to the earth, so that the motion is essentially a shear. If the shear spring constant is k_s, then the potential energy of a spring is $\frac{1}{2}k_s(\Delta\ell)^2$, where $\Delta\ell$ is the net horizontal displacement of the two ends of the shear spring. Find the frequencies and amplitude ratios for the two normal modes.

4. Two identical thin uniform rods, each of mass m and length ℓ, are rigidly joined to form a symmetric T.

   The system is initially at rest on a smooth horizontal table. An impulse $\vec{J}$ is applied perpendicular to rod AB at point B. Just after the impulse has struck,

   a) What is the velocity of the center of the mass?
   b) What is the total kinetic energy of the system, in terms of J?
   c) What is the instantaneous axis of rotation of the system?



5. A uniform sphere of mass m, radius b, and $I_{cm} = 2/5\ mb^2$ is initially set spinning with an angular velocity ω_o about a horizontal axis. It is initially in a position infinitesimally above the surface of a horizontal table and it has no translational motion. The spinning sphere is then allowed to come into contact with the table. There is a gravitational acceleration g and a coefficient of friction μ between sphere and table.

   a) How much time will elapse before the sphere is rolling without slip?
   b) What is the angular velocity after the sphere is rolling without slip?
   c) What fraction of its initial kinetic energy was lost while slipping? Where did it go?

6. Consider a rocket to be used for space flight. Let m_o denote the mass of the rocket without fuel and m_f denote the mass of the unburned fuel, giving the rocket a total mass $M \equiv m_o + m_f^o$ initially. Let v_e be the velocity of the exhaust gases relative to the rocket and let $\frac{dm_f}{dt}$ be the (constant) rate of fuel combustion (defined as a positive quantity). Neglect air friction, gravity, and all other external forces.

a) Find the thrust (propulsive force) on the rocket and its maximum speed. Assume we have a chemical rocket, with $v_e \approx 10^4$ m/sec.

b) Now consider photon propulsion, with the advantage of having $v_e \approx c$. We want to give our space travelers the advantage of a large time dilation, so let us choose

$$\gamma \equiv (1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$$

equal to 10. Here v is the rocket final speed.

c) Of course, we want our travelers to land safely and return to earth, so actually we will need 3 more such stages for the total of two accelerations and two decelerations. What is the ratio of the mass of the returning rocket remnant to the original total mass of rocket and fuel leaving earth?

Physics Qualifying Exam | UCI | May 15, 1972
11:15 - 1:15 p.m.

ELECTRICITY AND MAGNETISM

Do 4 Problems

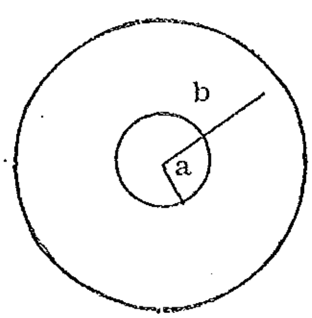
1. A parallel-plate capacitor is charged at a constant rate I (coulombs/sec). The capacitor has circular plates of radius a and a separation h. Neglect edge effects.
   a. Find the total energy (electric plus magnetic) as a function of time. Include only that portion of the magnetic energy which is inside the capacitor volume.
   b. Find the Poynting vector at a distance r from the center of the capacitor
   c. Find the rate of change of energy inside r.
   d. Does the result of b agree with c?
   e. If I is not constant in time, is the answer to d the same? Explain.

2. Find a surface charge distribution on the spherical surface r = a which will produce the potential

$$V = A(x^2 - y^2) + Bx$$

in the region $r < a$. What is the potential in the region $r > a$? [There is no charge anywhere except on the surface $r = a$.] Hint: express V in spherical coordinates, and then in spherical harmonics.

3. Consider a conducting disk of radius b. The conductivity is σ. If the disk has an initial charge density distribution at

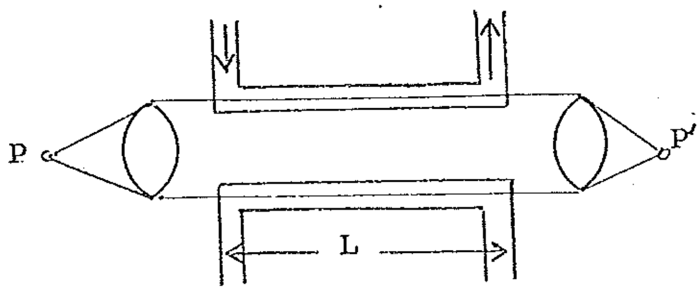


$$t = 0 \text{ of } \rho = \rho_0 \qquad r < a$$
$$= 0 \qquad r > a$$

find $\rho(r,t)$ at all other times.

