

Two-Headed Giraffes on Tricycles

Two branes, a long throat, and wrapped S^3 s
(RS from warped compactifications)

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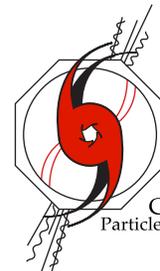
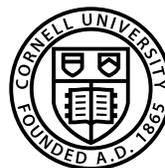
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Prompt

Warped compactifications of type IIB string theory from ten dimensions down to four dimensions play a central role in string theory. At the same time, five-dimensional warped geometries are a powerful tool for addressing the hierarchy problem and modeling the physics of the electroweak scale. This A Exam question considers the relationship between these two classes of warped geometries.

1. Describe warped compactifications of type IIB string theory, following the first three sections of [1]. Explain which 10-dimensional stress-energy sources are responsible for creating the warping.
2. Explain how placing D3-branes at the tip of a cone over an Einstein manifold X_5 creates the warped geometry $\text{AdS}_5 \times X_5$. (Note that \mathbb{R}^6 is a cone over S^5 , so that the geometry serving as the canonical example of AdS/CFT is a special case of this result.)
3. Now sketch how with the inclusion of additional sources one can obtain a ten-dimensional solution containing a finite warped throat region. Focus on the Klebanov-Strassler solution truncated at a large value of the radial coordinate of the cone.
4. Finally, connect this compactification to RS models by explaining which aspects of the string compactification correspond to IR and UV branes. This leads us to the question of primary physical interest: what sort of ‘substructure’ does string theory provide for what appear to be singular objects in the five-dimensional description?

Note: the Klebanov-Strassler solution is somewhat involved, and for the purpose of this exam it is sufficient to focus on the physics in the infrared (the “tip”) and at large radial distance, avoiding some of the complexity of the intermediate region.



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1 Introduction

I know. Compared to the other two 50+ page write ups I've submitted for this exam, this looks really scrawny. The irony is that I spent the most time preparing this prompt.

Well, if you're still reading then this paper's irreverent title hasn't dazzled you into passing me without actually reading this document. It was worth a shot. I guess we should get to some physics.

Two of the biggest developments in theoretical physics in the late 1990s were (i) the Randall-Sundrum (RS) model of a warped extra dimension and (ii) the AdS/CFT conjecture connecting D dimensional gravity to $(D - 1)$ dimensional gauge theory. The RS model was introduced as a 'phenomenological' (a word which we define relative to string theory) model of naturalizing the electroweak hierarchy, though it was quickly realized that it could be interpreted holographically as the five-dimensional dual of a strongly-coupled four-dimensional gauge theory. These two ideas went on to dominate a significant fraction of theoretical high energy physics in the following decade.

Despite being heuristically very similar, it took some time before an actual string realization of the RS could be constructed. The issue here is that the RS1 model is a slice of AdS_5 space with boundaries formed by two solitonic branes. For the most part, phenomenologists needn't worry about the ultraviolet completion of the 5D theory that produced these branes (e.g. are these branes the same thing as stringy Dp branes?) in order to use the effective theory to discuss TeV-scale phenomenology. It turned out that the scaffolding required to microscopically (i.e. string-ily) support such a 'low energy' theory in a meaningful way (i.e. with a well-understood gauge dual) is non-trivial. This required (1) knowing how to deform Maldacena's original $\text{AdS}_5 \times S^5$ construction to include the RG running in the gauge theory, (2) breaking the excess supersymmetries, and (3) generating the warped region with appropriate cutoffs corresponding to the Randall-Sundrum IR and UV branes.

In this A-exam we will *heuristically* describe how this structure can be realized in string theory. We will see that the IR region can be described by Klebanov and Strassler's "warped deformed conifold" [2] while the UV region warping is described by the construction by Giddings, Polchinski, and Kachru [1]. We will further remark that this construction is holographic to a so-called 'duality cascade' that is cut-off by a confinement scale that is given by the conifold deformation.

