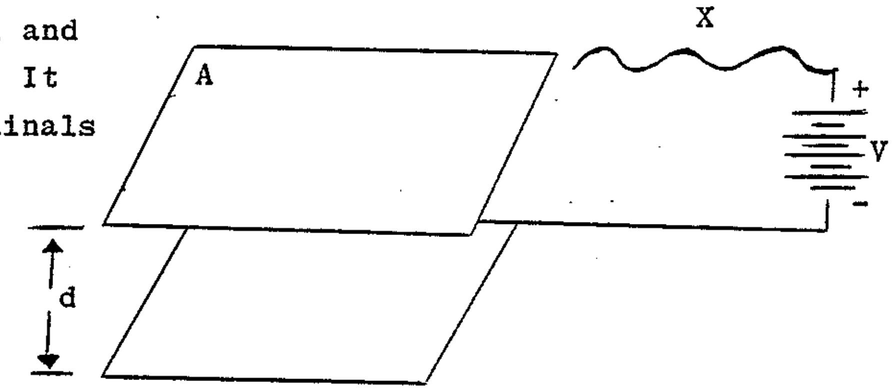
UCI PHYSICS QUALIFYING EXAMINATION - SPRING 1978

PART I - Electricity and Magnetism

- Time: 0900 - 1200 = 3 hours, May 22
- Do 4 of the 6 problems
- The 4 problems will count equally
- Two reference books permitted
- Use the supplied bluebooks for all work you wish to have graded. Identify the book with your number only. Label problems clearly. You may use scratch paper separately or in a denoted section of the bluebook(s).

Part I - Electricity and Magnetism

1. A gas-filled parallel plate ionization chamber has plates of area A separated by d and a capacitance $C = A\epsilon_o/d$. It is connected to the terminals of a battery as shown.



(a) What charge and energy are taken from the battery in the charging process? What charge and energy are stored in the capacitor? What is the electrostatic force on one plate?

(b) An ultra-violet photon creates an electron-positive ion pair midway between the plates at t = 0. Both the electron and positive ion undergo frequent gas atom collisions and so move parallel to the electric field with small constant drift velocities, $|v_e| \approx 1000 |v_+|$. An instrument connected in series at X measures charge $q(t) = \int_o^t i(t)dt$. Find $q_e(t)$ and $q_+(t)$ that result from the motion of the electron and positive ion. Show results on a labelled sketch. Do not include image charges or space charge effects in your analysis.

2. In general, when one produces a beam of ions or electrons, the space charge within the beam causes a potential difference between the axis and the surface of the beam. A 10 m A beam of 50 keV protons ($v = 3 \times 10^6$ m/sec) travels along the axis of an evacuated beam pipe. The beam has a circular cross section of 1 cm diameter. Calculate the potential difference between the axis and the surface of the beam, assuming that the current density is uniform over the beam diameter.

3. Show that a charge, acted upon by static electric forces only, cannot be in stable equilibrium (this is called "Earnshaw's theorem"). Using this result, show that the maximum value of the absolute value of a function of a complex variable which is regular in a closed region always lies on the boundary of that region (this is called "the maximum modulus principle").

4. An infinite sheet of current, located in the xz-plane, has a current density J (amperes/m) given by

$$J = J_z = J_o\, e^{i\omega t}$$

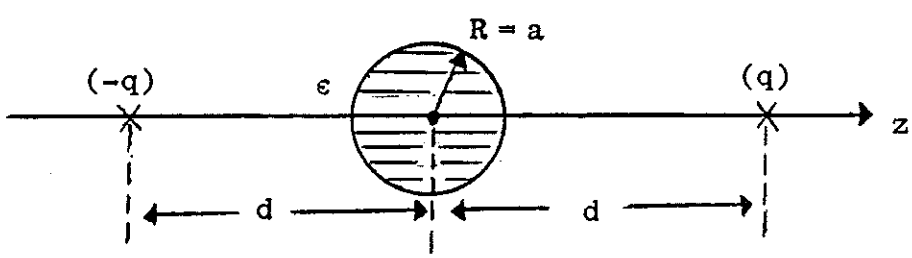
where J_o is constant over the x z plane.

(a) Use symmetry arguments to determine the directions of the electric field $\vec{E}$ and the magnetic field $\vec{B}$.

(b) Obtain $\vec{B}$ from the integral form of ampere's law, showing that the displacement-current term does not contribute. What is the wavelength of $\vec{B}(\vec{r},t)$?

(c) What is the power per unit area radiated from one side of the current sheet?

5. Consider two point charges of equal magnitude and opposite sign located at distances z = ±d from a dielectric sphere of radius a (dielectric constant ε) positioned at the origin



5. (continued)

(a) If $\varepsilon = 1$ find an expression for the potential everywhere in space (express potential in terms of Legendre polynomials, check value at the origin).

(b) If $\varepsilon > 1$, find expressions for the form of the potential inside and outside the dielectric sphere.

(c) Use appropriate boundary conditions to find the potential everywhere inside the sphere.

(d) Find the induced surface charge density, and verify that the total induced surface charge is zero.