[Hint: Use the continuity equation and $J = \sigma E$.]

4. The core (radius a) of a coaxial cylindrical cable is surrounded by an insulating sheath of conductivity σ_1, outer radius b, and a second layer of conductivity σ_2 extending to the outer metallic conductor of radius c. Find the resistance per meter of cable between the core and the outer conductor.

5. 

Light from a source of frequency f is led through the system shown. If the upper conduit carries a liquid having index of refraction n moving with velocity u, and the lower conduit carries the same liquid at rest, what is the minimum value of u that will cause destructive interference at P'.

6. The ionosphere can be considered as an ionized medium containing N essentially free electrons per unit volume. Show that if a linearly polarized wave propagates in the ionosphere in a direction parallel to that of the small uniform magnetic field H produced by the earth, its plane of polarization will be rotated through an angle proportional to the distance traveled by the wave. Calculate the constant of proportionality. [Hint: calculate the effective index of refraction for right and left circularly polarized light propagating parallel to H. Express the linear polarized light as the sum of right and left circularly polarized light. Express your final answer in terms of $\delta n = n_+ - n_-$ where $n_\pm$ are the two indices of refraction. You need not explicitly evaluate δn.]

Physics Qualifying Exam UCI May 15, 1972
2:45 - 5:45 p.m.

QUANTUM MECHANICS (3 hours)

Open Book.

Do any 5 problems.

1. Two identical spin $\frac{1}{2}$ particles of mass m are confined by a one dimensional potential well of length L. ($V(x) = 0$ for $0 < x < L$ and infinite elsewhere). There is a mutual interaction between the particles of the form $V_{12}(x_1,x_2) = \lambda\delta(x_1-x_2)$. To first order in λ, find the energies of the lowest lying singlet and triplet states.

2. A particle with intrinsic spin $S = \frac{1}{2}$ is rotating on a rigid rod. The Hamiltonian of the system is

$$H = \frac{\hbar^2}{2I} L^2 + A\bar{L}\cdot\bar{S}$$

$\bar{L}$, $\bar{S}$ are the orbital and spin angular momentum operators respectively. A and I are constants.

a) What are the energy levels of H?

b) At time $t = 0$ the state of the system is described by the ket vector $|\ell, m_\ell; \frac{1}{2}, m_s\rangle$ with $\ell = 1$, $m_\ell = 1$, and $m_s = -\frac{1}{2}$. What is the probability of finding the system in the same state at a later time t.?

3. The lowest four eigenstates $|i\rangle$, $i = 1,2,3,4$ of a certain Hamiltonian H_o have energies (in unspecified units) of -300, -200, -200 and -100 respectively. A perturbation H_1 is added to H_o. The matrix elements of H_1 between the above four states are (in the same unspecified units).

j \ i	1	2	3	4
1	1	1	3	2
2	1	2	$-\sqrt{6}$	0
3	3	$-\sqrt{6}$	1	4
4	2	0	4	0

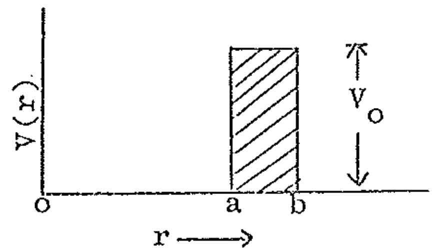
Physics Qualifying Exam UCI May 16, 1972
9:00 - 10:00 a.m.

QUANTUM MECHANICS (1 hour)

Closed Book.

Do 2 Problems.

1. A particle of mass m and angular momentum $\ell = 0$ is trapped in the following potential:



The energy of the particle is $E_o \ll V_o$.

a) Estimate the lifetime of this state.

b) Now assume the particle is incident on this potential from the outside with energy E_o. Estimate the differential cross section.

2. A certain molecule is made out of <u>two</u> atoms of mass M each. The potential acting between the atoms is

$$V(r) = V_o\left(\left(\frac{r_o}{r}\right)^{12} - 2\left(\frac{r_o}{r}\right)^{6}\right).$$

with V_o and r_o given. Discuss quantitatively the low lying spectrum of this molecule.

