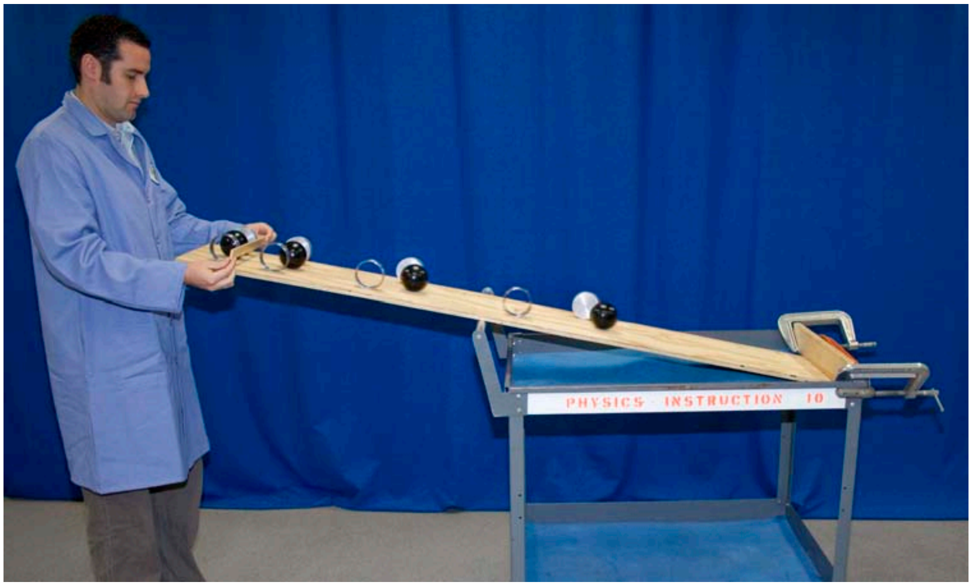
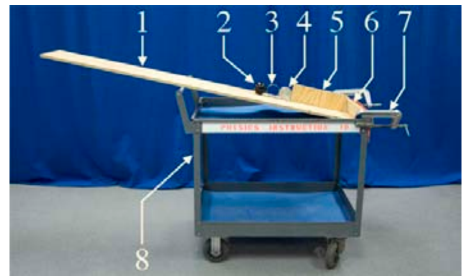
RING, DISC AND SPHERE 1Q10.30

Mechanics | Rotational Dynamics | Moment of Inertia





Concept:

Employing the conservation of mechanical energy between the top and bottom of the incline gives

$$Mgh = \frac{1}{2}M\,v^2 + \frac{1}{2}I\left(\frac{v}{R}\right)^2,$$

where h = height, M = mass, v = center of mass velocity, I = moment of inertia about the center of mass, and R = radius. But, I can be expressed as $I = \beta MR^2$, where β characterizes the geometry of the given rolling object. Substitution of this expression into the equation for energy conservation gives

$$v = \sqrt{\frac{2gh}{1+\beta}} \text{ where } \beta_{sphere} = \frac{2}{5},\ \beta_{disc} = \frac{1}{2} \text{ and } \beta_{ring} = 1.$$

Note that the solution is *independent* of the object's mass and size, and the winner of a rolling race can be predicted by simply knowing β. So the order of finish is the sphere, closely followed by the disc and then the ring.

Equipment:

1. Inclined Plane
2. Sphere
3. Ring
4. Disc
5. Starting Block
6. Finishing Block
7. (2) C-Clamps
8. Cart with Raised Handle

Procedure:

1. Hold the starting block near the top of the inclined plane so that it blocks all three objects from rolling.
2. Quickly move the starting block away from you and up to start the objects rolling down the incline.
3. Notice that the order of finish is sphere, disc and then ring.