6. Find the convection and displacement currents and the resulting magnetic field everywhere in space due to a slowly moving, uniformly dense, spherical cloud of charge. Let the cloud have a radius a, total charge e, and let it move along +z axis with velocity $v \ll c$.

UCI PHYSICS QUALIFYING EXAMINATION - SPRING 1978

PART II - Quantum Mechanics I (Quantitative)

- Time: 1300 - 1730 = 4.5 hours, May 22
- Do all 6 problems
- The 6 problems will count equally
- Two reference books permitted
- Use the supplied bluebooks for all work you wish to have graded. Identify the book with your number only. Label problems clearly. You may use scratch paper separately or in a denoted section of the bluebook(s).

PART II - Quantum Mechanics I (Quantitative)

1. a) Consider a typical set of 2j + 1 angular momentum eigenstates $|jm\rangle$; as usual

$$\vec{J}^2|jm\rangle = j(j+1)|jm\rangle$$

$$J_z = |jm\rangle = m|jm\rangle$$

($\vec{J}$ = angular momentum, and we set $\hbar = 1$)

Define the operator $T = \sum_{ijk} \epsilon_{ijk} J_i J_j J_k = \vec{J} \cdot (\vec{J} \times \vec{J})$.

Are the states $|jm\rangle$ eigenstates of T? If so, give the eigenvalues.

If not, construct from the $|jm\rangle$ at least one eigenstate of T.

b) The Pauli spin matrices are a two-dimensional representation of the angular momentum operators J_x, J_y, J_z. Give the analogous three-dimensional representation of these operators.

2. Let the magnetic moment operator of a particle of mass m be $\vec{\mu} = -g \frac{e}{2mc} \vec{J}$ where $\vec{J}$ is its spin operator. The polarization $\vec{\sigma}$ of the particle is the expectation value of its spin, and let $\vec{p}$ be the expectation value of its momentum. If the particle is acted upon by a uniform magnetic field $\vec{B}$ perpendicular to $\vec{p}$ and $\vec{\sigma}$, and $\vec{p}$ is turned thru and angle θ_p, find the angle θ_σ thru which $\vec{\sigma}$ is turned.

3. A particle of mass m is constrained to move on the surface of the sphere $r = a$, but is otherwise free.

a) What is the energy difference between the ground state and the first excited state of this system?

Gravity is now turned on, i.e. the potential energy $V = mgz = mga\cos\theta$ is added to the Hamiltonian.

b) The pattern of energy levels is completely different in the "weak gravity" limit $g \ll g_o$ and the "strong gravity" limit $g \gg g_o$. Evaluate the critical value g_o, omitting dimensionless constants of order unity.

c) What is the energy difference between the ground state and the first excited state in the "strong gravity" limit?

4. a) A particle of mass m moves in the Yukawa potential $V(r) = -\lambda \frac{e^{-\mu r}}{r}$. Use the variational method, with a trial wave function $\Psi \sim e^{-\alpha r}$, to estimate the critical value λ_o of the parameter λ; λ_o is defined by

$$\lambda < \lambda_o \rightarrow \text{no bound states present}$$

$$\lambda > \lambda_o \rightarrow \text{at least one bound state present}$$

b) Suggest simple trial functions to represent the 1p and 2s states and indicate briefly the reasoning behind your choice.

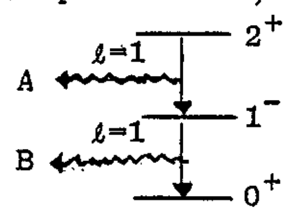
5. The electrostatic potential of a neutral atom of atomic number Z is to a good approximation a screened Coulomb potential

$$V(r) = \frac{Ze}{r} e^{-(r/a)}$$

where the exponential represents the shielding of the nuclear charge by the atomic electrons. The shielding length a, according to the Thomas-Fermi theory of the atom, is approximately $a_o/Z^{1/3}$, where a_o is the hydrogen Bohr radius.

Find the total scattering cross-section, using the Born approximation, for non-relativistic electrons scattering from such an atom.

6. A nucleus is initially in an unpolarized 2^+ state. It decays to a 1^- state by emitting a p-wave spinless particle A, and this state immediately decays to the 0^+ ground state by emitting another p-wave spinless particle B, as sketched below.



Note that a table of Clebsch-Gordan coefficients is given on the next page.

a) If the axis of polar coordinates is chosen along the direction of emission of particle A, what is the probability that the intermediate 1^- state has m $(=J_z/h)$ equal to +1? to zero? to -1?

b) What is the angular distribution $W(\theta)$ (i.e., the so-called angular correlation) of particle B referred to this same coordinate system?

c) Explain briefly the significance of the requirement in part a) that the 1^- state decay "immediately". For example, what is likely to happen in practical cases, if the intermediate state has too long a lifetime?

CLEBSCH-GORDAN COEFFICIENTS AND SPHERICAL HARMONICS

Note: A √ is to be understood over every coefficient; e.g., for -8/15 read $-\sqrt{8/15}$.