We will not attempt to properly acknowledge original literature. A brief half-hearted literature review is provided in Appendix B. We will assume that the reader has the appropriate background in Randall-Sundrum model so that we will not discuss 'phenomenology.' Further, we will assume that the reader has a background in AdS/CFT and string theory appropriate to the level at which they would like to follow this paper. And really, between you and me, *someone* ought to know string theory.

Unlike my other A-exam write ups, this is meant to be a *qualitative* paper meant to supplement my oral examination... which will also be qualitative. As for any detailed derivations, I take the same stance that my [Russian born] freshman math professor took about homework¹: "*Homework? I sheyit on your homework!*"

¹We later discovered that he meant to say that he makes a 'spread sheet' of our homework grades.

2 The big picture

Let's provide a framework by summarizing the 'big picture' in bullet form. (This is the A-exam equivalent of a montage².)

- **Maldacena.** The original AdS/CFT correspondence related a gravitational theory on $\text{AdS}_5 \times \text{S}^5$ to $\mathcal{N} = 4$ superconformal field theory. This is way too symmetric. Because we know that the isometry group of the gravity theory is related to the internal symmetry of the gauge theory, we would like to find ways to modify the S^5 into a less symmetric space where still have a handle on the gauge theory³.
- **KW.** Klebanov and Witten found that if the extra dimensions form a conifold, $\text{AdS}^5 \times \text{T}^{1,1}$, then one can break most of the supersymmetries. This turns out to be dual to $\mathcal{N} = 1$ superconformal field theory. We'll say a few more words (but not that much more) about $\text{T}^{1,1}$ below. This will be as far as we will break the supersymmetry. As long as the theory is superconformal, however, there will be no RG running and we will not have anything RS-like (where we recall that the dual picture of RS associates the warping with RG flow)
- **KT.** Klebanov and Tseytlin found that adding fluxes into the mix (from wrapped $D5$ branes) generates the necessary back reaction to produce the desired 'warped throat'. We are thus reduced to $\mathcal{N} = 1$ super Yang-Mills. This is pretty good, but the geometry contains a naked curvature singularity at the tip of the cone. In RS language we would say that there is no IR brane.
- **KS.** Klebanov and Strassler then smoothed out the tip of the conifold by blowing up the S^3 at the tip to produce the **deformed conifold**. We'll discuss this much more below, but the point was that this gets rid of the naked singularity and provides the desired structure for the IR brane and has a remarkable description in terms of a 'cascading' super QCD theory. One remaining item for a "realistic⁴" string realization is that the UV end of the conifold must be attached to a compact manifold. (It's much easier to work with *noncompact* conifolds as the tips of compactified conifolds.)
- **GKP.** Finally, Giddings, Polchinski, and Kachru included the deformed conifold as an appendage to a Calabi-Yau flux compactification. As we shall see, GKP construction also provides a way to generate the hierarchy in scales between the IR (KS tip) and the UV (compact manifold) branes.

3 The conifold

Calabi-Yau manifolds are typically non-singular, but at special parameter values they can be forced to develop singularities. One fairly generic class of singularity are conifolds, which are

²<http://www.southparkstudios.com/clips/collections/rock/185666>

³This is a non-trivial step since symmetries are precisely what help us get a handle on the gauge theory.

⁴Yes, these quotation marks are dripping with sarcasm.

defined by

$$\sum_i (z^i)^2 = 0 \tag{3.1}$$

for $z \in \mathbb{C}^4$. Note that here ‘squared’ really does mean *squared*, not modulus-squared. (This will be important for R symmetry.) This equation defines a cone, which can be seen straightforwardly from noting that any solution can be mapped to another solution via $z^i \rightarrow \lambda z^i$ for each i . Further, the base of the cone is topologically $S^2 \times S^3$, which can be seen less-straightforwardly by writing out the above equation in terms of real coordinates. The Ricci flat and Kähler metric on this space is that of a cone,

$$ds^2 = dr^2 + r^2 d\Omega_{T^{1,1}}^2 \tag{3.2}$$

We’re not going to go into the gory details of $d\Omega_{T^{1,1}}^2$, but an important relation is that the conical radius, r , is related to the radius of the S^3 , ρ , via

$$r \sim \rho^{2/3}. \tag{3.3}$$

In the KS solution the conifold is ‘deformed’ to smooth out the singularity at the tip. “Deformed” is actually a technical term for blowing up the S^3 at the tip, as illustrated on the left of Fig. 1. The defining equation for the deformed conifold is

