Linear Algebra

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■ Vectors

■ Vectors are given as lists

```
In[79]:= v := {a, b, c}

In[80]:= w := {x, y, z}

In[81]:= k v

Out[81]= {a k, b k, c k}
```

```
In[82]:= v + w

Out[82]= {a + x, b + y, c + z}
```

- **Functions are applied to each element in a vector**
  Scalar Dot Product

```
In[83]:= v . w

Out[83]= a x + b y + c z
```

- **Vector element**

```
In[84]:= v[[3]]

Out[84]= c
```

- **Ways to generate vectors**

```
In[85]:= Array[d, 6]

Out[85]= {d[1], d[2], d[3], d[4], d[5], d[6]}

In[86]:= Table[i + 6, {i, 5}]

Out[86]= {7, 8, 9, 10, 11}
```

■ Matrices

- **Matrices are given by**

```
In[87]:= m = {{m11, m12}, {m21, m22}}

Out[87]= {{m11, m12}, {m21, m22}}

In[88]:= MatrixForm[m]
```

Out[88]//MatrixForm=

$$\begin{pmatrix} m11 & m12 \\ m21 & m22 \end{pmatrix}$$

- **To generate a matrix**

```
In[89]:= A := Array[f, {3, 3}]
```

```
In[90]:= MatrixForm[A]
```

Out[90]//MatrixForm=

$$\begin{pmatrix} f[1,1] & f[1,2] & f[1,3] \\ f[2,1] & f[2,2] & f[2,3] \\ f[3,1] & f[3,2] & f[3,3] \end{pmatrix}$$

```
In[91]:= T := Table[3 i - 3 + j, {i, 3}, {j, 3}]

In[92]:= H := MatrixForm[T]

In[93]:= H
```

Out[93]//MatrixForm=

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

```
In[94]:= d := DiagonalMatrix[{a1, a2, a3}]

In[95]:= MatrixForm[d]
```

Out[95]//MatrixForm=

$$\begin{pmatrix} a1 & 0 & 0 \\ 0 & a2 & 0 \\ 0 & 0 & a3 \end{pmatrix}$$

```
In[96]:= iden := IdentityMatrix[3]

In[97]:= MatrixForm[iden]
```

Out[97]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
In[98]:= MatrixForm[Transpose[T]]
```

Out[98]//MatrixForm=

$$\begin{pmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{pmatrix}$$

```
In[99]:= T

Out[99]= {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}

In[100]:= T⟦2, 3⟧

Out[100]= 6
```

Third row

```
In[101]:= T[[3]]

Out[101]= {7, 8, 9}
```

Third column

```
In[102]:= Transpose[T][[3]]

Out[102]= {3, 6, 9}

In[103]:= Range[10]

Out[103]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

■ Matrix multiplication

```
In[104]:= m

Out[104]= {{m11, m12}, {m21, m22}}

In[105]:= p := {{p11, p12}, {p21, p22}}

In[106]:= MatrixForm[m . p]
```

Out[106]//MatrixForm=

$$\begin{pmatrix} m11\ p11 + m12\ p21 & m11\ p12 + m12\ p22 \\ m21\ p11 + m22\ p21 & m21\ p12 + m22\ p22 \end{pmatrix}$$

■ Matrix products with vectors

```
In[107]:= x := {x1, x2}

In[108]:= MatrixForm[m . x]
```

Out[108]//MatrixForm=

$$\begin{pmatrix} m11\ x1 + m12\ x2 \\ m21\ x1 + m22\ x2 \end{pmatrix}$$

```
In[109]:= MatrixForm[x . m]
```

Out[109]//MatrixForm=

$$\begin{pmatrix} m11\ x1 + m21\ x2 \\ m12\ x1 + m22\ x2 \end{pmatrix}$$

```
In[110]:= y := {y1, y2}

In[111]:= x . m . y

Out[111]= (m11 x1 + m21 x2) y1 + (m12 x1 + m22 x2) y2

In[112]:= Inverse[m]
```

Out[112]= $$\left\{\left\{\frac{m22}{-m12\ m21 + m11\ m22}, -\frac{m12}{-m12\ m21 + m11\ m22}\right\}, \left\{-\frac{m21}{-m12\ m21 + m11\ m22}, \frac{m11}{-m12\ m21 + m11\ m22}\right\}\right\}$$

In[113]:= **Det[m]**

Out[113]= $-m12\, m21 + m11\, m22$

In[114]:= **Det[T]**

Out[114]= 0

- **The trace is given by the sum over diagonal elements**

In[115]:= **tr :=** $\sum_{i=1}^{3}$ **T[[i, i]]**

In[116]:= **tr**

Out[116]= 15

■ Eigenvalues and Eigenvectors

In[117]:= **Eigenvalues[T]**

Out[117]= $\left\{0,\ \frac{3}{2}\left(5-\sqrt{33}\right),\ \frac{3}{2}\left(5+\sqrt{33}\right)\right\}$

In[118]:= **evt := Eigenvectors[T]**

In[119]:= **evt**

Out[119]= $\left\{\{1,\ -2,\ 1\},\ \left\{-\frac{15-\sqrt{33}}{-33+7\sqrt{33}},\ \frac{4\left(-6+\sqrt{33}\right)}{-33+7\sqrt{33}},\ 1\right\},\ \left\{-\frac{-15-\sqrt{33}}{33+7\sqrt{33}},\ \frac{4\left(6+\sqrt{33}\right)}{33+7\sqrt{33}},\ 1\right\}\right\}$

In[120]:= **<< "LinearAlgebra`Orthogonalization`"**

In[121]:= **nevt := N[evt]**

In[122]:= **nevt**

Out[122]= {{1., -2., 1.}, {-1.28335, -0.141675, 1.}, {0.283349, 0.641675, 1.}}

In[123]:= **GramSchmidt[nevt]**

Out[123]= {{0.408248, -0.816497, 0.408248}, {-0.78583, -0.0867513, 0.612328},
{0.464547, 0.570796, 0.677044}}

In[124]:= **Normalize[nevt[[1]]]**

Out[124]= {0.408248, -0.816497, 0.408248}

In[125]:= **Projection[nevt[[1]], nevt[[2]]]**

Out[125]= {0., 0., 0.}

In[127]:= **cp := CharacteristicPolynomial[T, eig]**

```
In[128]:= cp
```

Out[128]= $18\,\text{eig} + 15\,\text{eig}^2 - \text{eig}^3$

```
In[129]:= Roots[cp == 0, eig]
```

Out[129]= $\text{eig} == \frac{3}{2}\left(5 - \sqrt{33}\right) \,||\, \text{eig} == \frac{3}{2}\left(5 + \sqrt{33}\right) \,||\, \text{eig} == 0$

```
In[130]:= Factor[cp]
```

Out[130]= $-\text{eig}\,(-18 - 15\,\text{eig} + \text{eig}^2)$

```
In[131]:= m

Out[131]= {{m11, m12}, {m21, m22}}
```

■ Linear Equations m x = vec

```
In[132]:= ? LinearSolve

        LinearSolve[m, b] finds an x which solves the matrix equation m.x==b.

In[133]:= vec := {vec1, vec2}

In[134]:= LinearSolve[m, vec]
```

Out[134]= $\left\{\frac{\text{m22 vec1} - \text{m12 vec2}}{-\text{m12 m21} + \text{m11 m22}}, \frac{-\text{m21 vec1} + \text{m11 vec2}}{-\text{m12 m21} + \text{m11 m22}}\right\}$