Scaling of critical velocity for bubble raft fracture under tension

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(Dated: 9 September 2011)

The behavior of materials under tension is a rich area of both fluid and solid mechanics. For simple fluids, the breakup of a liquid as it is pulled apart generally exhibits an instability driven, pinch-off type behavior. In contrast, solid materials typically exhibit various forms of fracture under tension. The interaction of these two distinct failure modes is of particular interest for complex fluids, such as foams, pastes, slurries, etc.. The rheological properties of complex fluids are well-known to combine features of solid and fluid behavior, and it is unclear how this translates to their failure under tension. In this paper, we present experimental results for a model complex fluid, a bubble raft. As expected, the system exhibits both pinch-off and fracture when subjected to elongation under constant velocity. We report on the critical velocity $v_c$ below which pinch-off occurs and above which fracture occurs as a function of initial system width $W$, length $L$, bubble size $R$, and fluid viscosity. The fluid viscosity sets the typical time for bubble rearrangements $\tau$. The results for the critical velocity are consistent with a simple scaling law $v_c\tau/L \sim R/W$ that is based on the assumption that fracture is nucleated by the failure of local bubble rearrangements to occur rapidly enough.
I. INTRODUCTION

The rheological behavior of complex fluids exhibits a rich array of phenomenon due to their combination of solid-like and fluid-like properties [for example, see Bird et al. (1977); Edwards et al. (1991); Sollich et al. (1997)]. The response of these materials to applied stresses and strains raises interesting questions in areas ranging from yielding [Mason et al. (1996); Rouyer et al. (2005); Cohen et al. (2006); Moller et al. (2006, 2008); Xu and O’Hern (2006)], to shear-localization [Coussot et al. (2002); Salmon et al. (2003); Lauridsen et al. (2004); Clancy et al. (2006); Katgert et al. (2008)], to creep flow [Vincent-Bonnieu et al. (2006)], to stick-slip behavior [Coussot et al. (2002); Lauridsen et al. (2002)] and jamming [Cates et al. (1998); Liu and Nagel (1998, 2001); Trappe et al. (2001)]. The richness of their rheological response is one reason for the wide range of technological applications. Complex fluids include pastes, slurries, foams, granular materials, emulsions, and colloidal suspensions. Their applications range from fire fighting, printing, paints, foods, drugs and drug delivery, cosmetics, and oil recovery. Therefore, understanding the fundamental elements of their flow behavior and the transition between different flow regimes is critical.

One of the central issues for complex fluids is the transition between predominately fluid-like behavior that is flow dominated and solid-like behavior. Recently, the study of this transition has focused on shear geometries, such as cylindrical and planar Couette flow, and cone and plate geometries. A major question in these geometries is the fundamental mechanisms governing shear localization in which flowing and non-flowing regions of materials coexist (see Dennin (2008); Schall and van Hecke (2010) and the references therein). The coexistence of these two-states raises interesting questions in the context of jamming in complex fluids, such as the possibility of a dynamic transition between a "solid phase" and a "liquid phase" as a function of applied stress or strain. Such a transition may best be described by constituent relations that explicitly treat each region of the material as existing in a separate phase. This is in contrast to the use of constituent relations that are continuous as a function of external parameters, such as a Bingham plastic model. Given the wide range of shear localization that has been observed experimentally [examples include the work in Kabla and Debrégeas (2003); Coussot et al. (2002); Salmon et al. (2003); Lauridsen et al. (2004); Gilbreth et al. (2006); Clancy et al. (2006); Becu et al. (2006); Katgert et al.
(2008); Fall et al. (2009)], and the success of various theoretical models of shear localization [examples include the work in Varnik et al. (2003); Barry et al. (2011); Moller et al. (2008); Cox and Wyn (2008); Cheddadi et al. (2008); Denkov et al. (2009)], understanding this transition between fluid and solid behavior is critical. The wide-range of behavior points to another major question: to what degree is the fundamental physics governing the rheology of complex fluids common to different complex fluids. In addressing all of these questions, it has proven useful to focus on experimental systems that are relatively simple and serve as models for general behavior in complex fluids.

