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Classical Mechanics
Qualifying Exam
Spring 2016

You may consult only *Classical Mechanics* by Goldstein, Safko, and Poole. Do all four (4) problems. The exam is worth 50 points. Problem No. 4 is relativistic; the other three problems are non-relativistic. Write directly on this exam; do not use a blue book. Every other page has been left blank to provide extra work space. In order to receive credit you must show all of your work.

DO NOT OPEN THIS EXAM UNTIL YOU ARE TOLD TO DO SO

FOR ADMINISTRATIVE USE ONLY:

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1. **[14 points]** Consider two pendulums. The one on the left consists of a massless rigid rod of length ℓ and a bob of mass $2m$. The one on the right consists of an identical massless rigid rod, but its bob has mass m. When the pendulums hang straight down at rest, they are separated by a distance also equal to ℓ. The pendulum bobs are connected by a spring whose unstretched length is ℓ and whose spring constant is adjusted so that:

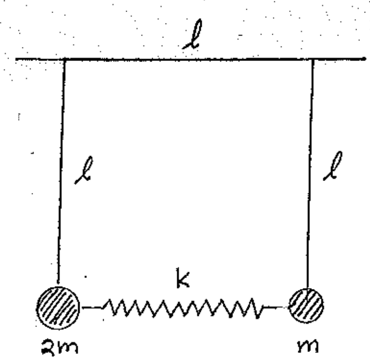
$$k = mg/\ell$$

(a) Find the angular frequencies of the normal modes of this system for small oscillations. HINT: Although you should use the pendulum angles θ_1 and θ_2 as the variables in this problem, note that in terms of Cartesian coordinates, the potential energy associated with the spring can be approximated as:

$$V = \frac{1}{2}k(x_2 - x_1)^2$$

where x_1 and x_2 are the x-coordinates of the two pendulum bobs, respectively (independent of y_1 and y_2). Of course there is also the gravitational potential energy, which is independent of x_1 and x_2.

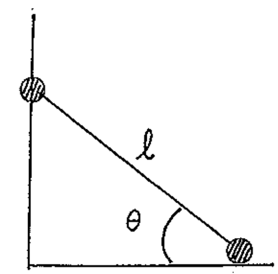
(b) For the higher angular frequency only, find the normal mode (θ_1, θ_2), with θ_1 normalized to 1.



2. **[12 points]** A particle of mass m is constrained to slide down a vertical pole. Another particle of mass m is constrained to move along the floor. The two particles are connected by a massless rigid rod of length ℓ. The rod makes an angle θ with the floor. There is no friction in the problem and the acceleration of gravity is g. The system is released from rest when $\sin\theta = 4/5$.

(a) What is the initial horizontal acceleration of the particle on the floor?

(b) What is the velocity of the particle on the vertical pole when $\sin\theta = 2/5$?

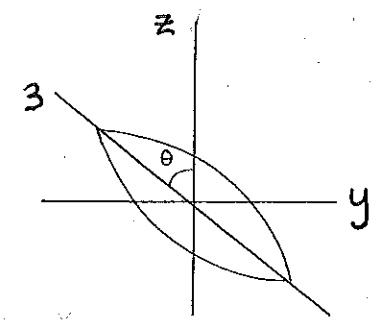


3. [**14 points**] A rigid body is a figure of revolution; the 3-axis is the axis of symmetry. There are no torques or gravity in this problem. The center of mass of the body is fixed at the origin. At time $t = 0$ the usual Euler angles (ϕ, θ, ψ) of the body are given by $\phi = 0$, $\theta = \arcsin\frac{4}{5}$, and $\psi = 0$. The angle θ between the body 3-axis and the laboratory z-axis remains fixed. At $t = 0$ the body has its angular velocity vector in laboratory Cartesian coordinates equal to $(0, -3\omega_0/5, 4\omega_0/5)$.

(a) Find the spin rate $\dot{\psi}$ of the body about its symmetry axis.

(b) Find the precession rate $\dot{\phi}$ of the body about the laboratory z-axis.

(c) If $I_1 = I_2 \equiv I_0$, find I_3 in terms of I_0.



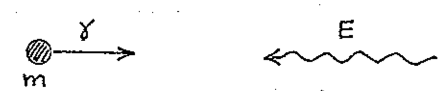
4. **[10 points]** In this problem all collisions take place in one dimension. An electron (mass m) of Lorentz factor $\gamma = 13/5$ moving along +x collides head on with a photon of energy E moving along −x. After the collision the electron is still moving along +x, but with Lorentz factor $\gamma' = 5/3$ and the photon energy is E'.

(a) After the collision, is the photon moving along +x or −x? Justify your answer.

(b) Find E in terms of m and c.

(c) Find E' in terms of m and c.

(d) Suppose that a photon with the same energy E as found in Part (b) but now moving along +x from the left overtakes and collides with an electron again with initial Lorentz factor $\gamma = 13/5$ moving along +x as above. In what direction will the photon be travelling after the collision? Justify your answer. You are not required to give the energy of the photon after the collision.



Physics Qual - Statistical Mechanics

(Spring 2016)

I. A Bose gas is made of particles that for low frequencies obey the dispersion law $\omega(k) = c\,k^n$, where c is some positive constant and n a positive integer. Think of the gas as being made up of N non-interacting particles in a large box of volume V, and at a constant temperature T.