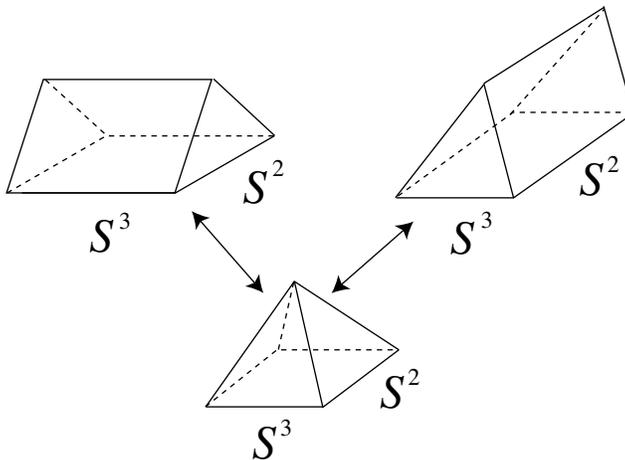


Figure 1: Deformed versus resolved conifold. From [3]

$$\sum_i (z^i)^2 = \epsilon^2. \tag{3.4}$$

We will discuss the origin of this deformation below. For the record, smoothing out the singularity by instead blowing up the S^2 is called ‘resolving’ the conifold. We will not make use of this latter case.

4 Sources of warping and running

Let us now discuss how we generate the warped throat region which plays the role of the RS1 bulk.

4.1 Avoiding a no-go theorem

As one can guess from Maldacena's $\text{AdS}_5 \times \text{S}^5$ construction, this warping comes from the inclusion of extra sources in the stress-energy tensor. We would ultimately like to generate an warped spacetime metric with an RS-like region of the form

$$ds_{10}^2 = e^{2A} dx_{\text{Mink.}}^2 + e^{-2A} (dr^2 + r^2 d\Omega_{\mathbb{T}^{1,1}}^2). \quad (4.1)$$

The Einstein equation for takes the form

$$R_{MN} = \kappa_{10}^2 \left(T_{MN} - \frac{1}{8} g_{MN} T \right) \quad (4.2)$$

where the stress-energy tensor has contributions from the usual supergravity fields in addition to any localized objects $T_{MN} = T_{MN}^{\text{su-gra}} + T_{MN}^{\text{loc}}$. Plugging in the metric ansatz and taking the trace, we obtain an equation for the warp factor

$$\tilde{\nabla} e^{4A} = e^{2A} \frac{G_{mnp} G^{mnp}}{12 \text{Im}\tau} + e^{-6A} [\partial_m \alpha \partial^m \alpha + \partial_m e^{4A} \partial^m e^{4A}] + \frac{\kappa_{10}^2}{2} e^{2A} (T_m^m - T_\mu^\mu)^{\text{loc}}. \quad (4.3)$$

This is the ugliest equation that I have the intestinal fortitude to discuss in this paper. G_{mnp} is a three-form flux (related to the ‘usual’ IIB supergravity fields via $G_{(3)} = F_{(3)} - \tau H_{(3)}$ with τ the axion/dilaton) where we have introduced the notation of lower-case Roman letters from the middle of the alphabet for internal manifold indices. The last term is a trace over the stress-energy tensor of the localized sources.