Foams have proven to be a very important model complex fluid [Kraynik and Hansen (1987); Kraynik (1988); Stavans (1993); Prud’homme and Khan (1996); Weaire and Hutzler (1999)]. One of the main reasons is the relative simplicity of the micro-scale physics. Foams are gas bubbles with liquid walls. In general, the main microscale forces are well understood in terms of surface tension of the fluid and internal pressure of the bubbles. This has produced a range of successful theoretical models of foam that focus on different aspects of the underlying physics [see for example, Kraynik and Hansen (1986); Okuzono et al. (1993); Durian (1995); Hutzler et al. (1995); Jiang and Glazier (1996); Brakke (1996); Asipauskas et al. (2003); Cox (2005); Janiaud et al. (2006); Denkov et al. (2009)]. In this paper, we focus on a experimental model foam system that is quasi two-dimensional: bubble rafts.

Bubble rafts are single layers of bubbles floating on the surface of water. They have been of interest since Bragg first proposed them as a model for crystalline and amorphous systems [Bragg (1942); Bragg and Nye (1947)]. One of the advantages of the bubble raft system is the ability to directly track the motion of the bubbles under applied strains or stresses. Bubble rafts have been instrumental in studying a number of the phenomenon already discussed for complex fluids. In this paper, we focus on a different class of deformations than the standard shear deformations: failure modes of a foam under applied tension.

A significant difference between solids and fluids is their response to applied tension that results in extension of the material. Typically, fluids exhibit an instability induced pinch-off. The details of the pinch-off are dependent on the fundamental properties of the fluid, such as surface tension and viscosity. However, the general behavior consists of the shrinkage of a region of the fluid until it reaches a critical width and the material breaks into one or
more pieces [Shi et al. (1994); Eggers (1997)]. In contrast, most bulk solids exhibit some form of fracture under applied tension [Bouchbinder et al. (2010); Broberg (1999)]. Again, the details depend on the material, and can include plastic or brittle fracture. The general behavior involves cracks nucleating in the bulk or at the boundaries and propagating through the material, breaking it into pieces. The question for foams, and other complex fluids, is whether or not one or both of these failure mechanisms occur, and can it be predicted which will dominate.

For bubble rafts, initial studies reported in Arciniaga et al. (2011) established that both pinch-off and fracture were possible under conditions of a constant pulling speed. A dominate factor in determining which mode occurred was the speed at which the system was pulled. However, the system size also played a role. In this paper, we focus on the transition from pinch-off to fracture and demonstrate that this transition is controlled by the pulling speed, systems size, and time for bubble rearrangements. We present a scaling argument for the critical speed at which the system makes the transition from pinch-off to fracture.

II. EXPERIMENTAL METHODS

We utilize two experimental geometries in this study. First, the failure of the system under tension is studied using a rectangular trough in which a pair of opposite walls are pulled apart at constant speed using stepper-motors. A detailed description of the apparatus is provided in Arciniaga et al. (2011). The pulling velocities range from 0.01 mm/s to about 200 mm/s. The initial distance between the moving walls determines the initial length $L$ of the system. The maximum length is limited by the overall size of the trough, the pulling speed, and the pulling distance required to generate failure. The walls are made of polycarbonate, and in general the bubbles wet the walls. Therefore, to fix the initial width, small pieces of polycarbonate of the desired width are attached to the moving walls. This creates a step edge across which the bubbles do not cross. Therefore, a discrete set of initial widths were studied. The combinations of initial width and length provide us with 15 distinct sets of initial geometries.

The dynamics of the raft are recorded using a conventional charge-coupled device (CCD) camera (Pixelink Corporation). Custom developed MATLAB programs are used to perform
FIG. 1. Illustration of the spatial organization of a typical mono-disperse and a typical poly-disperse raft are shown in Fig. 1 (a) and (c), respectively. The Fig. 1 (b) and (d) are the 2D Fourier transform corresponding to (a) and (c). The Fourier transform is used to define the characteristic bubble size.

The image processing to characterize the individual bubble size and failure mode. The failure mode is defined by the evolution of the total void area in the bubble raft as a function of time, as in Arciniaga et al. (2011). For pinch-off, no voids are generated, and the void area is zero. For fracture, the void area grows in time until failure is reached. For some regions of parameter space, there is a mixed-mode where voids nucleate but self-heal (void area returns to zero) before failure. For the purposes of this study, we focus on the onset of fracture, and
FIG. 2. Plot of the angular averaged power spectrum density (PSD) vs. the wavelength of the raft structure. The PSD of the poly-disperse raft (red line) has a wider distribution than the PSD of the mono-disperse raft. The characteristic diameter of the bubbles is defined to be the peak of the PSD.

define the critical velocity, $v_c$, to be the velocity at which the void area is observed to be non-zero at failure.