3. A proton of mass M and kinetic energy E is captured by a heavy spin zero nucleus, Z, leading to the ground state of nucleus (Z + 1) and photon, γ, of energy $E - \Delta$. The spin of the nucleus (Z + 1) is $\frac{1}{2}$. The total cross section for this process is $\sigma(E)$. In terms of $\sigma(E)$ what is the cross section for the reaction

$$\gamma + (Z + 1) \longrightarrow (Z) + p$$

at an incident energy E'. You may treat the motion of the proton nonrelativistically and ignore any recoil of the heavy nuclei.

The matrix elements of H_1 connecting these four states to all higher states are negligible.

To lowest nonvanishing order find the corrections to the energies of these states.

4. The Hamiltonian of a free particle in one dimension is

$$H = P_{op}^2/2m$$

Let $P_{op}(t)$ and $X_{op}(t)$ be the Heisenberg picture operators for the momentum and position at time t. Evaluate

a) $[P_{op}(t), X_{op}(t')]$
b) $[X_{op}(t), X_{op}(t')]$

5. A particle of mass m and charge e is in the ($n_x = 0$, $n_y = 0$, $n_z = 1$) state of an isotropic harmonic oscillator of natural frequency ω.

a) What is the probability of the particle decaying into the ground state and emitting a photon in the direction θ,φ ?
b) What is the lifetime of this state?

You may assume that $\frac{\hbar\omega}{mc^2} << 1$. Explain why this is a useful assumption for the above problem.

6. The motion of a Dirac particle in a certain potential is governed by the Hamiltonian

$$H = c\ \bar{\alpha}\cdot\bar{p} + \beta mc^2 + \beta U(\bar{r})$$

with $$\bar{\alpha} = \begin{pmatrix} 0 & \bar{\sigma} \\ \bar{\sigma} & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

(1 is the 2 x 2 unit matrix and the σ's are the usual Pauli matrices). N. B. There is a β in front of U(r).

a) Obtain the nonrelativistic limit of the above Hamiltonian.
b) Do the same for the Hamiltonian

$$H = c\bar{\alpha}\cdot\bar{p} + \beta mc^2 + \gamma_5 U(r)$$

where $$\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

7. A proton of mass M is bound in some fixed potential with binding energy E_B. The wave function of this bound state is $\psi(r)$. A fast electron (mass μ) with a large momentum k_i ($\hbar^2 k_i^2/2\mu >> E_B$) is incident on this system. You may assume that the electron interacts only with the proton.

What is the cross section for knocking out the proton in direction $\hat{k}_p$ and for the electron to emerge with momentum $\vec{k}_f$? Express your answer in terms of the Fourier transform of $\psi(\bar{r})$

$$\varphi(\bar{q}) = \frac{1}{(2\pi)^{3/2}} \int e^{i\bar{q}\cdot\bar{r}}\psi(\bar{r})\,d\bar{r} \quad .$$

Physics Qualifying Exam UCI May 16, 1972
4:00 - 6:00 p.m.

GENERAL

Answer any 10 questions

1. We all say that the photon has zero rest mass. Experimentally how well is the rest mass known? Describe an experiment to measure the photon rest mass.

2. What is a parton? A quark?

3. I measure the resistivities of potassium and silicon as functions of temperature near $300^{\circ}K$. Why do the two $\rho(T)$ curves have slopes of opposite sign? For each material what is the functional form of $\rho(T)$ near $0^{\circ}K$?

4. While floating near the moon in your space suit, you encounter an extraterrestial being also in a space suit. You hold out your right hand to shake and he holds out his left. Why do you not shake hands with him? Describe how you might communicate with him, using only unpolarized electromagnetic radiation, and learn if it is safe to shake hands.

5. The concentration of CO_2 in the earth's atmosphere is increasing at a rate of almost one part per million per year. The resulting direct effect on the earth's energy balance is called the greenhouse effect. Describe it.

6. A stick of length L_1, measured in its rest frame, slides along an ice surface with velocity v (very close to c) in the direction of its length. It approaches a hole in the ice a distance L_2 across, in the frame of the ice, where L_2 is much less than L_1. However, the stick is moving so fast that

$$L_2 \gg L_1 \sqrt{1 - \frac{v^2}{c^2}} ,$$

and consequently the stick falls in. How would an observer on the stick describe this?

7. Describe the transistor and briefly state its principles of operation.

8. You receive an article in which the author proposes to build an EASER (electron amplification by stimulated emission of radiation), that is, an electron analog of the laser. Why do you immediately throw the article away?

9. Relate Cherenkov radiation and the mechanism causing sonic boom in aircraft. Could Cherenkov radiation be seen with gravity waves?

