HADRONIC PENGUIN PARADE \( (b \to s \gamma) \)

flip tanedo ph1277e@cornell.edu

**Goal:** Summary of Dominant \& Next-to-Dominant Diagrams Contributing to the \( C_7 \) \& \( C_8 \) Operators in RS.

\[ \text{Dominant} \]

\[ C_a^7 \]

\[ \begin{aligned} &\text{This diagram does not exist in } \mu \to e \nu \\
&\text{graph is similar to } 1\text{MIH}^\pm, \text{ except there is no Goldstone cancellation.} \\
&\text{Integral is nasty, gives } \approx 0.5 \\
&\left[ \text{see } v_1 \text{ of } k \to e \nu \right] \\
&\sim (10^{-1}) \tilde{Q}_u (0.5) \\
&\uparrow \frac{1}{3} \\
&\text{mass } m_s \end{aligned} \]

\[ \text{Subdominant} \quad (\text{ref to } k \to e \nu) \]

\[ C_a^8 \]

\[ \begin{aligned} &\text{This is suppressed by an algebraic cancellation for the brane-localized } H. \\
&\text{One ends up with a factor of } (m_{W'})^2 \\
&\left[ \text{see latest } k \to e \nu \text{ paper} \right] \\
&\sim (10^{-1})(m_{W'})^2 (0.65) \sim 6.5 \times 10^{-4} \\
&\frac{1}{10^{-2}} \end{aligned} \]

\[ \begin{aligned} &\text{This is the dominant contribution to } a \text{ in } \mu \to e \nu \\
&\sim (10^{-3}) \tilde{g}_\mu \ln \frac{R}{r} (-0.31) \\
&\uparrow \approx 0.73 \\
&\text{includes } \frac{3}{2} \text{ in expression} \\
&\text{note: } \text{ext mass } m_s \approx \text{int mass } m_s \text{. blc.} \\
&\text{intermediate state is a } b \text{c mode: } \text{no } (m_{b/c}) < 10^{-4} \text{ suppression} \end{aligned} \]
REMKS: the gauge couplings are enhanced by $g_{SM}^2 \to g_{SM}^2 \log R/e$

some SM couplings:

$g_{ZWW} = \frac{g}{\sqrt{2}} \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta \right) = 0.26$
$g_{ZWW} = \frac{g}{\sqrt{2}} \left( -\frac{2}{3} \sin^2 \theta \right) = -0.11$
$g_{ZAA} = \frac{g}{\sqrt{2}} \left( -\frac{1}{2} + \frac{1}{3} \sin^2 \theta \right) = -0.315$
$g_{ZQQ} = \frac{g}{\sqrt{2}} \frac{1}{8} \sin^2 \theta = 0.057$

use:

$G_F = \frac{\alpha}{8 M_W^2} = 1.166 \times 10^{-5}/GeV^2 \Rightarrow \alpha = 0.65$

$\sin \theta = \frac{1}{2} \Rightarrow \sin \theta = 0.46$

These are all negligible. OMINH$^\pm$ & OMINH must have an external mass insertion. However, because the intermediate state is brane-to-brane, it must be a zero mode (by boundary conditions). Thus instead of a $10^{-7}$ suppression, these get a $10^{-3}$ suppression.

The OMINH diagram is tiny because of a cancellation between the physical Higgs & the neutral Goldstone. The easiest way to see this is to promote the (Higgs + Goldstone) to a complex field:

$\Delta \to \Delta \to \sum_{x=H,\phi} \left( M_h^2 - M_{\phi}^2 \right) \Delta^2 \sim 10^{-4}$

This, then, has mass inser.
2 diagrams w/ 2 mass insertions
at least 1 internal since consecutive external mass insertions → zero modes
so: 3M12, (2+1)M12, (1+2)M12, \( z \rightarrow z^5 \)

\[
3M12 \approx (10^{-1})^3 \frac{g_{WW}^2 g_{ZZ}^2 r_{WZ} \ln \frac{r_{WZ}}{r_{Z}}}{0.63} \quad k \rightarrow \infty \text{ value}
\]

\[
(2+1)M12 \approx (10^{-1})^3 \frac{g_{WW}^2 g_{ZZ}^2 r_{WZ} \ln \frac{r_{WZ}}{r_{Z}}}{8.5} \quad k \rightarrow \infty \text{ value}
\]

\[
(1+2)M12 \approx (10^{-1})^3 \frac{g_{WW}^2 g_{ZZ}^2 r_{WZ} \ln \frac{r_{WZ}}{r_{Z}}}{0.63} \quad k \rightarrow \infty \text{ value}
\]