Notation:

		J	J	...
		M	M	...
m_1	m_2	Coefficients		
m_1	m_2			
.	.			
.	.			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

1/2 × 1/2

		1
		+1
+1/2	+1/2	1

		1	0
		0	0
+1/2	-1/2	1/2	1/2
-1/2	+1/2	1/2	-1/2

		1
		-1
-1/2	-1/2	1

1 × 1/2

		3/2
		+3/2
+1	+1/2	1

		3/2	1/2
		+1/2	+1/2
+1	-1/2	1/3	2/3
0	+1/2	2/3	-1/3

		3/2	1/2
		-1/2	-1/2
0	-1/2	2/3	1/3
-1	+1/2	1/3	-2/3

		3/2
		-3/2
-1	-1/2	1

1 × 1

		2
		+2
+1	+1	1

		2	1
		+1	+1
+1	0	1/2	1/2
0	+1	1/2	-1/2

		2	1	0
		0	0	0
+1	-1	1/6	1/2	1/3
0	0	2/3	0	-1/3
-1	+1	1/6	-1/2	1/3

		2	1
		-1	-1
0	-1	1/2	1/2
-1	0	1/2	-1/2

		2
		-2
-1	-1	1

2 × 1

		3
		+3
+2	+1	1

		3	2
		+2	+2
+2	0	1/3	2/3
+1	+1	2/3	-1/3

		3	2	1
		+1	+1	+1
+2	-1	1/15	1/3	3/5
+1	0	8/15	1/6	-3/10
0	+1	6/15	-1/2	1/10

		3	2	1
		0	0	0
+1	-1	1/5	1/2	3/10
0	0	3/5	0	-2/5
-1	+1	1/5	-1/2	3/10

		3	2	1
		-1	-1	-1
0	-1	6/15	1/2	1/10
-1	0	8/15	-1/6	-3/10
-2	+1	1/15	-1/3	3/5

		3	2
		-2	-2
-1	-1	2/3	1/3
-2	0	1/3	-2/3

		3
		-3
-2	-1	1

2 × 1/2

		5/2
		+5/2
+2	1/2	1

		5/2	3/2
		3/2	+3/2
+2	-1/2	1/5	4/5
+1	+1/2	4/5	-1/5

		5/2	3/2
		+1/2	+1/2
+1	-1/2	2/5	3/5
0	+1/2	3/5	-2/5

		5/2	3/2
		-1/2	-1/2
0	-1/2	3/5	2/5
-1	+1/2	2/5	-3/5

		5/2	3/2
		-3/2	-3/2
-1	-1/2	4/5	1/5
-2	+1/2	1/5	-4/5

		5/2
		-5/2
-2	-1/2	1

3/2 × 1/2

		2
		+2
+3/2	+1/2	1

		2	1
		+1	+1
+3/2	-1/2	1/4	3/4
+1/2	+1/2	3/4	-1/4

		2	1
		0	0
+1/2	-1/2	1/2	1/2
-1/2	+1/2	1/2	-1/2

		2	1
		-1	-1
-1/2	-1/2	3/4	1/4
-3/2	+1/2	1/4	-3/4

		2
		-2
-3/2	-1/2	1

3/2 × 1

		5/2
		+5/2
+3/2	+1	1

		5/2	3/2
		+3/2	+3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	-2/5

		5/2	3/2	1/2
		+1/2	+1/2	+1/2
+3/2	-1	1/10	2/5	1/2
+1/2	0	3/5	1/15	-1/3
-1/2	+1	3/10	-8/15	1/6

		5/2	3/2	1/2
		-1/2	-1/2	-1/2
+1/2	-1	3/10	8/15	1/6
-1/2	0	3/5	-1/15	-1/3
-3/2	+1	1/10	-2/5	1/2

		5/2	3/2
		-3/2	-3/2
-1/2	-1	3/5	2/5
-3/2	0	2/5	-3/5

		5/2
		-5/2
-3/2	-1	1

UCI PHYSICS QUALIFYING EXAMINATION - SPRING 1978

PART III - Mathematical Physics

- Time: 9:00 - 12:00 (3 hours), May 23
- Do 4 of the 5 problems
- The problems will count equally
- Two reference books permitted
- Use the supplied bluebooks for all work you wish to have graded. Identify the book with your number only. Label problems clearly. You may use scratch paper separately or in a denoted section of the bluebook(s).

PART III - Mathematical Physics

1. a) Consider the ordinary differential equation:

$$xy'' - (1 + x)y' + 2(1 - x)y = 0$$

i) Find the first three terms in a power series expansion of two independent solutions about the origin.

ii) Do these solutions have singularities at any finite point of the complex x-plane?

b) i) Define the "characteristic curves" of a second order linear partial differential equation in two variables. Explain the classification of the latter into hyperbolic, parabolic and elliptic equations.

ii) What sort of boundary curve and what boundary conditions thereon, are suitable for the equation:

$$\frac{\partial^2 \Psi}{\partial x^2} + 1000 \frac{\partial^2 \Psi}{\partial y^2} = 27 \frac{\partial \Psi}{\partial y}$$

iii) If Ψ and $\frac{\partial \Psi}{\partial y}$ are specified in $0 \leq x \leq 1$ for the equation:

$$\frac{\partial^2 \Psi}{\partial x^2} - \frac{\partial^2 \Psi}{\partial y^2} = 0$$

over what (if any) range of x and y, can Ψ be determined from the equation?