The bubble raft is formed by flowing compressed nitrogen gas through a needle under a soap solution surface. The bubble size is fixed by the pressure, the needle diameter and the needle depth under the solution. We are able to manufacture both mono-disperse and poly-disperse bubble rafts by this technique. The mono-dispersed bubble raft produces a highly ordered crystalline 2D structure within well-defined domains that are separated by grain boundaries. The poly-dispersed bubble raft produces a highly disordered amorphous 2D structure which is created by fixing the nitrogen pressure, the needle diameter, but oscillating the needle depth under the solution. The images of the typical mono-disperse and poly-disperse bubble rafts are presented in Fig. 1 (a) and (c). Figure 1 (b) shows a 2D Fourier transform pattern indicative of the ordered, hexagonal structure for the mono-dispersed bubble raft; whereas Fig. 1 (d) shows a 2D Fourier transform indicative of the amorphous structure of the poly-disperse raft. We define the characteristic bubble radius from the peak wavevector of the angular averaged Power Spectral Density (PSD) of the 2D Fourier Transform pattern (Fig. 2). It is worth noting that as expected, the PSD peak is broader in the poly-disperse raft than in the mono-disperse raft.

The final experimental control parameter is the viscosity of the soap solution used to
generate the bubbles. We focused on two different soap solutions: 5% Miracle Bubble, 15% glycerol, and 80% deionized water by volume ratio and 5% Miracle Bubble, 32% glycerol, and 63% deionized water by volume. The 32% glycerol solution has about twice the viscosity of the 15% solution. A third solution was studied, 5% Miracle Bubble, 64% glycerol, and 63% deionized water by volume. However, at these high concentrations of glycerol, the differential evaporation of water caused significant changes in the viscosity of the solution on the time-scale of the experiments. Future work will be required to reliably study the higher viscosity systems.

The viscosity was varied as a method of impacting the typical time for a T1 event. A T1 event is a topological rearrangement in a foam in which two neighboring bubbles move apart and the space is filled by two bubbles that were previously next-nearest neighbors. To characterize the typical time for a T1 event to occur, we used oscillatory flow. The bubble raft is formed between two parallel plates, one of which is held fixed and the other is oscillated sinusoidally at constant amplitude and frequency. For sufficiently large amplitudes and low frequencies, isolated T1 events are generated. (See Lundberg et al. (2008) for details of the apparatus and the dynamics of T1 events under oscillation). The importance of T1 events in general for flow are discussed in a number of references. Some examples are Weaire and Hutzler (1999); Dennin (2004); Cohen-Addad et al. (2004); Vaz and Cox (2005); Wang et al. (2007); Durand and Stone (2006); Cox and Wyn (2008). For the data reported in this paper, the T1 time is measured using an amplitude of 10 mm with an oscillation frequency of 0.05 Hz. The average T1 time is measured in the amorphous bubble raft using a characteristic bubble radius of 0.5 mm. We confirmed that the the typical time for a T1 event was independent of the oscillation amplitude and frequency, within a range of values around 10 mm and 0.05 Hz, respectively.

III. SCALING BEHAVIOR

To derive a scaling law for the critical pulling speed, $v_c$, we start with the assumption that the T1 events control the crossover from plastic flow (pinch-off) to fracture through the nucleation of voids. This is based on the experimental observation that the number of bubbles in the system is preserved. Therefore, fracture is not caused by bubbles popping,
and the only other option is bubbles losing contact with their neighbors, as occurs in a T1 event. The argument presented here is expected to apply to the poly-disperse raft, for which grain boundaries are not significant. The dynamics in mono-disperse rafts is expected to be dominated by the motion of grain boundaries. In this case, there are similarities to the T1 dominated situation, in that the motions in a grain boundary can be decomposed into chains of T1 events. However, the motions are highly correlated, and this is expected to impact any scaling behavior.