(a) Compute the average energy, entropy and specific heat for such a system. In particular, determine the leading low temperature behavior.

(b) Give three physical examples for the above system, justifying your argument.

II. A dilute gas made up of N spin-$\frac{1}{2}$ particles with magnetic moment μ_B is maintained at a constant temperature T. The gas is immersed in an *inhomogeneous* magnetic field $B(\vec{x})\,\hat{z}$. Calculate the particle density and pressure as a function of $\vec{x}$.

III. Metallic sodium can be thought of as being made up of a lattice of sodium atoms and a gas of free electrons (each sodium atom contributes on the average one free electron).

a) What is the classical value for the specific heat of a piece of sodium containing N atoms?

b) Explain why the classical expression for the contributions of the lattice and the electron gas to the specific heat are both incorrect at room temperature.

c) What is the correct expression for the specific heat at room temperature?

[Useful facts: The atomic number of sodium is 23, the density of sodium is $0.97 gm/cm^3$, the mass of the proton is $1.67 \times 10^{-24} gm$.

IV. Show that in two dimensions an ideal Bose-Einstein gas does *not* exhibit Bose-Einstein condensation at finite temperatures.

EM Qualifying Exam Spring 2016

1. A circuit is made of straight wires of length L arranged as a square. Each wire has a resistance R. A current of magnitude $I = I_0 \cos \omega t$ is driven through the wire. How much power will this consume? Specifically address the two limiting cases $\omega = 0$ and $R = 0$.

2. A magnet is in the shape of a cylinder of length L and radius R. It is initially oriented along the x-axis. The magnetism is uniform, of magnitude M_0, and directed along the axis of the cylinder.

(a) Find an exact expression for the magnetic field everywhere.

(b) Find the leading expression for the magnetic field far away from the magnet.

The magnet is now rotated about an axis passing through its center and perpendicular to the axis of the cylinder. For concreteness assume the axis of rotation to be the z-axis. The angular velocity is ω and ω may be taken to be small.

(c) Find the leading expression for the electric field far away from the magnet to leading order in ω.

3. A circular wire of radius a lies in the $x - y$ plane; its center is at $x = y = z = 0$. It carries a current I which flows clockwise if viewed from the positive z axis.

There is another circular wire which is identical to the first, except that its center is at $x = x_0, y = y_0, z = z_0$. It also lies in the $x - y$ plane, its radius is also a, and it also carries a current I which flows clockwise as viewed from the positive z-axis. We will assume that the loops are small compared to the distance between the centers i.e. $x_0, y_0, z_0 \gg a$.

Find the force and torque exerted by the first wire on the second.

4. An insulating sphere of radius R carries a charge distribution $\rho = \rho_0 r^2$ as measured from its center. A second sphere of radius R carries a charge distribution $\rho = \rho_0 r^3$ as measured from its center. The two spheres are placed touching each other. For concreteness, we will assume that the first sphere is centered at $x = -R, y = z = 0$, and the second sphere is centered at $x = R, y = z = 0$. Find the E-field everywhere.

Quantum Mechanics Qual (Spring 2016)

I. The excited electronic configuration of the Helium atom $(1S)^1(2S)^1$ can exist either as a singlet, or as a triplet state. Which state has the lower energy, and why? Write down an expression which represents the triplet-singlet energy splitting in terms of the single-electron orbitals $\psi_{1s}(\mathbf{x})$ and $\psi_{2s}(\mathbf{x})$.

II. The unperturbed Hamiltonian for a two-state system is represented by

$$H_0 = \begin{pmatrix} \epsilon_1 & 0 \\ 0 & \epsilon_2 \end{pmatrix} .$$

A time-dependent perturbation is added of the form

$$V_t = \begin{pmatrix} 0 & \lambda \cos \omega t \\ \lambda \cos \omega t & 0 \end{pmatrix} ,$$

with λ a real parameter.
(a) Assuming that at $t = 0$ the system is in the state $(1, 0)$ with energy ϵ_1, find, using time-dependent perturbation theory, an expression for the probability that the system transitions to the state $(0, 1)$ for $t > 0$. Do the calculation assuming $\epsilon_1 - \epsilon_2$ is *not* close to $\pm\hbar\omega$.
(b) What problem arises if $\epsilon_1 - \epsilon_2$ *is* close to $\pm\hbar\omega$?

III. Using the Fermi Golden Rule, give an expression for the transition rate relevant for the photoelectric effect. Write your final answer for the differential rate in terms of the photon's wavevector $\mathbf{k}_i$, its polarization $\epsilon_\lambda(\mathbf{k_i})$, and the final electron momentum $\mathbf{p}_f$.

IV. Consider two spatially localized spin-$\frac{1}{2}$ particles coupled by a transverse exchange interaction, immersed in an inhomogeneous magnetic field. The Hamiltonian is

$$H = -J\,(S_1^+ S_2^- + S_1^- S_2^+) + h_1 S_1^z + h_2 S_2^z$$

Here h_1 and h_2 are proportional to the external magnetic fields at the two sites, and J measures the strength of the spin exchange coupling.
a) Find the eigenvalues of H.
b) If both spins are *up* at time $t = 0$, what is the probability that they will both be *down* at a later time t ?