Let us remark upon some salient features. First of all the first term tells us that the fluxes source the warping. We will say more about the origin of these fluxes in terms of $D3$ branes. Both this term and the second term are positive definite. If we integrate both sides of the equation over the compact manifold, then the left-hand side vanishes as a total derivative, and the first two terms on the right hand side give a positive definite contribution. Back in the 1980s—when physicists didn't really understand D branes and so the localized term was neglected—this was interpreted as a no-go theorem against warped compactifications. In the modern era we are comfortable including D brane sources.

4.2 Negative tension

If one includes a Dp brane wrapped on a $(p-3)$ cycle on the compact manifold \mathcal{M}_6 , the third term turns out to be proportional to

$$(T_m^m - T_\mu^\mu)^{\text{loc}} = (7-p) T_p \delta(\mathcal{M}_6). \quad (4.4)$$

What we find is that for $p < 7$, we need *negative tension* branes. Such objects happen to show up fairly generically, e.g. $O3$ orientifold planes. It is interesting at this point to remark, however, that even phenomenologists could have guessed something like this would happen since the simple RS scenario requires negative tension branes to satisfy the Einstein equation. (We won't derive this either, but this time it's not because the author is incapable of doing so.) The main difference, however, is that the RS scenario required both branes to have negative tension. In the KS/GKP construction the microscopic structure at the IR does *not* contain negative tension; instead, the negative tension comes only from the compactified manifold that provides the microscopic structure for the UV brane (the end of the throat).

4.3 $D3$ s at the tip

Klebanov and Witten placed N $D3$ branes at the conifold singularity to generate a warped metric

$$ds^2 = h^{-\frac{1}{2}}(r)(dx_{\text{Mink.}}^2) + h^{\frac{1}{2}}(r) (dr^2 + r^2 d\Omega_{T^{1,1}}^2), \quad (4.5)$$

where $h(r) = 1 + R^4/r^4$, with $R^4 \sim N\alpha'^2/\text{Vol}(\Omega_{T^{1,1}})$. This is shown heuristically in Fig. 2. The backreaction of the $D3$ branes pull out the AdS throat, but we note that there is a singularity at the origin. Further, the warped geometry is pure AdS_5 so that the corresponding gauge theory is conformal. We would like to resolve these issues. Er, technically we would like to *deform* them.

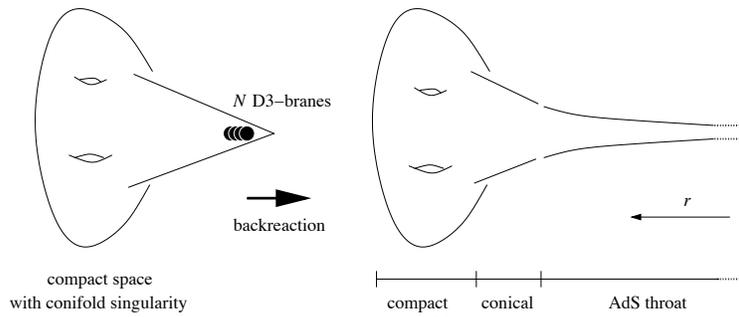


Figure 2: The warped conifold throat from $D3$ branes placed on the conical singularity. From [4]

4.4 Fractional $D3$ s

In addition to the N $D3$ branes at the conifold tip, we may also include M $D5$ branes that have managed to get themselves wrapped all over the S^2 . Since this two-cycle shrinks in the deformed conifold, these $D5$ branes look like $D3$ branes with fractional charge. We thus call them fractional $D3$ branes. It turns out that these $D5$ s will be responsible for turning on the flux that sources the ‘additional’ warping that breaks conformality.