\[
3M12_5 \approx (10^{-1})^3 \frac{g_{WW}^2 g_{ZZ}^2 r_{WZ} \ln \frac{r_{WZ}}{r_{Z}}}{0.07} \quad k \rightarrow \infty
\]

\[
(2+1)M12_5 \approx (10^{-1})^3 \frac{g_{WW}^2 g_{ZZ}^2 r_{WZ} \ln \frac{r_{WZ}}{r_{Z}}}{8.5} \quad k \rightarrow \infty \text{ value}
\]

\[
(1+2)M12_5 \approx (10^{-1})^3 \frac{g_{WW}^2 g_{ZZ}^2 r_{WZ} \ln \frac{r_{WZ}}{r_{Z}}}{0.63} \quad k \rightarrow \infty
\]

Remarks: For \( \mu \rightarrow e \), we had a numerical coincidence:

\[
\alpha(Y^3_e) = 2M12 + 3M12 + 3M12_5 + (2+1)M12 + (1+2)M12_5
\]

\[
\text{numerically cancel}
\]

→ But: This was all moot because \( \alpha(Y^3_e) \ll \alpha(Y^2_e) \)

In \( b \rightarrow s \gamma \), such a numerical coincidence doesn't seem to hold since the values of the couplings are different for quarks. But anyway, the couplings are still \( \alpha(i) \) and are small compared to the gluon diagrams.

Note: I just copied the tree's integer values - \( b \rightarrow s \gamma \) may differ due to the fermion localization, but don't expect a \( \alpha(\alpha) \) change.

I checked numerically.
GWON DIAGRAMS W/ & MASS INSERTIONS
THE GRAPHS ARE IN 1-1 CORRESPONDENCE

acciouNG TO YUHUN : @ 3 TeV
\( \alpha_s \sim 0.1 \Rightarrow g_s^2 \sim 1.2 \)
\( \Rightarrow g_s^2 \ln \frac{r}{R} \sim 4 \) \( \Rightarrow g_s^2 \ln \frac{R}{r} \)
ALSO A DYNKIN FACTOR, \( C_r = \frac{9}{3} \) FROM \( T^4 \)

\[
3MIG (+G^5) \sim (10^{-1})^3 \frac{g_s^3 \ln \frac{R}{r} C_r}{c_s} \sim \frac{1}{200}
\]

\[
2MIG (+G^5) \sim (10^{-5})^3 \frac{g_s^2 \ln \frac{R}{r} C_r}{c_s} \sim \frac{1}{20}
\]

\[
1MIG (+G^5) \sim (10^{-1})^3 \frac{g_s^2 \ln \frac{R}{r} C_r}{c_s} \sim \frac{1}{20}
\]

HERE WE'VE USED THE \( k=\alpha \) INTEGRAL VALUES AS AN ESTIMATE.
NOTE THAT THE 2MIG DIAGRAM IS ONLY A FACTOR OF FOURSOME SMALLER THAN THE DOMINANT CHARGED HIGGS DIAGRAM.

\( C_r \) need to be careful, in old notes we wrote \( C_r = \frac{3}{2} \).
ZEWL \[\frac{1}{\mu^5}\]

**DIAGRAMS**

\[\sim (10^{-1})^3 \frac{g^2 \ln \frac{p'}{p}}{\ln \frac{Q}{m}} (-0.05)\]

Why is this so small? Derivatives acting on a profile?

**H+H MIXING**

\[\sim (10^{-1})^3 \frac{g^2 \ln \frac{p'}{p}}{\ln \frac{Q}{m}} (-0.23)\]
ALIGNMENT DIAGRAMS (I MASS INSERTION)

**DOMINANT:**

\[ \mathcal{M}^5 \]

\[ \mu \begin{array}{c}
\text{IMIG} \\
\text{OMIG}
\end{array} \]

\( \sim (10^{-1}) \frac{g_s^2 \ln R/R}{\ln \Lambda} \quad C_R \)

\((\mu \text{IGs})\) \quad \text{From } \mu \text{ to } e^\pm: \text{ the analogous } \mu \text{IGs lepton diagrams are } \sim 50\% \text{ of the } \mu \text{IGs diagrams.}

**C_{\mu} \text{ SUBDOMINANT}**

\[ \mu \mu \]

\[ \text{This, along with } \mu \text{IGs, was the dominant misalignment contribution to } H \rightarrow e^\pm. \]

\( \sim (10^{-1}) \frac{g_s^2 \ln R/R}{\ln \Lambda} (0.23) \)

7.4 \quad \text{before align.}

**1MIG (25):**

\[ g_s^2 \ln R/R \rightarrow g_2 \mu^2 \ln R/R \]

\( \gamma \text{ small.} \)