10. In the Fizeau experiment light is propagated through a moving medium, such as water flowing through a pipe. What is the speed of propagation measured in the laboratory and what does this result require of the "ether"?

11. Describe how the Hanbury Brown-Twiss intensity interferometer (R. Hanbury Brown and R. Q. Twiss, 1956) can be used to measure stellar diameters. How does it differ from the Michelson stellar interferometer?

12. Write a sentence or two defining each of 3 of the following:
    a) Roton
    b) Nematic liquid crystal
    c) Strangeness
    d) Dilution refrigerator
    e) Neutrino
    f) Tardyon.

May 16, 1972
10:15-12:15 AM

Physics Qualifying Exam UCI

THERMODYNAMICS AND STATISTICAL MECHANICS

Open book.

Answer 4 questions.

1. A gas satisfies the van der Waals equation of state

$$(p + a/V^2)\ (V - b) = RT \quad .$$

   a. Find its entropy and internal energy in terms of C_v.

   b. Using an expansion in $1/V$ compute $C_p - C_v$ up to terms of order $1/V^3$.

2. The energy difference between the ^1S_o and ^3S_o states of He is 159,843 cm^{-1}. Taking into account the angular-momentum degeneracies evaluate the fraction of excited (triplet) atoms in Helium gas at 6000^oK.
[Constants: $k = 1.38 \times 10^{-16}$ erg/^oK = 8.6×10^{-5} eV/^oK; 1 eV corresponds to a wave number 8066 cm^{-1}]

3. Show that the chemical potential of a photon gas is zero.

4. An electron in a magnetic field $\vec{H}$ has an energy $\pm\ \mu_B H$, depending on the spin orientation relative to H.

   Calculate the spin paramagnetic susceptibility of a completely degenerate Fermi gas at absolute zero.

5. The canonical partition function for a certain system of N particles at a temperature T and volume V is

$$Q_N(V,T) = \frac{1}{N!}\left(\frac{V}{\lambda^3}\right)^N \left\{1 + 2^N e^{-V/\lambda^3}\right\}$$

$$\lambda = \sqrt{(2\pi\hbar^2)/(mkT)}$$

   Using either the canonical or grandcanonical partition function (grandcanonical much, much easier), find the equation of state of this system. Sketch a P-V isotherm and discuss its salient features.

6. Find the density distribution of a gas in a cylinder of radius R and length ℓ rotating around its axis with angular velocity ω, assuming that the total number of particles is N.

May 16, 1972
1:45 - 3:45 PM

Physics Qualifying Exam UCI

MATHEMATICAL PHYSICS

Closed Book
Do 5 Problems

1. a. Show that the "harmonic oscillator" wave-functions

$$\Psi_n(x) = (-1)^n (2^n n! \pi^{\frac{1}{2}})^{-1/2} e^{x^2/2} \frac{d^n}{dx^n} e^{-x^2}$$

are, up to a constant, their own Fourier transforms.

b. What is the meaning of the relation:

$$\sum_{n=0}^{\infty} \Psi_n(x)\Psi_n(y) = \delta(x-y) \quad ?$$

c. Assume that any function f, such that $\int_{-\infty}^{\infty} |f(x)|^2 dx < \infty$,

can be expanded into a series with respect to the Ψ_n.
$f(x) = \sum_{n=0}^{\infty} C_n \Psi_n(x)$, such that

$$\lim_{N\to\infty} \int_{-\infty}^{\infty} |f(x) - \sum_{n=0}^{N} C_n \Psi_n(x)|^2 dx = 0 \quad .$$

Compute the Fourier transform

$$\tilde{f}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-ikx} dx$$

in terms of the $\{C_n\}$ and show that

$$\int_{-\infty}^{\infty} |f(x)|^2 dx = \int_{-\infty}^{\infty} |\tilde{f}(k)|^2 dk$$

2. Use the method of steepest descent to derive Stirling's formula

$$\Gamma(\lambda+1) = \sqrt{2\pi\lambda}\,(\lambda/e)^{\lambda}\{1+O(1/\lambda)\}$$

with $\Gamma(\lambda+1) = \int_0^{\infty} x^{\lambda} e^{-x} dx$

3. Show that

$$\int_0^{2\pi} \frac{dt}{(a+b\ \cos t)^2} = \frac{2\pi a}{(a^2-b^2)^{3/2}} \qquad (a>b>o)$$

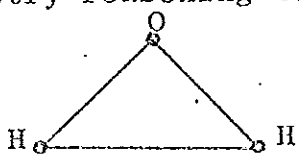
4. Use Fermat's principle to show that light-rays starting from the origin in a (two-dimensional) medium with index of refraction

$$\frac{v}{c} = \frac{1}{\sqrt{A - y}} \qquad (A = \text{constant})$$

are parabolas.