2. Consider the integral equation

$$f(x) = g(x) + \lambda \int_a^b K(x,x') f(x') dx'$$

2. (continued)

written in operator form as

$$|f\rangle = |g\rangle + \lambda\, \kappa\, |f\rangle$$

i) Write out the Neumann series for this equation.

ii) What is the qualitative form of the Fredholm solution and what advantage does this solution have over the Neumann series?

iii) For what sort of kernel κ would you use the Wiener-Hopf method?

b) Find eigenfunctions and eigenvalues λ of

$$f(x) = \lambda \int_{-1}^{+1} \left[1 + \sqrt{\tfrac{3}{2}}\,(x+y) + 6\,xy\right] f(y)\, dy$$

3. Find the first two terms in the asymptotic expansion for large real ν of

$$F(\nu) = \int_0^1 \exp\,[i\nu x^2]\,dx$$

4. a) i) Find the Fourier transform of the polynomial

$$P_n(x) = \sum_{m=0}^{n} a_m x^m$$

ii) Give a rigorous definition, within the context of the theory of distributions (generalized functions), of the "functions" which occur in the solution of (i).

b) Define

$$\varphi(x,y) = \sum_{m=-\infty}^{\infty} \exp\,[-\pi(m+y)^2 x]$$

for $x>0$, and one particular theta function by

$$\theta(x) = \varphi(x,0).$$

Find the Fourier series expansion of $\varphi(x,y)$ of the form

$\sum_{n=-\infty}^{\infty} a_n(x)e^{2\pi iny}$ and hence derive a functional relation

between $\theta(x)$ and $\theta\left(\frac{1}{x}\right)$.

5. i) A coin is tossed n times with probability x of heads and 1-x of tails. Find the probability that heads turns up exactly k times in the n independent trials.

ii) Let y_i be independent random variables taking the values 0 and 1 with probability 1-x and x respectively. Define

$$Z_n = \frac{1}{n}\sum_{i=1}^{n} y_i$$

a) Write down the probability distribution of Z_n.
Now define

$$B_n(f,x) = \langle f(Z_n) \rangle \text{ where}$$

f(x) is any continuous function of x in [0,1] and $\langle\ \rangle$ denotes expectation (or mean) value.

b) Derive an expression for $B_n(f,x)$ in terms of $f\left(\frac{k}{n}\right)$ (k is an integer between 0 and n) and show that, for fixed f, $B_n(f,x)$ is a polynomial in x of degree n.

c) Use the central limit theorem to find a simple form for $B_n(f,x)$ as $n \to \infty$.

d) Indicate, briefly and without being rigorous, how the previous results can be used to show that any continuous function can be uniformly approximated by polynomials. (Weierstrass's theorem).

UCI PHYSICS QUALIFYING EXAMINATION - SPRING 1978

PART IV - Classical Mechanics

- Time: 1:00 - 5:30 PM, (3.5 hours), May 23
- Do 5 of the 6 problems
- The problems will count equally
- Two reference books permitted
- Use the supplied bluebooks for all work you wish to have graded. Identify the book with your number only. Label problems clearly. You may use scratch paper separately or in a denoted section of the bluebook(s).

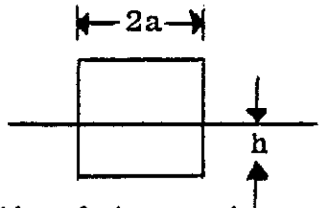
PART IV - Classical Mechanics

1. Estimate numerical values for the following quantities, and indicate how you arrive at your estimate.
   a) The speed, in m/sec, of a comet at perigee = 0.1 A.U.
   b) The magnetic field, in gauss, near the ground underneath a typical power transmission line feeding the Los Angeles area, due to the line only.
   c) The speed, expressed as a fraction of the velocity of light, of a 200 GeV/c proton.
   d) The pressure, in Newton's/m^2, of a 100 mi/hr wind blowing on the side of a building.
   e) The amount of energy, in joules, required to raise an isolated metal sphere of 1 meter radius to a million volts.

2. Three masses m_1, m_2 and m_3 move subject to their mutual gravitational attraction. If the masses are initially at the vertices of an equilateral triangle of side a, and if their initial velocities are suitably chosen, their locations will continue to define a (rotating) equilateral triangle of side a. Express ω, the angular frequency of rotation of the triangular configuration about its center of mass, as a function of the masses, of the side a, and of the gravitational constant G. (It is interesting to note that several asteroids, called Trojan asteroids, appear to form such a configuration with Jupiter and the Sun.)

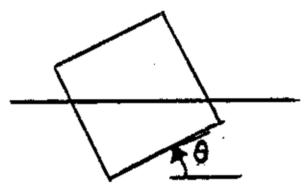
3. A long beam with a square cross-section (side = 2a) floats on water. The beam has density s; the water has density 1.

a) The beam is in equilibrium with one side of the square parallel to the water surface, as shown below.



Calculate h, the depth of immersion.

b) For certain densities s, there is a second equilibrium position, in which a trapezoidal portion of the cross-section is submerged, as shown below.