Each fractional $D3$ is a source of a unit of F_3 flux (the field strength of the RR 2-form $C_{(2)}$ in the IIB supergravity action) through the $T^{1,1}$ tricycle. This, in turn, causes the five-form flux through $T^{1,1}$ to depend on the radial coordinate r . A heuristic way to understand this is to

recall in Maldacena's $\text{AdS}_5 \times \text{S}^5$ construction, the 't Hooft coupling depended on the number of $D3$ branes in the stack by

$$\lambda = g_{\text{YM}}^2 N. \quad (4.6)$$

The effect of the r -dependence of $F_{(5)}$ is to change the effective N to

$$N_{\text{eff}}(r) \sim N + g_s M^2 \log\left(\frac{r}{r_0}\right). \quad (4.7)$$

This introduces precisely the logarithmic deviation from AdS that we want to break conformal invariance in the dual theory.

We can motivate this in a slightly more technical way. The self-dual five-form flux $F_{(5)}$ is sourced by the $H_{(3)}$ and $F_{(3)}$ fluxes to stabilize moduli and any localized sources. It turns out that including the fractional $D3$ branes causes the $F_{(5)}$ flux to pick up a radial dependence via

$$\int_{\text{T}^{1,1}} F_{(5)} = \frac{1}{2} (\alpha')^2 \pi N_{\text{eff}}(r), \quad (4.8)$$

with N_{eff} given above. The integration constant r_0 is a characteristic UV scale. The point is that this turns our conifold into a **warped conifold** (i.e. warped in addition to the AdS curvature) with an r -dependent warp factor,

$$ds_{10}^2 = e^{2A(r)} dx_{\text{Mink}}^2 + e^{-2A(r)} (dr^2 + r^2 d\Omega_{\text{T}^{1,1}}^2). \quad (4.9)$$

Using some sort of voo-doo, Horcrux, or other kind of black magic, one can integrate equation involving $F_{(5)}$ to obtain an expression for the warp factor,

$$e^{-4A(r)} \sim \frac{(\alpha')^2}{r^4} \left[g_s N + (g_s M)^2 \log\left(\frac{r}{r_0}\right) + (g_s M)^2 \right], \quad (4.10)$$

where we've dropped various prefactors on each term. It is suggestive to write this in the form

$$e^{-A(r)} \sim 1 + \frac{R_{\text{UV}}^4 + R_{\text{IR}}^4 \log(r/r_0)}{r^4} = 1 + \frac{R_{\text{IR}}^4 \log(r/r_s)}{r^4} = 1 + \frac{R_{\text{eff}}^4(r/r_s)}{r^4}, \quad (4.11)$$

where the R s are the radius of curvature at the UV and IR scales. In the last step we defined a 'slowly varying' radius of curvature $R_{\text{eff}}(r) \sim \pi \alpha'^2 g_s N_{\text{eff}}(r)$. Comparing to (4.5) we see that the warping indeed deviates from AdS by the logarithmic factor so that the space is now only approximately $\text{AdS}_5 \times \text{T}^{1,1}$.

5 Flux this tip

Let us now continue on to the KS and GKP solutions. At this point we should note that we have a warped solution thanks to our fluxes, what we have not yet done is provided a large and stable hierarchy. It is sometimes under appreciated that warping and hierarchy are not the same thing: warping provides an [exponentially] good way to generate hierarchies, but only if such hierarchies

can be stabilized. In toy constructions of string realizations of RS-like models where $D3$ branes played the role of the Randall-Sundrum IR and UV branes (e.g. [5]) this was manifested by somehow forcing one $D3$ brane to stay some finite distance away from the rest of its friends at the bottom of the cone. In ‘phenomenology’ this manifests itself as the need to have some kind of Goldberger-Wise-like mechanism to fix the radion⁵. In our stringy construction this issue will be related to fixing the IR scale, which we’ve already alluded comes from deforming the conifold tip. One could then ask where the deformation came from. The answer, as usual, is flux (which in turn came from the fractional $D3$ s discussed earlier). This isn’t actually new information: the whole point of flux compactifications is that one can generate potentials for the moduli; ϵ^2 is just another modulus.