For a foam, or the bubble raft, the application of external strain causes the bubbles to initially stretch. Eventually, the bubbles are forced to undergo topological rearrangements with neighbors moving apart from each other. When two other bubbles fill in the space created by the bubbles moving apart, the rearrangement is considered a complete T1 event. This is the source of plastic flow in the system. As discussed, the typical time for a T1 event is controlled by the material properties of the system. If the bubbles move apart faster than a critical speed, bubbles are unable to move into the space created from the bubbles moving apart. In this case, the T1 event is unable to complete, and a hole is nucleated in the system. This hole serves as the initiation of fracture. This process is illustrated schematically in Fig. 3.

The fact that the number of bubbles in the system is preserved has an additional consequence. Because the bubble volumes are essentially constant as well, the flow is area-preserving. This means that the divergence of the velocity field is zero. From this, we can assume the following:

\[ |\frac{\partial v_x}{\partial x}| \sim |\frac{\partial v_y}{\partial y}| \]  \hspace{1cm} (1)

Taking the geometry defined in Fig. 3 (lower, left corner), we have the speed in the pulling direction \(v_x\), the speed perpendicular to pulling \(v_y\), the initial length in the pulling direction \(L\), and the initial width perpendicular to pulling \(W\). Additionally, we have the typical time for a T1 event to occur \(\tau\) and the typical bubble radius \(R\). Using the fact that the flow is incompressible, we have a relationship between \(v_x\) and \(v_y\):

\[ \frac{v_x}{L} \sim \frac{v_y}{W}. \]  \hspace{1cm} (2)

In general, \(v_x\) is set by the pulling speed \(v\), and the size of \(v_y\) is set by this relation. We
FIG. 3. Illustration of the role of T1 events in the nucleation of fracture zones. A T1 event occurs when two neighboring bubbles are pulled apart, and two next-nearest neighbors fill the gap and become nearest neighbors. This is the sequence illustrated by the top-right image labeled “no fracture”. In contrast, when the bubbles are moving faster than a critical speed associated with the time for a T1 event to occur, the next-nearest neighbors can not respond in time, and a hole is nucleated in the system. This is illustrated by the lower-right image labeled “fracture”.

are interested in the case when the typical velocity of bubbles moving apart is too great for a T1 event to complete. We will focus on when this occurs for \( v_y \), and the transition from pinch-off to fracture should occur when

\[
v_y \sim \frac{R}{\tau}.
\]  

(3)

Converting this to a condition on \( v_x \), and taking this value of \( v_x \) to be the critical pulling speed \( v_c \), we get

\[
v_c \sim \frac{v_y L}{W} \sim \frac{RL}{\tau W}.
\]  

(4)

This suggests that if we scale \( v_c \) by \( \tau \) and \( L \), we will find that the critical velocity is inversely proportional to the initial width of the system:

\[
\frac{v_c \tau}{L} \sim \frac{R}{W}.
\]  

(5)
FIG. 4. Typical images for pinch-off and fracture of the bubble raft. (a) - (c) are images of a poly-disperse bubble raft with an characteristic bubble radius 0.33 mm. (d) - (f) are images of a mono-disperse bubble raft with the characteristic bubble radius 0.33 mm. The scale bar on the top of each image corresponds to a length of 20 mm. The initial length in each case is 60 mm, and the initial width is 80 mm. Images (a) and (d) are the initial state of the rafts. Images (b) and (e) provide an example of fracture. Images (c) and (f) are examples of pinch-off. The pulling velocities are 2.57 mm/s for (d)(f), 3.42 mm/s for (a)(c)(e), and 4.29 mm/s for (b).

It is worth making two comments on this result. First, this is consistent with our initial results reported in Arciniaga et al. (2011). However, this derivation adds two important elements. It provides a justification for scaling the critical velocity with $L$ and the bubble radius with $W$. Without a physical justification, this choice of scaling is relatively arbitrary. Second, there are a number of time scales that could be considered as relevant for scaling the velocity. This specifically selects the typical time for a T1 event to occur.

IV. RESULTS

Fig. 4 shows a typical poly-disperse (Fig. 4 (a) - (c))) and mono-disperse (Fig. 4 (d) - (f)) bubble raft in its initial state ((a) and (d)) and at different states of failure. The initial raft length and width are 60 mm and 80 mm, respectively, for both cases. The pulling velocities for Fig. 4 are 2.57 mm/s for (d)(f), 3.42 mms for (a)(c)(e), and 4.29 mm/s for
Two features are illustrated by these images. First, for both cases, the higher pulling speed results in fracture. This was already reported in Arciniaga et al. (2011). Second, the boundaries of the highly-ordered, mono-disperse case are not nearly as smooth as the poly-disperse case at failure. This is especially evident when comparing the two final pinch-off states.