Calculate the tangent of the inclination θ as a function of the density s.

c) What is the range of densities s for which such a tilted equilibrium position, with a trapezoidal submerged portion, exists? Remark: In this problem we do not distinguish between stable and unstable equilibrium; both are referred to as equilibrium.

4. A system undergoes small oscillations about equilibrium; expressed in terms of normal coordinates $q_k(t)$, the kinetic and potential energies are

$$T = (1/2) \sum_{k=1}^{n} \dot{q}_k^2 \qquad V = (1/2) \sum_{k=1}^{n} \lambda_k q_k^2$$

4. (continued)

a) What are the normal frequencies of oscillation?
A constraint is now imposed on the system, namely $\sum_{k=1}^{n} A_k q_k = 0$.
The kinetic and potential energies are unaffected.

b) Write the explicit equation whose roots now give the new frequencies of oscillation.
Show from this equation that the new frequencies interleave the old frequencies; i.e., if the old frequencies are $\omega_1 > \omega_2 > \ldots > \omega_n$, and the new frequencies are $\Omega_1 > \Omega_2 > \ldots > \Omega_{n-1}$, then show that $\omega_1 > \Omega_1 > \omega_2 > \Omega_2 > \ldots > \Omega_{n-1} > \omega_n$.

5. A circular metal plate of radius a is rigidly clamped around its perimeter. The differential equation describing small oscillations of the plate is

$$\nabla^4 u + \alpha \frac{\partial^2 u}{\partial t^2} = 0$$

where $\nabla^4 u = \nabla^2 (\nabla^2 u)$ and α is a constant involving the elastic properties of the metal. The boundary conditions at r = a are

$$u = 0 \text{ and } \frac{\partial u}{\partial r} = 0$$

(i) Consider the equivalent mathematical problem of solving

$$\nabla^4 u - k^4 u = 0$$

inside the unit circle, with $u = 0$, $\frac{\partial u}{\partial r} = 0$ at $r = 1$.
Let the smallest eigenvalue of this equation be k_o^4. What is the frequency, ω_0, of the lowest mode of our plate in terms of k_o, a, and α?

5. (continued)

(ii) By noting the factorization $\nabla^4 - k^4 = (\nabla^2 - k^2)(\nabla^2 + k^2)$ give two linearly independent solutions of $\nabla^4 u - k^4 u = 0$ which are appropriate to the problem.

(iii) By imposing the boundary conditions of (i), find a transcendental equation whose roots are the eigenvalues k.

(iv) If the perimeter of the plate is free rather than rigidly clamped, what are the boundary conditions?

6. Consider the Lagrangian

$$L = m\, e^{\gamma t}\, \frac{1}{2}\, (\dot{x}^2 - \omega^2 x^2)$$

for the motion of a particle of mass m in one dimension (x). The constants m, γ and ω are real and positive.

(a) Find the equation of motion

(b) Interpret the equation of motion by stating the kinds of forces to which the particle is subject.

(c) Find the canonical momentum, and from this construct the Hamiltonian function.

(d) Is the Hamiltonian a constant of the motion? Is the energy conserved? Explain.

(e) For the initial condition $x(0) = 0$ and $\dot{x}(0) = + v_0$, what is $x(t)$ asymptotically as $t \to \infty$?

UCI PHYSICS QUALIFYING EXAMINATION - SPRING 1978

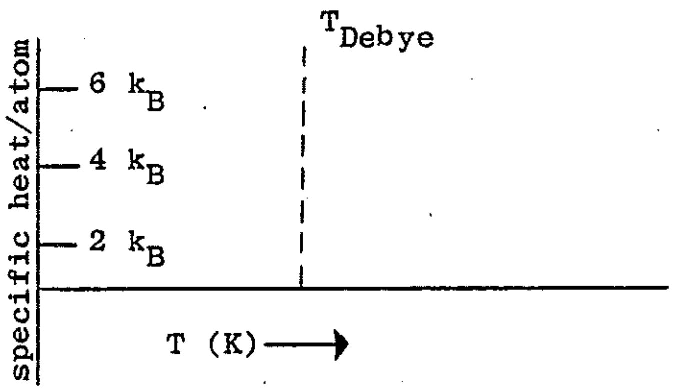
PART V - Thermodynamics and Statistical Mechanics

- Time: 0900 - 1200 = 3 hours, May 24
- Do all 4 problems
- Problems count equally
- CLOSED BOOK
- Use the supplied bluebooks for all work you wish to have graded. Identify the book with your number only. Label problems clearly. You may use scratch paper separately or in a denoted section of the bluebook(s).

PART V - Thermodynamics and Statistical Mechanics

1. Acoustic properties of dielectric solids dominate their thermodynamic behavior and other properties such as photo-conducting resistance. Diamond is a monoatomic dielectric solid of carbon having 10^{21} atoms cm^{-3}; on the graph below, sketch in, roughly, its specific heat (per atom) as a function of absolute temperature.

a)



b) How is T_{Debye} related to the Debye frequency, ω_D?

c) If the acoustic velocity at low frequencies is 5×10^5 cm/sec, approximately what is the value of ω_D?