Let’s dig into the structure of the IR a little bit. We define the $T^{1,1} = S^2 \times S^3$ structure of the deformed conifold in terms of two ~~three-eyes~~ tricycles. The A tricycle is nonvanishing at the tip and corresponds to the S^3 that we blow up in our deformation. This vanishes as $z \rightarrow 0$ and is defined as the S^3 defined by setting the z_i to be real. We shall take the B tricycle to be the dual of the A tricycle in the sense that the compact manifold can be thought of as a “Cartesian product” of the two cycles. More concretely, the B tricycle is the S^2 of the $T^{1,1}$ along with the AdS radial direction; it is defined by taking $z_{1,2,3}$ imaginary and z_4 real in (3.4).

The fractional $D3$ branes place M units of $F_{(3)}$ flux on the A cycle. The field equations (alternatively, $D3$ charge conservation) then require that the $H_{(3)}$ flux must also be supported on the dual B cycle with, say, $-K$ units of flux;

$$\int_A F_{(3)} = 4\pi^2 \alpha' M \qquad \int_A G_{(3)} = -4\pi^2 \alpha' K. \qquad (5.1)$$

Poincaré duality then gives a superpotential

$$W = \int_{\mathcal{M}_6} G_{(3)} \wedge \Omega = (2\pi)^2 \alpha' \left(M \int_B \Omega - K \tau \int_A \Omega \right). \qquad (5.2)$$

This tells us, for reasons that I define to be outside the scope of this paper (probably more black magic), that the ϵ^2 describing the size of the A tricycle is

$$z = \int_A \Omega. \qquad (5.3)$$

This then (again, *abra cadabra*) tells us that the integral over the dual B tricycle is

$$\int_B \Omega = \frac{\epsilon^2}{2\pi i \epsilon^2} \log \epsilon^2 + \text{holomorphic}. \qquad (5.4)$$

The Kähler covariant derivative of the superpotential with respect to this complex coordinate is then (*expelliarmus!*),

$$D_z W \sim \alpha' \frac{M}{2\pi i} \log z - i \frac{K}{g_s} \qquad (5.5)$$

⁵Perhaps a more pedestrian manifestation of this question is whether or not RS models *truly* generate a hierarchy of scales or have we just chosen misleading coordinates?

so that in the $K/Mg_s \gg 1$ limit, we obtain an exponentially small

$$\epsilon^2 \sim \exp(-2\pi K/Mg_s). \quad (5.6)$$

This now tells us that the flux compactification with N $D3$ branes at the conifold singularity and M fractional $D3$ branes gives a deformed conifold at the IR tip whose characteristic scale is *exponentially* smaller (warped) than the UV scale set by the compact manifold.

6 RS, is that you?

A heuristic picture of the warped compactification that we have described is presented in Fig. 3. As

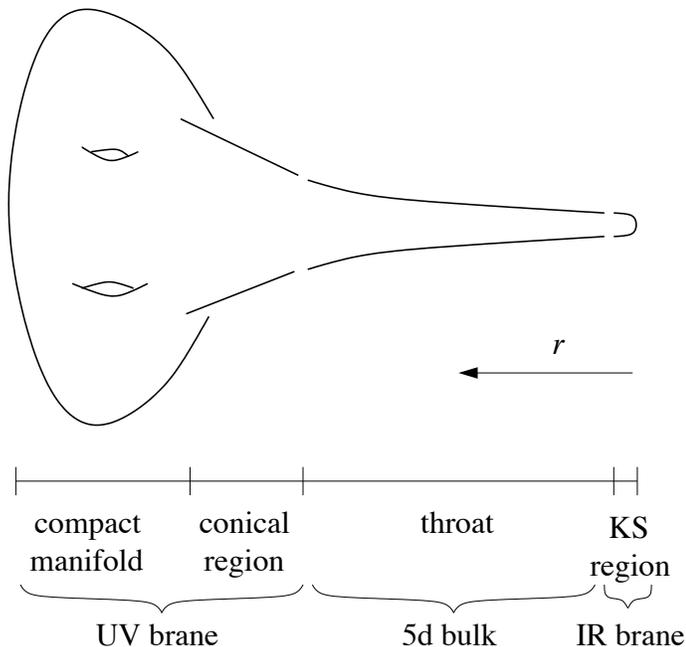


Figure 3: A realization of the Randall-Sundrum model (RS1) on the throat. From [4]

