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QUALIFIER EXAM
Statistical Mechanics
Spring 2013

This is an open book exam. You may refer to *Statistical Mechanics* by R.K Pathria. There are blank pages available to provide sufficient work space. To receive credit you must show all of your work. Make an effort to present clear and organized solutions. Little or no credit will be given for obscure mathematics.

DO NOT OPEN THIS EXAM UNTIL YOU ARE TOLD TO DO SO

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Statistical Mechanics

1. **(a)** Evaluate the partition function and the major thermodynamic properties of an ideal gas consisting of N_1 molecules of mass m_1 and N_2 molecules of mass m_2, confined to a space of volume V at temperature T. Assume that the molecules of a given kind are mutually indistinguishable, while those of one kind are distinguishable from those of the other kind.

   (b) Compare your results with the ones pertaining to an ideal gas consisting of $(N_1 + N_2)$ molecules, *all of one kind*, of mass m, such that $m(N_1 + N_2) = m_1N_1 + m_2N_2$.

2. Consider the effusion of molecules of a Maxwellian gas through an opening of area A in the walls of a vessel of volume V.

   (a) Show that, while the molecules inside the vessel have a mean kinetic energy $\frac{3}{2}kT$, the effused ones have a mean kinetic energy $2kT$, T being the *quasistatic* equilibrium temperature of the gas.

   (b) Assuming that the effusion is so slow that the gas inside is always in a state of quasistatic equilibrium, determine the manner in which the density, the temperature, and the pressure of the gas vary with time.

3. Assuming the dispersion relation $\omega = Ak^s$, where ω is the angular frequency and k the wave number of a vibrational mode existing in a solid, show that the respective contribution toward the specific heat of the solid *at low temperatures* is proportional to $T^{3/s}$.

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CLASSICAL MECHANICS QUALIFYING EXAM

Spring 2013

You may consult only *Classical Mechanics* by Goldstein, Safko, and Poole. Do all four (4) problems. The exam is worth 50 points. Problem No. 2 is relativistic; the other three problems are non-relativistic. Write directly on this exam; do not use a blue book. Every other page has been left blank to provide extra work space. In order to receive credit you must show all of your work.

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1. **[15 points]** In this problem, all given quantities are dimensionless and numerical answers are expected for all parts of the problem. A particle of mass $m = 1$ moves in the field of a central potential. The equation for the orbit is:

$$\frac{1}{r} = \frac{1}{8}\left(e^{2\theta} + e^{-2\theta}\right) = \frac{1}{4}\cosh 2\theta$$

At $t = 0$, the particle is at $r = 4, \theta = 0$ (or $x = 4, y = 0$ in rectangular coordinates) and its velocity is $v = 3/4$ in the $+y$-direction.

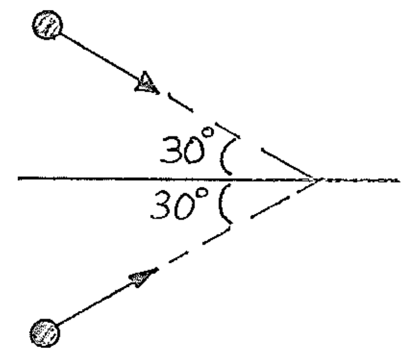
(a) Find the force associated with the potential. Your answer should contain r (the distance from the origin) and numerical quantities only. No other symbols should appear in your answer.

(b) Find the magnitude of the particle's velocity when its distance from the origin is $r = 2$.

(c) Find the magnitude of the angular momentum of the particle.

2. **[10 points]** Two protons with identical Lorentz factors γ are moving in the x-y plane when they collide. Just before the collision, one proton is moving with a velocity vector at an angle of 30° ($\pi/6$ radians) above the x-axis; the other has its velocity vector at the same angle below the x-axis. Find the minimum value of γ that will result in the creation of an antiproton via the reaction:

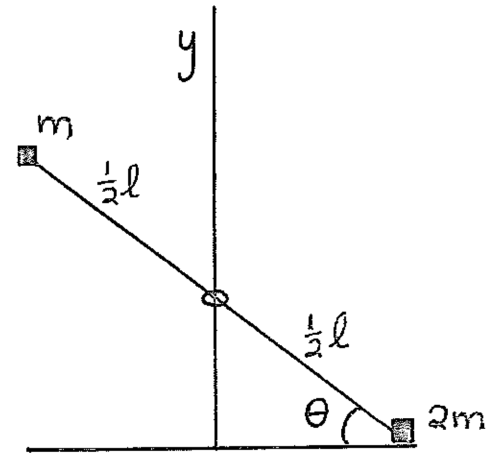
$$p + p \rightarrow p + p + p + \bar{p}$$