Before focusing on the full scaling result, it is useful to consider the critical velocity as a function of the initial raft width for mono-disperse and poly-disperse bubble rafts with a range of characteristic bubble sizes, but the same soap solution (i.e. time for T1 events). This is shown in Fig. 5. For ease of comparison, the critical velocity is scaled by the initial length and the initial width is scaled by the characteristic bubble radius. Three different characteristic sizes of the bubbles are used for both the mono-disperse and poly-disperse raft. As predicted, the data collapse separately for the mono-disperse and poly-disperse systems. A number of interesting features are evident. First, the data for the mono-disperse bubble raft exhibits greater fluctuations (see Fig. 5(b)), especially for larger initial widths. This is consistent with the rougher profiles at pinch-off. The highly discrete nature of the mono-disperse raft suggests that significantly more data is required for proper averaging. Also, the results in Fig. 5(c) emphasize that the critical velocity for the crystalline structure is lower than for the amorphous structure. This suggests that it is easier to separate a typical grain boundary than to nucleate a void through a failed T1 event. Finally, we expect that $v_c \sim R/W$.

Figure 6 shows log-log plots for $v_c$ versus $W/R$ for both the poly-disperse (Fig. 6(a)) and mono-disperse (Fig. 6(b)) cases. In this case, the poly-disperse case agrees with the expected $1/W$ behavior. Interestingly, the mono-disperse case is consistent with power-law behavior, but with an exponent closer to -1.7 than -1.0. It remains to be seen if understanding the collective behavior of the grain boundaries can explain this.

The final element we tested was the dependence of the critical velocity on soap solution. The critical velocity vs. the initial width for 15% and 32% glycerol are presented in Fig. 7. Both measurements are preformed using a poly-disperse bubble raft with a characteristic bubble radius of about 0.5 mm. The critical velocity vs. the initial raft width of the 32% solution shows a similar trend with 15% solution, but the critical velocities for all points are lower. Dimensional analysis is sufficient to suggest scaling the critical velocity by a typical time-scale. There are a number of potential time-scales, but our scaling argument suggests
FIG. 5. $v_c/L$ vs. $W/R$ for different bubble sizes and structures. (a) is for poly-disperse rafts and (b) is for mono-disperse rafts. The symbols distinguish systems with different characteristic radii: (solid squares) 0.3 mm, (solid circles) 0.5 mm, and (solid triangles) 0.75 mm. The initial lengths and widths ranged from 40 mm to 80 mm and 40 mm to 120 mm, and (c) provides a comparison between the poly-disperse and mono-disperse scaled plots.
FIG. 6. Log-log plots for $v_c/L$ vs. $w/R$ for both (a) poly-disperse and (b) mono-disperse. Both systems are consistent with power-laws ($v_c/L \propto (w/R)^m$), though the mono-disperse case is significantly noisier for wider systems. The power-law exponent $m$ is given in each figure. For the poly-disperse case, $m = -1$, as predicted. For the mono-disperse case, the exponent is $m = -1.7$, presumably due to dynamics of the grain boundaries.

FIG. 7. The critical velocity vs. the initial bubble raft width with the different volume ratio of the glycerol: 15% glycerol (black square) and 32% glycerol (red circle). The data are for an initial $L$ of 60 mm and a bubble radius of 0.5 mm. An increase in viscosity results in a consistently lower critical velocity for fracture.

that the typical time for a T1 event is the relevant time-scale.

As described in Sec. II, the characteristic T1 time is measured in a parallel oscillatory geometry. The T1 time corresponds to the three stages of the processes: the detaching of
FIG. 8. The histogram of the time for T1 events to occur in the (a) 15% and (b) 32% glycerol solutions. The T1 event time is measured by counting the time for the single T1 event in the oscillatory geometry. The histograms of 15% and 32% glycerol solution reveal an increase of the characteristic T1 time from 0.36 s to 0.47 s as the viscosity is increased.

the connected bubbles; the intermediate state of an empty space surrounded by four nearby bubbles; and then the attaching for the other two bubbles. The whole T1 process can be observed clearly in the oscillatory shearing bubble raft with a low frequency and a large amplitude setting. There is a distribution of T1 times, and our results are presented in Fig. 8(a) for the 15% glycerol ratio solution and (b) for the 32% glycerol solution. The mean of the distribution is taken as the characteristic T1 time. We find a mean time of 0.36 s and 0.47 s respectively for the 15% glycerol solution and 32% glycerol solutions.