2. A material is brought from temperature T_i to temperature T_f by placing it in contact with a series of N reservoirs at temperatures $T_i + \Delta T$, $T_i + 2\Delta T, \ldots, T_i + N\Delta T = T_f$. Assuming that the heat capacity of the material, C, is temperature independent, calculate the entropy change of the total system, material plus reservoirs. What is the entropy change in the limit $N \to \infty$ for fixed $T_f - T_i$?

3. The energy levels of a three-dimensional rigid rotor of moment of inertia I are given by

$$E_{J,M} = \frac{\hbar^2}{2I} J(J+1)$$

where $J = 0, 1, 2, \ldots; M = -J, -J+1, \ldots, J$. Consider a system of N rotors:

a) Using Boltzmann statistics, find an expression for the thermodynamical internal energy of the system.

b) Under what conditions can the sums in part (a) be approximated by integrals? In this case calculate the specific heat C_v of the system.

4. A dilute gas of atoms having $S = 1$, $L = 1$ and $J = 1$ is located in a uniform magnetic field, B. Using quantum mechanics and a assuming that $\mu_{Bohr} B \ll k T$, find:

a) The energy levels for the three spin projections along the field direction.

b) The contribution to the specific heat due to the magnetic moment.

UCI PHYSICS QUALIFYING EXAMINATION - SPRING 1978

PART VI - Quantum Mechanics II (Qualitative)

- Time: 1300-1430 = 1.5 hours, May 24
- Do all 4 problems
- Problems count equally
- CLOSED BOOK
- Use the supplied bluebooks for all work you wish to have graded. Identify the book with your number only. Label problems clearly. You may use scratch paper separately or in a denoted section of the bluebook(s).

PART VI - Quantum Mechanics II (Qualitative)

1. Suppose an electron is in a state described by the wave function

$$\psi = \frac{1}{\sqrt{4\pi}} (e^{i\varphi}\sin\theta + \cos\theta)\, g(r)$$

where $\int_0^\infty |g(r)|^2 r^2 \, dr = 1$

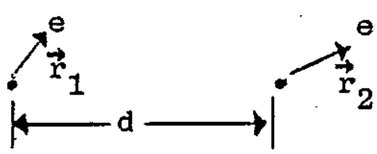
and φ, θ are the azimuth and polar angles, respectively.

(a) What are the possible results of a measurement of the z-component of the angular momentum L_z of the electron in this state?

(b) What is the probability of obtaining each of the possible results in part (a)?

(c) What is the expectation value of L_z?

2.



Consider the following model for the Van der Waals force between two atoms: Each atom consists of one electron bound to a very massive nucleus by a potential $V(r_i) = \frac{1}{2} m \omega^2 r_i^2$. Assume that the two nuclei are d ($d \gg \sqrt{\hbar/m\omega}$) apart along the x-axis and that there is an interaction $V_{12} = \beta \frac{x_1 x_2 e^2}{d^3}$ (Ignore the fact that the particles are indistinguishable).

(a) Consider the ground state of the entire system when $\beta=0$. Give its energy and wave function in terms of $\vec{r}_1$ and $\vec{r}_2$.

(b) Calculate the lowest non-zero correction to the energy (ΔE) and to the wave function due to V_{12}.

(c) Calculate the r.m.s. separation along the x direction of the two electrons, to lowest order in β.

2. (continued)

$$\psi_o(x) = \langle x|0\rangle = \left(\sqrt{\frac{m\omega}{\pi\hbar}}\right)^{\frac{1}{2}} e^{\frac{-x^2 m\omega}{2\hbar}} \; ; \; \psi_1(x) = \langle x|1\rangle =$$

$$= \left(\sqrt{\frac{m\omega}{\pi\hbar}}\;\frac{2m\omega}{\hbar}\right)^{\frac{1}{2}} x\, e^{\frac{-x^2 m\omega}{2\hbar}} \; ;$$

$\langle n|x|m\rangle = 0$ for $|n-m| \neq 1$; $\langle n-1|x|n\rangle = (n\,\hbar/2m\,\omega)^{\frac{1}{2}}$,

$\langle n+1|x|n\rangle = ([n+1]\,\hbar/2m\,\omega)^{\frac{1}{2}}$.

3. (a) Suppose the state of a certain harmonic oscillator with angular frequency ω is given by the wave function

$$\psi = N \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}\, \psi_n(x)\, e^{-in\omega t}, \quad \alpha = x_o\sqrt{\frac{m\omega}{2\hbar}}\, e^{i\varphi}, \quad N = e^{-\frac{1}{2}|\alpha|^2} .$$

Calculate the average position of the oscillator $\langle x\rangle$ in this state, and show that the time-dependence of $\langle x\rangle$ is that of a classical oscillator with amplitude x_o and phase φ.

(b) The Hamiltonian for a one-dimensional harmonic oscillator in a laser electromagnetic field is given by

$$H = \frac{p^2}{2m} + \frac{ep}{2m\omega} E_o \sin\omega t - \tfrac{1}{2}\, e\, E_o x \cos\omega t + \tfrac{1}{2}\, m\, \omega_o^2 x^2 .$$

Where ω_o, m, and e are the angular frequency, mass, and charge of the oscillator, and ω is the angular frequency of the radiation. Assume the laser is turned on at t = 0 with the oscillator in its ground state ψ_o. Treat the electromagnetic interaction as a perturbation, in first order, and find the probability for any time t > 0 that the oscillator will be found in one of its excited states ψ_n.