3. **[15 points]** A rigid rod of length ℓ has a massless slip ring at its midpoint that constrains the midpoint to move along the y-axis. A mass $2m$ is attached to the lower end of the rod and maintains contact with the ground (x-axis) at all times. A mass m is attached to the upper end of the rod. There is no friction in the problem. Initially the rod makes an angle $\theta = \arccos(3/5)$ with the horizontal. The system is released from rest.

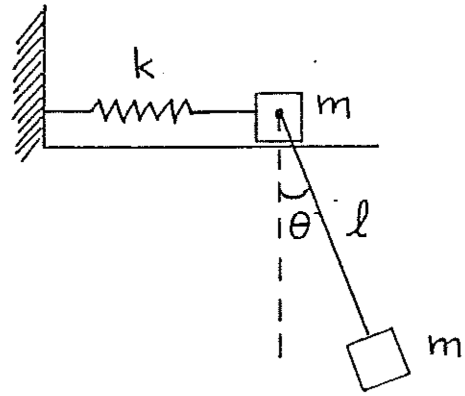
(a) Find the initial angular acceleration $\ddot{\theta}$ of the rod.

(b) Find the velocity of the slip ring when it strikes the floor.



4. **[10 points]** A mass m, constrained to move along a horizontal line, is attached to a spring of spring constant $k = m\omega_0^2$, which is in turn attached to a fixed wall. A second identical mass m is attached to the first by a massless rigid rod of length $\ell = g/\omega_0^2$, which pivots freely about the point of suspension, where g is the acceleration of gravity.

The equilibrium length of the spring is also ℓ and there is no friction in the problem. Find the frequencies of the two normal modes for small oscillations of this system. Express your answers in terms of ω_0 and numerical factors only.



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Electricity and Magnetism Qualifying Exam

Spring 2013

Do all six problems. In this exam we follow the conventions of Jackson's 3rd Edition, i.e., SI units for non-relativistic problems and cgs units for relativistic problems. You may use any intermediate results in Jackson that is appropriate. You do not need to write the numerical values of special functions.

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1. [10 points] A conducting sphere of radius a carries a total charge of q. It is surrounded by a dielectric material of permittivity $\epsilon = 2\epsilon_0$ out to radius b. Find the charge density at the outer surface r=b and the potential on the conducting sphere.

2. [15 points] An infinitely long, thin wire parallel to the z-axis at a position of $x=d>0$ and $y=0$ carries a current of I flowing in the positive z-direction. The y-z plane at $x=0$ is a conducting wall. Find the magnetic field at $x>0$ and the force per unit length acting on the wire.

3. [20 points] The potential on the surface of a hollow sphere is maintained as:

$$\Phi(r = a, \theta, \phi) = \begin{cases} V\,cos\theta(1 + sin\,\phi) & \text{for} \quad 0 \leq \theta \leq \frac{\pi}{2} \\ 0 & \text{for} \quad \frac{\pi}{2} < \theta \leq \pi \end{cases}$$

There is no charge inside or outside of the sphere. For $r<a$, find the potential as a power series of r and explicitly evaluate terms up to and including r. Find the total charge on the surface of the sphere.

4. **[15 points]** In this problem we are only interested in electric dipole radiation. Ignore all other forms of multipole radiation. Three charges, $+2q, -q, -q$, are attached to the origin by rigid rods all of length a and situated in the x-y plane as shown, with the positive charge along the $+y$-axis and the two negative charges each at an angle of 120° from the $+y$-axis.

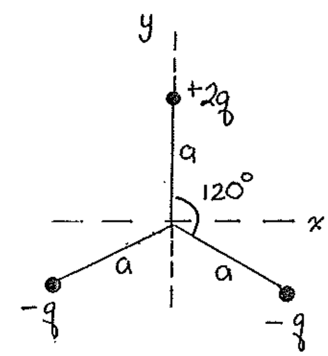
(a) Suppose the assembly is rotated about the z-axis at constant angular velocity ω. Find the time-averaged power per unit solid angle $dP/d\Omega$ received at a distant point on the $+z$-axis.

