A Dual Axion vs. Gravity

MAIN REFS, Dvali et al. { hep-th/0507215, 1312.7273 } BASIC FORMALISM, USE ν # (not in this talk)

OTHER REFS :

- Kallosh, (Linde)^2, Susskind hep-th/9502069
  ORIGINAL ATTEMPT, BUT "GAVE UP TOO SOON"
- TOPOLOGICAL NATURE of THIS PHENOMENON & RELATIONS TO 2d & 3d THEORIES ARE SPELLED OUT IN

  Luscher Phys. Lett B78 ('78) 465
  Dvali, Jackiw, Pi hep-th/0511175

Caveat : I will be sloppy w/ prefactors, signs, etc
Fortunately, it won't matter.

Invitation : the STRONG CP PROBLEM : why is $\langle G\tilde{G}\rangle \ll 1$?

- QCD GENERATES NON PERTURBATIVE $\theta_{QCD}\, G\tilde{G}$
- also picks up (arg det m_q) $G\tilde{G}$

} why is sum of these ≈ 0?

A nice solution: PQ GLOBAL SYMMETRY ... broken in every which way.

~~ANOMALOUS w.r.t. QCD → $\frac{a}{f}G\tilde{G}$~~

SPONTANEOUSLY BROKEN $\longrightarrow$ Goldstone boson : a, AXION

ANOMALOUS $\longrightarrow$ $\frac{a}{f} G\tilde{G}$ COUPLING , breaks SHIFT SYM

WORK OUT POTENTIAL for $a \rightarrow \langle a\rangle$ CANCELS θ!
"symmetry" reason for $\theta = 0$.

<u>a fly in the ointment</u> : black holes eat global charges

there is a 'folk theorem' that gravity breaks global symmetries non-perturbatively

↳ eg imagine throwing an electron into a black hole. electric charge (GAUGED) is preserved, but lepton # is not.

∃ hep-th "calculations" w/ path integrals over spacetime topology ... I HAVE NOTHING TO SAY ABOUT THIS.

from the point of view of an effective field theorist ... I DON'T KNOW WHAT TO DO.

Tim's question: AFTER SHIFTING AXION (eliminating $G\tilde{G}$)

$$\mathcal{L}_a = (\partial a)^2 + \frac{a}{f} G\tilde{G} - \underbrace{V(a)}$$

from gravity (EXPLICIT BREAKING OF PQ)

what is the form of $V(a)$?

eg DO I INCLUDE A $\sim M_{Pl}^2 a^2$ term?

... even <u>irrelevant</u> terms are dangerous because (f/M_{Pl}) is not that small & the expt'l bound is very tight: $\theta \lesssim 10^{-9}$.

it seems like the usual EFT parameterization doesn't work (OR AT LEAST GIVES A DISCOURAGING ESTIMATE)

Quick Review : EM / Poincaré duality

in absence of sources, $\vec{E} \leftrightarrow \pm \vec{B}$

form notation: $dF = 0$ $\leftrightarrow$ Bianchi
$d{*}F = {*}J$ $\leftrightarrow$ EoM from action principle

given a Gauge potential, A (1-form)

$$Z = \int \mathcal{D}A \; e^{iS[F]}$$ ← assuming no $A \wedge J$ term

$$= \int \mathcal{D}F \, \mathcal{D}V \; e^{i\int \frac{1}{2g^2} F\wedge {*}F \; + \; i\int V \wedge dF}$$

probably $d{*}F$

I want to write as PI over F

introduce LAGRANGE MULTIPLIER (1-form) to enforce BIANCHI IDENTITY
not (yet) dynamical, will become DUAL POTENTIAL

BUT NOW I CAN DO PI OVER $\mathcal{D}F$

EoM: $\frac{1}{g^2} {*}F + dV = 0$ (sloppy w/ signs, #'s)

then $G \equiv dV = -\frac{1}{g^2} {*}F$

↑ ~ dual field strength
... V is becoming dynamical
to make up for the fact that we're integrating out dynamical dof

$$Z = \int \mathcal{D}V \; e^{i\int -\frac{g^2}{2} G\wedge {*}G \; + \; V\wedge dF}$$

$V \wedge dF \longrightarrow V \wedge J_{MAG}$ ← magnetic monopoles

Caveats

ASSUMED NO ELECTRIC CURRENT → but duality still holds in the presence of electrons in the ELECTRIC theory? [ASK YURI)

hack: puncture your spacetime to remove worldlines of electrons (POINT PARTICLES)

then: you get topology when considering Wilson loops around such a world line (non-contractible)

↳ but this is a different talk.