indicated, the warped throat region corresponds to what phenomenologists would like to call the 5D bulk region. This region deviates from AdS by a logarithmic factor which breaks conformality. The IR brane is described by the Klebanov-Strassler deformed conifold which provides a structure which truncates the throat. On the other end the compact manifold itself (with its sources of negative tension) serve as a ‘UV brane’ to cutoff the throat behavior.

7 Remarks on the duality cascade

We now present a very brief sketch of the dual theory to the Klebanov-Strassler region of our warped throat. The space $T^{1,1} = SU(2) \times SU(2)/U(1)$ (i.e. *before* we deform the conifold) has an

$SU(2) \times SU(2) \times U(1)$ isometry. The local $U(1)$ is realized as a global R symmetry in the gauge theory. This theory contains two chiral superfields A_i and B_i in $SU(N) \times SU(N)$ representations $(\mathbf{N}, \bar{\mathbf{N}})$ and $(\bar{\mathbf{N}}, \mathbf{N})$ respectively. These chiral superfields transform as $(\square, 1)$ and $(1, \square)$ under the $SU(2)$ s above.

Adding the fractional $D3$ branes breaks the AdS symmetry in the gravity theory and hence conformal symmetry in the gauge theory. We can understand the effect of the fractional $D3$ s by considering them one at a time as 5D domain walls. As we pass a fractional $D3$ going up the throat the gauge symmetry changes from $SU(N) \times SU(N)$ to $SU(N) \times SU(N+1)$. Thus we see for M such fractional $D3$ branes the conifold UV (large r) gauge symmetry is $SU(M+N) \times SU(N)$.

Let us write the near-horizon warp factor (4.11) as $\exp(-4A(r)) \sim \log(r/r_s)/r^4$. Holography tells us to interpret this logarithm as $\log(\mu/\Lambda)$ for some renormalization scale μ and a fundamental scale Λ . We would now like to ask what happens as we flow to the infrared, i.e. $r \sim \mu \rightarrow$ small. The gauge and string couplings are related by

$$\frac{1}{\alpha_1(\mu)} + \frac{1}{\alpha_2(\mu)} = \frac{1}{g_s}, \quad (7.1)$$

where $\alpha_{1,2} = g_{\text{YM}_{1,2}}^2/4\pi$ refer to the couplings of the $SU(M+N)$ and $SU(N)$ groups respectively. The left-hand side is constant because (*Alakazam*) the dilaton is constant. On the other hand, the difference of these objects obey a logarithmic running,

$$\frac{1}{\alpha_1(\mu)} - \frac{1}{\alpha_2(\mu)} \sim \log\left(\frac{\mu}{\Lambda}\right). \quad (7.2)$$

As μ decreases, α_1^{-1} decreases and α_2^{-1} increases. Eventually, however, α_1^{-1} vanishes. In this case we may replace the theory with its Seiberg dual. Recall that Seiberg duality replaces an $SU(N_c)$ supersymmetric gauge theory with F quark flavors with an $SU(F - N_c)$ theory and F flavors (and baryons). For α_1 , $N_c = M + N$ and $F = 2N$. Thus Seiberg dualizing has transmogrified our gauge theory,

$$SU(M+N) \times SU(N) \quad \rightarrow \quad SU(N) \times SU(N-M). \quad (7.3)$$

This process can be repeated over and over again, as shown in Fig. 4. In fact, in the KT model it could be repeated *ad infinitum* all the way to the naked singularity. For the Klebanov-Strassler deformed conifold, the geometry provides a cutoff for this cascade at $r_s \sim \epsilon^{2/3}$. On the gauge side the theory confines. One can see this by considering the R symmetry groups of the KT and KS models. For $\epsilon = 0$, the conifold coordinates obey a $U(1)$ symmetry associated with $z_i \rightarrow e^{i\theta} z_i$. When $\epsilon \neq 0$, this is broken to $z \rightarrow -z$, which is precisely what one expects since gaugino condensation breaks $U(1)_R \rightarrow \mathbb{Z}_{2N}$, which in turn is broken to Z_2 by anomalies⁶.