What is the mechanical analog of this problem?

5. Find the normal modes of vibration of an H_2O molecule (see Figure). You may use either group - theoretical methods (the character table of C_{2v} is given below) or qualitative symmetry reasoning to determine the three modes.



C_{2v}:

E	C_2	σ_v	σ_v'
3	1	3	1

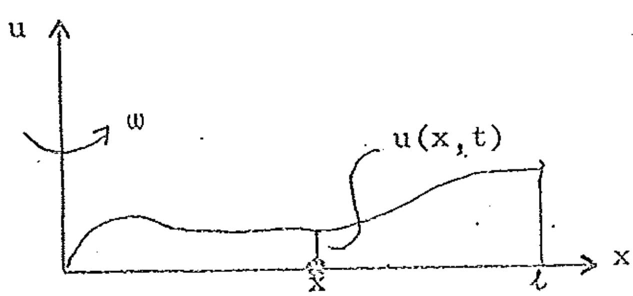
(Here C_2 leaves O in place, σ_v is the reflection leaving all atoms in place, σ_v' leaves only O in place.)

6. Discuss the conformal mapping

$$z = \frac{h}{\pi}\left(e^{\frac{\pi}{V} w} + \frac{\pi}{V} w\right)$$

with h and V constants. What electrostatic interpretation can you attribute to the h, V, Im w ?

7. Study the small vertical vibrations of a homogeneous, perfectly flexible string of length ℓ, fixed at one end, free at the other and rotating around the fixed end in a horizontal plane with fixed angular velocity ω.



(Continued)

7. (Continued)

a. Using the notation in the figure and assuming that the centrifugal force exerted at the point x, which is equal to $m\omega r^2$, with $m = \rho(\ell - x)$, $r = \frac{\ell + x}{2}$, is the only restoring force, show that the equation of motion is

$$\frac{\partial^2 u}{\partial t^2} = \frac{\omega^2}{2} \cdot \frac{\partial}{\partial x} \left[(\ell^2 - x^2) \frac{\partial u}{\partial x} \right] \qquad (1)$$

with boundary conditions

$$u(0,t) = 0 \ , \ u(\ell,t) = \text{finite} \qquad (2)$$

b. Find the normal modes of vibration.

c. Find the motion of the string if the initial conditions are

$$u(x,0) = \alpha x$$

$$\frac{\partial u(x,0)}{\partial t} = 0 \ .$$

Necessary formulas:

$$P_n(\xi) = \frac{1}{2^n n!} \frac{d^n(\xi^2 - 1)}{d\xi^n}$$ are solutions of the

equation

$$\frac{d}{d\xi}\left[(1-\xi^2)\frac{dy}{d\xi}\right] + n(n+1)y = 0$$

with boundary conditions:

$$y(\pm 1) = \text{finite} \ .$$

$$\int_{-1}^{+1} P_n(\xi)\, P_m(\xi)\, d\xi = \frac{2}{2n+1}\, \delta_{nm}$$

$$P_{2n-1}(0) = 0 \quad P_{2n}(0) = (-1)^n \frac{(2n-1)!!}{(2n)!!}$$

8. Use Rayleigh-Schrödinger perturbation theory to determine the change in the fundamental frequency of a rectangular membrane $0 \le x \le a$, $0 \le y \le b$, produced by a small nonuniformity of the mass-density of the form

$$\rho(x,y) = \begin{cases} \rho_o + \epsilon & 0 \le x < \frac{a}{2} \\ \rho_o & \frac{a}{2} < x \le a \end{cases} \ , \ 0 \le y \le b$$

ρ_o = constant

8. (Continued)

[Hint: The normal modes satisfy Helmholtz' equation

$$\nabla^2 u + \omega^2 u = 0 \quad \text{with} \quad \omega^2 = \frac{k}{\rho}$$

where k is the tension. Use the expansion

$$\frac{k}{\rho_o + \epsilon} = \frac{\omega^2}{1 + \frac{\epsilon}{\rho_o}} = \omega^2 \left(1 - \frac{\epsilon}{\rho_o} \right)$$

assuming $\frac{\epsilon}{\rho_o} << 1$, before applying perturbation theory].