THERE'S ALSO AN ISSUE ABOUT LOCALITY
magnetic sources are nonlocal (MONOPOLES) in ELEC. THY

↳ have to be careful in our axion story that our 'general' statements ~~are~~ about the form of the axion potential are really general for local interactions.

[P.4b ⇨]

ASIDE: more careful treatment of currents in the Dvali-Jackiw-Pi paper

↳ deals w/ issues of whether dual current is properly conserved, avoids issues by appealing to topology

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THE POINT: p-form gauge field $A^{(p)} \to (p+1)$-form field str. $F^{(p+1)} = dA^{(p)}$

Hodge duality: $*F^{(p+1)}$ is a $(D-p-1)$-form field strength for dual gauge pot, which must be a $(D-p-2)$-form

so $\boxed{A^{(p)} \longleftrightarrow V^{(D-p-2)}}$

<u>special case</u>: $p = D-2$, dual potential is a 0-form (<u>SCALAR</u>)

↳ in fact, this scalar has a <u>shift symmetry</u> (all that is left of 'gauge' invariance)

⇒ derivatively coupled. ⇐ Goldstone!

$$\boxed{\text{so in } D=4, \text{ our axion (0-form) is dual to a 2-form } B^{(2)}}$$

Kalb-Ramond antisym. 2-tensor <u>GAUGE</u> field

$B^{(2)} \to B^{(2)} + d\Lambda^{(1)}$

the original hope of the kallosh et al. paper was that this larger gauge symmetry could explain protection from gravity

Dvali is arguing that it sort of does, but not in the obvious way (we'll see how)

<u>special case 2</u>: $p = D-1$

then $F^{(D)}$ is a D-form, $*F^{(D)}$ is a 0-form
⇒ dual field strength is not the d of anything!

INTERPRETATION: the $(D-1)$-form never had any MASSLESS propagating degrees of freedom.

CAN ALSO SEE FROM COUNTING ON-SHELL DOF (taking into account GAUGE REDUNDANCY) ⇐ $\binom{D-2}{p-1}$

so 3-forms in $D=4$ are kind of weird
⇑
We will make use of this!

biggest caveat: ASSUMED MASSLESS GAUGE FIELDS

↳ "of course" since masses break the gauge group → cf HIGGS MECHANISM FOR 1-FORMS

but: we know the AXION PICKS UP A MASS
it has to because its vev must be stabilized to some value to cancel Θ

↳ is there any hope for a faithful dual description?

HOW DOES $B^{(2)}$ reflect axion mass?

Naive: $W^{(1)}$ eats $h^{(0)}$ ← breaks sym?

$$\begin{matrix} W^{(1)} & \text{eats} & h^{(0)} \\ \updownarrow & & \updownarrow \\ B^{(2)} & & ?? \end{matrix}$$

?? ← new dof? not in axion thy.

in general: (p+1)-form can eat p-form to acquire longitudinal polarization

so why not try 3-form?

$$\begin{matrix} C^{(3)} & \text{eats} & B^{(2)} & \to & \text{MASSIVE } C^{(3)} \\ \uparrow & & \uparrow & & \uparrow \\ 0 \text{ DOF} & & 1 \text{ DOF} & & 1 \text{ DOF} \end{matrix}$$

(DUAL TO MASSIVE AXION?)