8 Conclusion

We have briefly reviewed the Giddings-Kachru-Polchinski/Klebanov-Strassler ‘microscopic’ string construction of a warped compactification that mimics the ‘macroscopic’ structure of the Randall-

⁶It is unfortunate that the examinee did not have enough time to connect this to his A exam prompt on instantons.

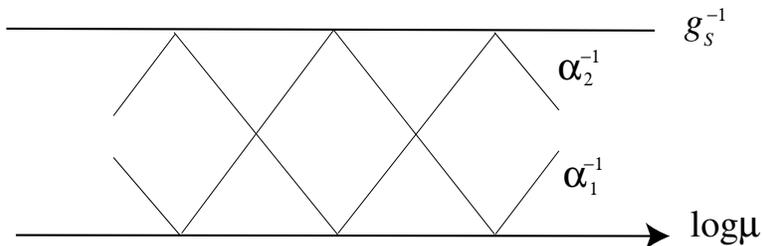


Figure 4: Cartoon of the duality cascade from [3].

Sundrum model of a warped extra dimension. We’ve focused on the supergravity side and have been unabashedly qualitative. (I admit that I am a little bashful about how qualitative I’ve been.)

Let us be up front about some of the many interesting topics that we did *not* discuss. These include

1. **Non-supersymmetric throats.** The construction that we have presented still is manifestly $\mathcal{N} = 1$ supersymmetric. While this typically carries virtues, it is still not quite the ‘minimal’ RS1 scenario that phenomenologists like to play with. Generally it is difficult to get a handle on the gauge and gravity theories in the absence of SUSY. Recent progress in this direction (which also contributed to the author’s interest in this topic) has been made by Kachru, Simic, and Trivedi [6].
2. **The duality cascade.** The KS solution can be understood purely from the gauge theory side. For an excellent review, see Strassler’s TASI lectures [7].
3. **Randall-Sundrum phenomenology.** We didn’t touch RS phenomenology one bit. This is partly because the author is rather fed up with it at the moment. However, there has been some recent interesting work building novel supersymmetric Randall-Sundrum holographic models. One particularly interesting direction involves having supersymmetry broken at a high scale only to re-emerge as an accidental symmetry as the theory couples to a superconformal sector during its RG flow [8].
4. **Randall-Sundrum *phenomenology*.** Perhaps more to the point of this work, a former Cornell theory group graduate student has performed a more thorough investigation of the ‘imprint’ of a stringy completion on ‘phenomenological’ RS models and their experimental signatures [9].

In this respect this A exam prompt has been a way for the author to familiarize himself with the string-side picture of warped compactifications with an eye on possible future projects at the intersection of string theory and phenomenology.

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A Notation and Conventions

4D Minkowski indices are written with lower-case Greek letters from the middle of the alphabet, μ, ν, \dots . Compact manifold coordinates are denoted by lower-case Roman letters from the middle of the alphabet, m, n, \dots . Occasionally we'll omit a tedious derivation by saying that a fact follows 'by magic.' We will indicate this by inserting magical incantations like *hocus pocus!*

B Brief Literature Review

Nothing in this paper represents original work, except the errors. The primary references were the text by Becker² and Schwarz [3], the original literature (KS [2], GKP [1]), and the dissertations by two recent Heidelberg graduates [10, 4]. I also benefitted from notes of Shamit Kachru's lectures at PiTP 2008⁷ and Stefan Sjors BSM Journal Club⁸. Finally, the primary catalyst for my humble understanding of this material came from edifying discussions with Liam McAllister and Sohang Ganhdi.

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