Useful information: The normalized oscillator wavefunctions $\psi_n(x)$ have the property that

$$\left(x + \frac{\hbar}{m\omega}\frac{d}{dx}\right)\psi_n = \sqrt{\frac{2\hbar}{m\omega}\cdot n}\;\psi_{n-1}, \quad \left(x - \frac{\hbar}{m\omega}\frac{d}{dx}\right)\psi_n = \sqrt{\frac{2\hbar}{m\omega}\cdot(n+1)}\;\psi_{n+1}$$

4. The independent-particle shell model of the nucleus consists of non-interacting neutrons and protons moving in a three-dimensional harmonic oscillator potential.

a) Draw an energy level diagram for the 5 lowest states of this extreme shell model. Indicate degeneracies.

If we now consider the presence of a strong, central spin-orbit potential:

b) For ^{18}Ne (10 protons) list the spins, parities and isotopic spins for all the states in the $(1\ d_{5/2})^2$, $(1\ d_{5/2}$, $2\ s_{1/2})$ and $(2\ s_{1/2})^2$ configurations.

c) Predict for both the ground state and the first excited state of ^{17}F (9 protons) the spins, parities, and magnetic moments.

UCI PHYSICS QUALIFYING EXAMINATION - SPRING 1978

PART VII - General Physics - The last part!

- Time: 1500 - 1800 = 3 hours, May 24
- Do 10 of the 13 problems
- Problems count equally
- CLOSED BOOK
- Use the supplied bluebooks for all work you wish to have graded. Identify the book with your number only. Label problems clearly. You may use scratch paper separately or in a denoted section of the bluebook(s).

PART VII - General Physics

1. Answer each of the following with, at most, a sentence.

   a) Give the principal decay mode of each:

   $$\mu^+,\ \pi^\circ,\ e^+,\ ^{14}C,\ ^{238}U.$$

   b) Give the spin and parity of each:

   $$\mu^+,\ \pi^+,\ ^2H,\ ^{12}C,\ \Delta^{++}.$$

   c) Give the approximate magnitudes of the:

      1) Proton energy at the center of the sun.
      2) Wavelength of a VHF TV signal.
      3) Experimental upper limit on the mass of the $\bar{\nu}_e$.
      4) Temperature at which ^3He exhibits superfluid properties.
      5) Critical magnetic field for superconducting lead at 0°K.

   d) What would you usually measure with:

      1) A Ge(Li) detector?
      2) An electron micro-probe?
      3) A Pitot tube?
      4) An electroscope?
      5) A Pirani gauge?

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2. a) Describe briefly an experiment that would determine whether the ν_μ is different from the ν_e (or the $\bar{\nu}_\mu$ different from the $\bar{\nu}_e$).

   b) Assuming that the experiment described in a) establishes that the ν_μ and ν_e are different, then there must be a quantum number that distinguishes muonic leptons from electronic leptons. Assign the lepton number as in the table below.

Lepton number	Particle							
	e^-	e^+	ν_e	$\bar{\nu}_e$	μ^-	μ^+	ν_μ	$\bar{\nu}_\mu$
L_e	1	-1	1	-1	0	0	0	0
L_μ	0	0	0	0	1	-1	1	-1

Describe an experiment that would distinguish which of the following two conservation laws is correct for a weak decay involving both muonic and electronic leptons:

a) $\sum_i L_e^{(i)}$ and $\sum_i L_\mu^{(i)}$ conserved separately.

("additive" conservation law)

b) $\sum_i (L_e^{(i)} + L_\mu^{(i)})$ and $\prod_i (-1)^{L_\mu^{(i)}}$ conserved

("multiplicative" conservation law)

3. SLAC has used the following scheme for producing high-energy, monochromatic photons:

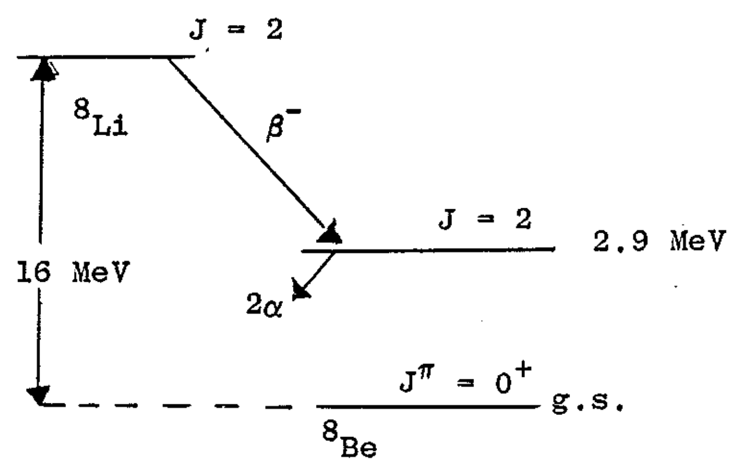
A beam of low-energy photons from a laser (visible light) is directed head-on into the 20 GeV electron beam. Find the energy of the scattered photons as a function of the angle they make with the electron beam and give the approximate value in GeV of the maximum photon energy.