(b) Does the plane of polarization of the radiation seen at the distant point along the $+z$-axis rotate about the line of sight or is it fixed in direction? If it is fixed in direction, what is the direction?

(c) Repeat Part (a) except for a distant point on the $+x$-axis.

(d) Repeat Part (b) except for a distant point on the $+x$-axis.

(e) If the original assembly is now rotated about the y-axis, find the time-averaged power per unit solid angle received at a distant point on the $+z$-axis.



5. **[15 points]** In all parts of the problem a charge q_1 moves through the laboratory relativistically with coordinates given by:

$$x = \frac{4}{5}ct, \quad y = 0, \quad z = 0$$

where t is the laboratory time. For each of the following parts find the *total* force acting on a second charge q_2 as measured in the laboratory at time $t = 0$. In particular find the Cartesian components of the total force on q_2 in terms of q_1, q_2, the distance a, fundamental constants, and numerical factors for each of the following situations. Plug in numerical values for β and γ so that these symbols do not appear in your answers.

(a) The charge q_2 is fixed in the laboratory at the point $y = a$ on the positive y-axis.

(b) The charge q_2 moves through the laboratory with coordinates given by:

$$x = \frac{4}{5}ct, \quad y = a, \quad z = 0$$

(c) The charge q_2 moves through the laboratory with coordinates given by:

$$x = 0, \quad y = a, \quad z = \frac{4}{5}ct$$

(d) The charge q_2 moves through the laboratory with coordinates given by:

$$x = 0, \quad y = a + \frac{4}{5}ct, \quad z = 0$$

6. **[15 points]** In this problem we are concerned with a rectangular waveguide filled with air. Furthermore we are only concerned with TE modes; we ignore TM modes. The (angular) cutoff frequency of the $TE_{0,1}$ and $TE_{1,1}$ modes are known to be related by $\omega_{1,1} = \frac{5}{4}\omega_{0,1}$.

(a) Find the dimensions a and b of the guide in terms of $\omega_{0,1}$ and c.

(b) Identify all TE modes with cutoff frequencies ω_c such that $\omega_{0,1} < \omega_c < \omega_{1,1}$ or show that no such modes exist. Note the strict inequalities.

(c) Find all TE modes with cutoff frequencies ω_c such that $\omega_{1,1} < \omega_c < 2\,\omega_{0,1}$ or show that no such modes exist. Note the strict inequalities.

(d) Can a TE mode with frequency $\omega = \frac{4}{5}\omega_{0,1}$ propagate? If so, identify the mode or modes. If not, explain why not.

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QUALIFIER EXAM
Quantum Mechanics
Spring 2013

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Quantum Qualifying Exam 2013

1. A particle moves on the surface of a sphere of radius R (the Hamiltonian is therefore the angular part of the free Hamiltonian). At time t=0, the particle is at the north pole of the sphere. Find an expression for the probability that the particle is at the south pole at time $t = t_0$. **You do not need to perform the sums or integrals that may occur.**

2. A particle of mass m moves in the potential

$$V = \begin{cases} \infty & x < 0 \\ 0 & 0 < x < L/2 \\ \epsilon & L/2 < x < L \\ \infty & x > L \end{cases} \tag{1}$$

a) Find an exact equation (or set of equations) for the ground state energy. **You do not need to solve this equation (or set of equations).**

b) Write down the solution for the ground state energy when $\epsilon = 0$.

c) Treat ϵ as a small parameter, and find the ground state energy to order ϵ.

d) Solve the exact equation in part (a) to order ϵ and compare to your result in part (c).

3. Consider the Hamiltonian

$$H = \frac{p^2}{2m} + \lambda x^4 \tag{2}$$

We will look for a variational solution of the form $\psi \propto e^{-\alpha x^2}$. Explain all steps to find the best variational solution. **You do not need to perform the integrals that may occur.**

4. Two spin 1/2 particles are coupled by a Hamiltonian

$$H = \lambda \vec{S}_1 . \vec{S}_2 \tag{3}$$

The system is initially in the ground state. It is then perturbed by a Hamiltonian

$$H_1 = \begin{cases} \lambda S_{1x} t & 0 < t < t_0 \\ 0 & \text{else} \end{cases} \tag{4}$$

For each of the excited states, find the probability that the system is in that state at late times. **Perform all required integrals.**