**OM1M5 / OM1HW5:**

These are all small because

\[ (0) \]

\[ = 0 \quad \text{by } g_2 \text{ acting on flat profiles} \]

\[ = 0 \quad \text{by } BC \text{ on IR brane} \]

**OM1W:**

\( \sim (10^{-1}) g_s^2 \ln R/R (0.1) \)
\[ C_B \text{ ANAPHOTIC} \]

DOMINANT

SAME AS THE ANALOGOUS l DIAGRAM

\[ \propto Y_u + Y_y Y_d \]

1MH \[ \text{I} \]

\[ [1, 2, 3] \text{ MIG U1 GROUND FROM GROUND, } 2 \frac{\sqrt{3}}{2} \]

XXX

\[ \text{THE DIAGRAMS WHERE THE GROUND IS EMITTED FROM THE VIRTUAL GROUND GO LIKE} \]

\[ \frac{1}{d_4} t^b t^c = \frac{3}{2} i t^a \]

\[ \text{WHEREAS THE DIAGRAMS WHERE THE GROUND IS} \]

\[ \text{EMITTED FROM THE VIRTUAL QUARK GO LIKE} \]

\[ t^b t^a t^b = -\frac{1}{6} \]

\[ \Rightarrow \text{ THESE DIAGRAMS ARE ENHANCED BY } \sim \Theta(16) \]

Note how the dominant things or quark diagrams add:

\[ a = I_{C_{7g}} \]

\[ \frac{3}{2} \left( g_s^2 \ln \frac{r_1}{E} \right)^2 \left( \frac{R' u}{m_z^2} \right)^2 I_{C_{8g}} \]
OTHER DIAGRAMS ($\mathcal{C}_8$)

There are no Hig couplings, so ignore those $C_7$ diagrams. The $\Upsilon\Upsilon\Upsilon\Upsilon$ gluon diagram dominates over the other Higgs diagrams for the same reasons as in Fig.

The $WZ$ diagrams are completely subdominant to the gluon diagrams. As we argued above, that the glue-from-glue diagrams are enhanced over the glue-from-quark diagrams.

$C_8$ Misalignment

$Z^0$ should be small, derivative acting on zero mode gluons.

Other diagrams:
- Gluon diagrams beat electroweak
- Glue-from-glue beat glue-from-quark
To correct Cheng's Appendix B

\[ L_{\text{neutral}} = g J^3 W^3 + \frac{1}{2} g' J_Y B^Y \]

\[ Z = c_w W^3 - s_w B \quad A = s_w W^3 + c_w B \quad \Rightarrow \quad W^2 = s_w A + c_w Z \]
\[ B^Y = c_w A - s_w Z \]
\[ g' = \frac{1}{t_w} g \quad \frac{1}{2} J_Y = J^{sm} - J^3 \]

\[ L_n = g J^3 \left( s_w A + c_w Z \right) + \frac{1}{2} t_w g \left( J_{sm} - J^3 \right) \left( c_w A - s_w Z \right) \]

\[ = \frac{g}{c_w} \left[ c_w^2 J^3 + s_w^2 J^3 - s_w J_{sm} \right] \cdot Z \]

\[ = \frac{g}{c_w} \left[ \frac{1}{4} \bar{u} \gamma u - \frac{i}{4} \bar{u} \gamma s u - \frac{1}{4} \bar{d} \gamma d + \frac{i}{4} \bar{d} \gamma s d - \frac{2}{3} s_w^2 \bar{u} \gamma u + \frac{1}{3} s_w^2 \bar{d} \gamma d \right] \cdot Z \]

\[ = \frac{g}{4 c_w} \left[ \left( 1 - \frac{8}{3} s_w^2 \right) \bar{u} \gamma u - \bar{u} \gamma s u \right. \]
\[ \left. - \left( 1 - \frac{4}{3} s_w^2 \right) \bar{d} \gamma d + \bar{d} \gamma s d \right] \cdot Z \]

\[ = P_L + P_R \quad \gamma S = P_R - P_L \quad \Rightarrow \quad A + B Y S = \left( A - B \right) P_L + \left( A + B \right) P_R \]

\[ g_{2u_u} = \frac{g}{c_w} \left( \frac{1}{2} - \frac{2}{3} s_w^2 \right) \quad g_{2u_u} = \frac{g}{c_w} \left( \frac{1}{2} - \frac{2}{3} s_w^2 \right) \]

\[ g_{2d_d} = \frac{g}{c_w} \left( -\frac{1}{2} + \frac{1}{3} s_w^2 \right) \quad g_{2d_d} = \frac{g}{c_w} \left( -\frac{1}{2} + \frac{1}{3} s_w^2 \right) \]