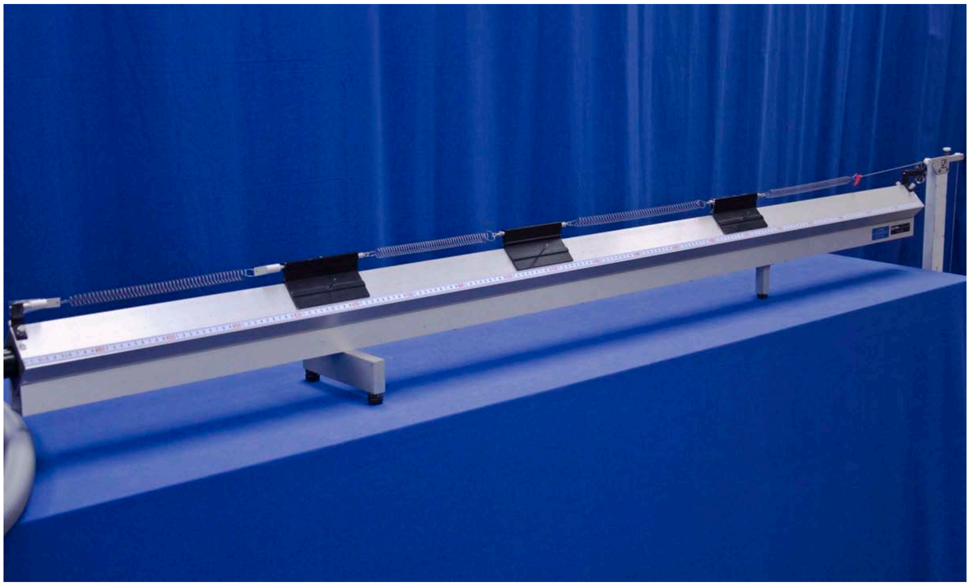
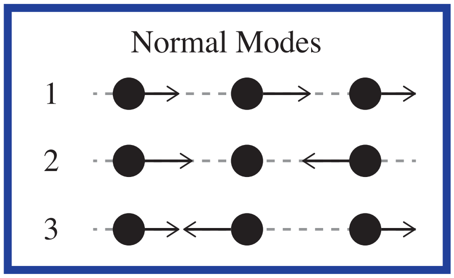
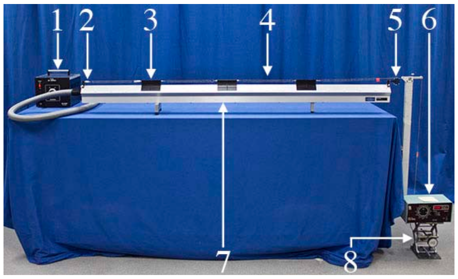
COUPLED HARMONIC OSCILLATORS 3A75.10

Oscillations and Waves   Oscillations   Normal Modes







Concept:

The three-glider system, with identical masses m and spring constants k, will have three normal modes characterized by the angular frequencies:

$$\left\{\omega_1^2, \omega_2^2, \omega_3^2\right\} = \left\{\left(2-\sqrt{2}\right)\frac{k}{m}, 2\frac{k}{m}, \left(2+\sqrt{2}\right)\frac{k}{m}\right\}$$

Mode 2 is the easiest to understand, since the middle glider remains stationary as the outer gliders oscillate about it, out of phase, at frequency ω_2 (see upper right figure). This is conceptually obvious since each outer mass behaves as an independent oscillator attached to two springs with effective spring constant $2k$. In Mode 1, all three gliders move in phase, at ω_1, but the middle glider has a displacement amplitude $\sqrt{2}$ greater than both of the outer gliders (see upper right figure). Mode 3 has the outer gliders moving out of phase with the middle glider. All three gliders oscillate in simple harmonic motion at ω_3, and again the middle glider has displacement amplitude $\sqrt{2}$ greater than both of the outer gliders. An excellent simulation can be found at http://www.falstad.com/coupled/.

Equipment:

1. Air Supply and Hose
2. (7) Glider-Spring Connectors
3. (3) Gliders ($m = 225$ g)
4. (4) Springs ($k = 4 \times 10^3$ g/s^2)
5. Bumper Pulley
6. Harmonic Motion Driver
7. Air Track
8. Lab Jack

Procedure:

1. Turn on the air supply to the air track and the drive switch on the front of the motor.
2. Set the driver frequency to 1.0 Hz to tune in Mode 2 (note that this is a linear frequency, $f = \omega/2\pi$). Notice that the glider system enters the second normal mode.
3. Adjust the driver frequency to explore the two other normal modes for the system ($f_1 = 0.5$ Hz, $f_3 = 1.3$ Hz).
4. Adjust the driver frequency to explore non-normal modes. Notice that the resulting motion is a superposition of the normal modes and beats should occur.