HOW IT WORKS

$\mathcal{L}_B = dB \wedge * dB$

GAUGE SYM: $B \to B + d\Lambda$

also has a GLOBAL SHIFT SYM

$$\boxed{B \to B + \Omega^{(2)}}$$

↑ any constant 2-form.

gauge this symmetry, introduce 3-form C

$C \to C + d\Omega$

↑ now promote to local 3-form

field strength $E = dC$

$$\mathcal{L} = \# m^2 (dB - C)^2 + \tfrac{1}{2} F^2$$

← squared = $\cdot \wedge * \cdot$

<u>Note</u> : we have <u>preserved</u> B's original gauge sym.
& also GIVEN A MASS TO THE B dof

by "being eaten" rather than "eating"

w/ no new dof.

is THIS REALLY DUAL TO MASSIVE AXION?

<u>same trick</u>: $\mathcal{L}$ multiplier, integ out dB

uf, I'm being sloppy

$$\mathcal{L} = \# m^2 (dB - C)^2 - \tfrac{1}{2} F^2 + \# \Lambda^2 a \, d{*}dB$$

axion = LAGR. MULTIPLIER

INTEGRATE OUT (dB)

CHECK

$$\mathcal{L} = \frac{\Lambda^4}{2m^2} (\partial a)^2 - \Lambda^2 a \, dC - \tfrac{1}{2} F^2$$

NOW INTEGRATE OUT C, get axion EOM

$$\Lambda^2 \partial^2 a + m^2 (a - \text{const}) = 0$$

indeed, massive axion.

ok: so what can we do with this?

our main goal is the strong CP problem.

We can reformulate in 3-form language
(NOTE: NOT DUALIZING – we're just using the geometric structure that already exists in QCD)

the θ term: $\theta \int$ ~~[illegible]~~ $G \wedge G$

note: cf KINETIC TERM
$\int G \wedge * G = \langle G, G \rangle$
REQUIRES METRIC

this term is metric-independent
so you know it's topological

in fact: $G \wedge G = \boxed{d\, C^{(3)} = E^{(4)}}$

CHERN SIMONS 3-form $F \wedge A - \frac{1}{3} A \wedge A \wedge A$

C is like a "composite" gauge field
for the FIELD STRENGTH E, whose electric field
is the CP violating term! NOTE: no propagating dof, but still a long-range field.

In fact: this is a 4D analog of Polyakov's argument that in 3D instantons (MONOPOLES) generate confinement.

so: QCD has a massless 3-form field (w/ no dof) that mediates a long range force $E^{(4)}$ in the vacuum. (imagine 2-brane sources @ infinity)

$\boxed{\text{the value of the field} \leftrightarrow \mathrm{Tr}\, G\tilde{G}}$

so CP problem is precisely the "long rangedness" of this field.

The Axion solution in dual picture

PROBLEM: $\langle E, E \rangle \neq 0$ (const)

$$\downarrow$$

$\langle C, C \rangle_{q \to 0} \sim 1/q^2$ ← as 3-form has massless pole.

(SAME AS KOGUT-SUSSKIND POLE IN U(1) PROBLEM FOR η')

how to turn off this long range force?

↳ Higgs it, SCREEN THE FORCE.

with what shall I higgs it with?
WE ALREADY SPELLED IT OUT!

↳ C can eat the dual axion!

BECOMES MASSIVE, turns off $E \leftrightarrow \boxed{\langle G\tilde{G} \rangle = 0}$

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alternate POINT of view.

Θ-dependent theory:

$\mathcal{L} = \frac{1}{\Lambda^4} E^2 + \dots$

↑ gives $q^2 = 0$ pole for C

NOW USE EFT + TOPOLOGY TO DETERMINE EXISTENCE of MASS GAP
→ sufficient for solution to strong CP
w/o details of higher Θ terms!

↑ this is key: we're not parameterizing grav. corrections — we're saying whether or not they break the axion solution.

INTRODUCE ANOMALOUS CURRENT

$$\partial_\mu J^\mu = F\tilde{F} = E \qquad \longleftrightarrow \partial^2 a \sim E$$

eg $J_\mu = f\partial_\mu a$

then [claim] ANOMALY GENERATES

$$\mathcal{L} = \frac{1}{\Lambda^4}E^2 + \frac{E}{\Lambda^2}\frac{\partial_\nu}{\partial^2}J^\mu \longleftrightarrow$$

$\hookrightarrow$ EOM: $\partial_\nu E = -\Lambda^2 \frac{\partial_\nu\partial_\mu}{\partial^2}J^\mu$

$$\boxed{\Box E = -\Lambda^2 E}$$

E PROPAGATES MASSIVE FIELD
$\partial^2 = 0$ POLE REMOVED
physics is θ - indep.

remark: J_μ didn't have to be PQ current

eg. another solution to strong CP problem is $m_q = 0$ for some quark.

then: $J_\mu = \bar{Q}\gamma_\mu\gamma_5 Q$

axion is replaced by η' !

