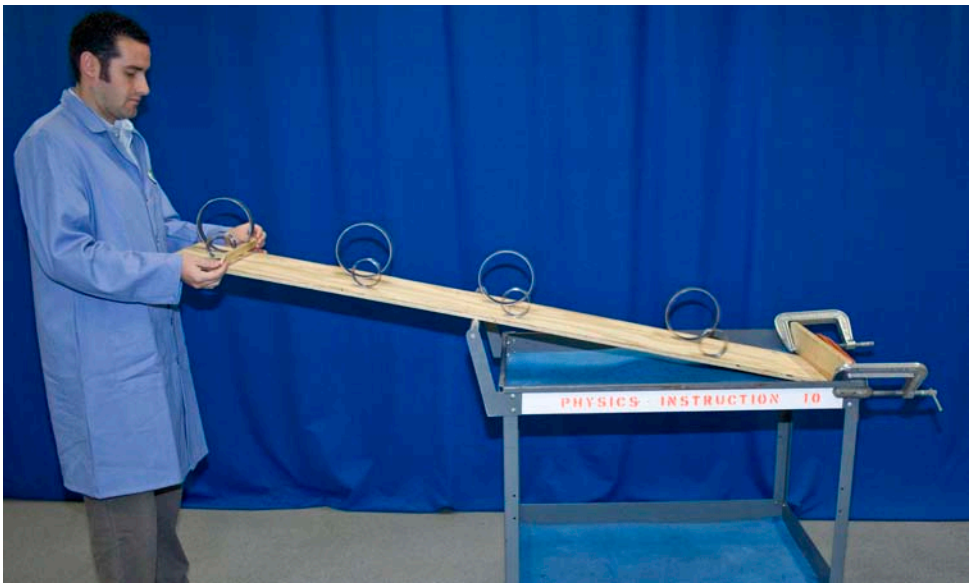
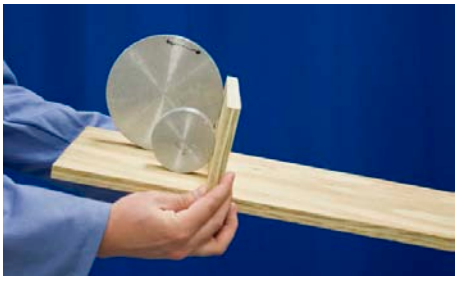
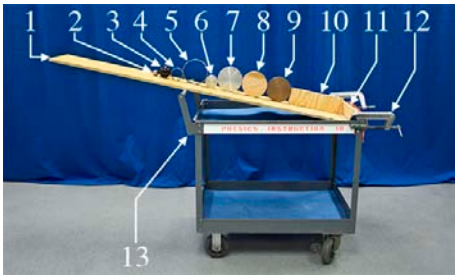
ALL DISCS ROLL THE SAME 1Q10.35

Mechanics | Rotational Dynamics | Moment of Inertia







Concept:

Employing the conservation of mechanical energy between the top and bottom of the incline gives

$$Mgh = \frac{1}{2} M v^2 + \frac{1}{2} I \left(\frac{v}{R} \right)^2,$$

where h = height, M = mass, v = center of mass velocity, I = moment of inertia about the center of mass, and R = radius. But, I can be expressed as $I = \beta MR^2$, where β characterizes the geometry of the given rolling object. Substitution of this expression into the equation for energy conservation gives

$$v = \sqrt{\frac{2gh}{1+\beta}} \text{ where } \beta_{sphere} = \frac{2}{5}, \beta_{disc} = \frac{1}{2} \text{ and } \beta_{ring} = 1.$$

Note that the solution is *independent* of the object's mass and size, and the winner of a rolling race can be predicted by simply knowing β.

Equipment:

1. Inclined Plane
2. Small Sphere
3. Large Sphere
4. Small Ring
5. Large Ring
6. Small Disc
7. Large Disc
8. Wooden Disc
9. Copper Disc
10. Starting Block
11. Finishing Block
12. (2) C-Clamps
13. Cart with Raised Handle

Procedure:

1. Choose the geometry of the objects you would like to race down the incline.
2. Hold the starting block near the top of the inclined plane so that it blocks the objects from rolling.
3. Quickly move the starting block away from you and up to start the objects rolling down the incline.
4. Notice that, as long as you are using objects with the same geometry, they reach the bottom of the incline at the same time.
5. Repeat as desired using objects with different geometries or uniform densities: small sphere vs. large sphere, small ring vs. large ring, small disc vs. large disc and large aluminum disc vs. wooden disk vs. copper disc.