Figure 9 shows the fully scaled critical velocity, $v_c \tau / L$ versus $W/R$, where we take $\tau$ as the average T1 time from Fig. 8. As expected, the data collapses for these two soap solutions. We attempted to test the scaling for even higher concentrations of glycerol. However, as mentioned in Sec. II, there were difficulties controlling the viscosity of the solution as a function of time. The data followed the general trend of a decreased $v_c$ and an increased $\tau$. However, the behavior as a function of $W$ was difficult to reproduce given the time scales involved. Additional work is required to determine if this was simply the result of evaporation changing the viscosity of the solution or if the scaling actually breaks down for sufficiently high viscosities as new mechanisms become important.
FIG. 9. Plot of $v_c\tau/L$ versus $W/R$ for 15% (black squares) and 32% (red circles) solutions. The data includes a range of initial lengths and widths, but a fixed bubble radius of 0.5 mm. The time $\tau$ is the average T1 event time presented in Fig. 8.

V. SUMMARY

By varying the initial size of the bubble rafts (length $L$ and width $W$), the bubble radii $R$, and the effective time for T1 events $\tau$, we have shown that the critical velocity $v_c$ for the transition from fracture to pinch-off is consistent with the relation: $v_c\tau/L \sim R/W$. The derivation of this relation assumes that the T1 events control the nucleation of voids, and the experimental agreement strongly suggests that T1 events play this central role. For amorphous, molecular plastic materials, there is evidence that the failure under tension is controlled by the dynamics of shear-transformation zones [Spaepen (1977); Falk and Langer (1998); Falk (1999); Falk and Langer (2011)], which are regions of non-affine molecular rearrangements. There is evidence from experiments in bubble rafts under shear that the T1 events represent the core of shear-transformation zones [Dennin (2004); Twardos and Dennin (2005); Lundberg et al. (2008)]. The combination of the results presented here and the previous results for shear geometries strongly suggests that a better understanding of the connection between shear-transformation zones and T1 dynamics will be important for understanding the failure of complex fluids under tension.

The comparison of the mono-disperse and poly-disperse systems raise some interesting questions. The most important issue is understanding the connection between the grain boundaries and isolated T1 events. The fact that the mono-disperse systems exhibit con-
sistently lower values for $v_c$ is clear evidence of a lower nucleation energy for voids along a grain boundary. However, there is also the different scaling exponent with bubble radius and system width. If one could explain both of these with either an interaction between T1 events or a version of a shear-transformation zone model, this would be an important step forward in our understanding of these systems.

One issue that requires additional study is the scaling with the time for a T1 event to occur. At this point, we have only tested two different systems, so further work is needed in this direction. Certainly, most of the relevant time-scales in the problem will be functions of each other. So, uniquely distinguishing between the time scale for T1 events and other time scales requires additional work. However, at this point, the results are certainly consistent with the T1 event time as the relevant scaling time. One experimental challenge is gaining direct control of the T1 event time independent of the bubble size. The use of glycerol to produce higher viscosity systems faced experimental challenges related to evaporation that were already discussed. Also, there is the possibility that at high enough solution viscosity, there are additional effects that enter. The critical velocity appears to become more independent of system size. This may be due to effects introduced by the very high viscosities, including changes in the bubble-bubble attractive forces.

Finally, it is worth noting that the failure of foams under over-pressure has also been studied [Arif et al. (2010)] with a focus on crack propagation. In this geometry, one also observes a velocity dependent transition between different failure modes. In this case, a transition from plastic deformation and brittle fracture is reported in Arif et al. (2010). Though there are important difference in the geometry and underlying microscopic mechanisms (such as film rupture) between the results reported here and in Arif et al. (2010), it is interesting to speculate on the potential connections between the different failure modes and the implications for complex fluids in general.

ACKNOWLEDGMENTS

We acknowledge the support of NSF-DMR-0907212 and Research Corporation. We also thank Corey O’Hern, Simon Cox, Peter Taborek, and Sascha Hilgenfeldt for useful discussions.
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