Gravity: so we showed how axion solution works, what about how it might FAIL?

START W/ AXION

$$\mathcal{L} = (\partial a)^2 + \frac{a}{f}E + \frac{1}{\Lambda^4}E^2$$

GO TO DUAL DESCRIPTION

we already saw

$$\mathcal{L} = \frac{1}{f}(C - dB)^2 + \frac{1}{\Lambda^4}E^2$$

GRAVITY MUST RESPECT GAUGE SYM $\begin{cases} C \to C + d\Omega \\ B \to B + \Omega \end{cases}$

the only way to spoil the (dual) axion story is to UNHIGGS the system!

HOW CAN THIS BE DONE?

ONLY: if $\exists$ a new 3-form to eat [part] of B

$\hookrightarrow$ only candidate is GRAV. CS term: $C_G = \Gamma d\Gamma - \frac{3}{2}\Gamma\Gamma\Gamma$

$$d\,C_G = \underset{(!)}{\cancel{R\wedge R}}^{\;R\wedge R} \equiv E_G$$

Then: $\mathcal{L} = \frac{1}{\Lambda^4}E^2 + \frac{1}{\Lambda_G^4}E_G^2 + \frac{1}{f}(\alpha C + \alpha_G C_G - dB)^2$

only a specific lin comb of C & C_G ARE EATEN!

$\Rightarrow$ REMAINING PART OF C STILL GIVES A LONG RANGE E $\Rightarrow$ physical Θ term!

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So what can we do?

Well, now the problem is: NOT ENOUGH FOOD FOR THE 3-FORMS!

SO SIMPLE PROPOSAL: find more 2-forms to feed them!

anomalous U(1)s that are PQ-like

in the model-building part of the paper, Dvali suggests neutrino #.

↳ WE WON'T DISCUSS, unfortunately

BUT the punchline is really neat:

the safety of the PQ solution to STRONG CP PROBLEM sets a bound on the LIGHTEST neutrino MASS

↳ relating 2 a-priori different types of physics!

DUALITY w/ ARB. AXION POTENTIAL

in dual thy w/ $\overset{\text{dual}}{\text{axion}}$ solution:

$$\mathcal{L} = \frac{\Lambda^4}{24} K\left(\frac{E}{\Lambda^2}\right) + 12 m^2 (dB - C)^2$$

HIGGSING STORY

$K = E^2/\Lambda^4$ to L.O. but QCD in general generates other NP terms!

do they spoil PQ??

DUAL

$$\mathcal{L} = \frac{\Lambda^4}{2M^2} (\partial a)^2 - \Lambda^2 V(a)$$

for DILUTE INSTANTON GAS APPROXIMATION:
$V(a) = \cos(a - \text{const})$
$K(F) = F \sin^{-1}(F) + \sqrt{1-F^2}$

s.t. $\dfrac{dV}{da} = -E = -\Lambda \left(K'(\text{const} - a)\right)^{-1}$ (inverse)

So: no matter what the form of K, E vanishes @ the minimum of the axion potential!

~~is this true~~ what constraint does this set on $V(a)$?

check: $\mathcal{L} = \frac{\Lambda^4}{24} K\left(\frac{E}{\Lambda^2}\right) + 12M^2(dB - C)^2 + \frac{\Lambda^2}{24} a \, \varepsilon^{\nu\lambda\rho\sigma} \partial_{[\nu} \partial B_{\lambda\rho\sigma]}$

is this $a\, d{*}dB$?

EoM: $\partial^2 a + M \,\mathrm{Inv}[K'(\text{const} - a)] = 0$

So: $V(\text{const} - a) = -\Lambda^2 \int \mathrm{Inv}[K'[\text{const} - a]]\, da$