4. A proposed experiment to measure the charge distribution of the pion involves the formation of "exotic atoms" $\mu^+ - \pi^-$ or $\mu^- - \pi^+$.

   a) What is the ground state energy in eV for the $\mu - \pi$ system if both are treated as point particles?

   b) Find the energy correction in eV for the ground state of the "atom" assuming that the pion is a uniformly charged sphere whose radius is 10^{-13} cm.

5. ^{8}Li is an example of a β-delayed particle emitter. The ^{8}Li ground state has a half-life of 0.85 s and decays to the 2.9 MeV level in ^{8}Be. The 2.9 MeV level then decays into 2 alpha particles with a half-life of 10^{-22}s.



   a) What is the parity of the 2.9 MeV level in ^{8}Be? Give your reasoning.

   b) Why is the half-life of the ^{8}Be 2.9 MeV level so much smaller than the half-life of the ^{8}Li ground state?

   c) Where in energy, with respect to the ^{8}Be ground state, would you expect the threshold for ^{7}Li neutron capture? Why?

6. a) Assuming that the two protons of the H_2^+ molecule are fixed at their normal separation of 1.06 Å, sketch the potential energy of the electron along the axis passing through the protons

b) Sketch the electron wave functions for the two lowest states in H_2^+, indicating roughly how they are related to hydrogenic wave functions. Which wave function corresponds to the ground state of H_2^+ and why?

c) What happens to the two lowest energy levels of H_2^+ in the limit that the protons are moved far apart?

7. The atomic number of Mg is Z = 12.

a) Draw a Mg atomic energy level diagram (not necessarily to scale) illustrating its main features, including the ground state and excited states arising from the configurations in which one valence electron is in the 3s state and the other valence electron is in the state $n\ell$ for n = 3, 4 and ℓ = 0, 1. Label the levels with conventional spectroscopic notation. Assume LS coupling.

b) On your diagram, indicate the following (give your reasoning!):

i) an allowed transition

ii) a forbidden transition

iii) an intercombination line (if any)

iv) a level which shows (1) anomalous and (2) normal Zeeman effect, if any.

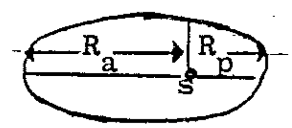
8. Find the threshold energy (kinetic energy) for a proton beam to produce the reaction

$$p + p \rightarrow \pi^{\circ} + p + p$$

with a stationary proton target.

9. An excellent camera lens of 60 mm focal length is accurately focussed for objects at 15 m. For what aperture (stop opening) will the diffraction blur of visible light be roughly the same as the defocus blur for a star (at ∞)?

10. A comet in an orbit about the sun has a velocity of 10 km/sec at apheliom and 80 km/sec at perihelion. If the earth's velocity in a circular orbit is 30 km/sec, and radius of its orbit is 1.5 x 10^8 km, find the aphelion distance R_a for the comet.



11. A particle X has two decay modes with partial decay rates $\gamma_1(\text{sec}^{-1})$ and $\gamma_2(\text{sec}^{-1})$.

a) What is the inherent uncertainty in the mass of X?

b) One of the decay modes of X is the strong interaction decay:

$$X \rightarrow \pi^+ + \pi^+$$

What can you conclude about the isotopic spin of X?

12. A non-relativistic electron of mass m, charge -e, in a cylindrical magnetron moves between a wire of radius a at a negative electric potential $-\varphi_o$ and a concentric cylindrical conductor of radius R at zero potential. There is a uniform constant magnetic field B parallel to the axis. Use cylindrical coordinates r, θ, z. The electric and magnetic vector potentials can be written

$$\varphi = -\varphi_o \frac{\ln r/R}{\ln a/R}, \quad \vec{A} = \frac{1}{2} Br\, \hat{\theta} \quad (\hat{\theta} \text{ a unit vector in direction of increasing } \theta).$$

a) Write the Lagrangian and Hamiltonian functions.

b) Show that there are three constants of the motion, write them down, and discuss the kinds of motion which can occur.

c) Assuming that an electron leaves the inner wire with zero initial velocity, there is a value of the magnetic field B_c such that for $B \leq B_c$ the electron can reach the outer cylinder, and for $B > B_c$ the electron cannot reach the outer cylinder. Find B_c and make a sketch of the electron's trajectory for this case. You may assume that $R \gg a$.

13. Give short answers including one sentence of reasons for the following ten questions:

a) How old is the universe?

b) What is Hubble's constant?

c) When was the most part of the Helium in the universe made?

d) When were the heavy elements (of which we are made) formed?

e) Is the universe closed?

f) What was the temperature of the universe when the black body radiation decoupled?

g) What is the baryon density of the universe?

h) How many photons are there per baryon?

i) How many neutrinos are there per baryon?

j) How big was the big bang?

"The big bang? Believe me, it was very, very, very, very, very, *very* big."