P121: Hint for Griffiths Question 9.6

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Write your wave $\tilde{f}(z,t)$ as in equation (9.25):

$$\tilde{f}(z,t) = \begin{cases} \tilde{A}_I e^{i(k_1 z - \omega t)} + \tilde{A}_R e^{i(-k_1 z - \omega t)} & \text{for } z < 0 \\ \tilde{A}_R e^{i(k_2 z - \omega t)} & \text{for } z > 0 \end{cases}$$

We must now apply boundary conditions at $z = 0$. We know that f must be continuous at zero. This leads to the same constraint as the case of a massless knot done in the book:

$$\tilde{A}_I + \tilde{A}_R = \tilde{A}_T$$

So now the trickier part is writing down the appropriate boundary condition for the derivative of f, which replaces equation (9.27). ((This is part (a) of the question.))

Here's the key hint that you should pick up from reading the bottom half of page 371: $\mathbf{F} = m\mathbf{a}$. Unlike the case of a massless knot done in the book, the difference in the derivatives of f from the left and from the right can be related to a force. Please note that $\partial f/\partial z|_{0^\pm}$ does not have the units of force (what's the appropriate constant?).

This should give you a constraint equation. On one side you'll have factors of $ik_{1,2}$ brought down by the space derivatives, and on the other side you'll have a factor of $-\omega^2$ from the double time derivative of acceleration. You will note that the equation is complex.

Part (b) is asking you to use both of these constraints to determine A_T and δ_T. You need to use the statement that the second string is massless. This means that $v_2 = \sqrt{T/\mu_2} = \infty$. But by equation (9.24), $v_1/v_2 = k_2/k_1$. Hence $k_2/k_1 = 0$. This will simplify some of the algebra you need to do.

At the end of the day, however, there is quite a bit of tedious algebra to be done. Of course, you should **understand the physics** before worrying about the math. At this point you should have through about most of the physics you'll want to glean from this problem and you'll be left with trying to determine an amplitude and phase.

Some hints on the math: make life a little easier and rename lumps of (real) quantities that stick together in the algebra, for example, (mk_1/μ_1). At some point you'll get a equations of the form:

$$\tilde{A}_T = (\text{Complex expression}) \cdot \tilde{A}_{T,I}$$

This is what you need! Now you can rewrite all the complex things in the form $Ae^{i\theta}$:

$$A_R e^{i\delta_R} = \left(A_0 e^{i\delta_0}\right) A_{T,I} e^{i\delta_{T,I}}$$

Where I have written A_0 and δ_0 to refer to the "complex expression" above.

Anyway, you should give your answer for part (b) in two ways: A_R and δ_R in terms of the incident amplitude and phase, and then again in terms of the transmitted amplitude and phase. (This corresponds to taking the subscript to be I or T in the equations above. In case you were wondering, the answers will be different, do you understand physically why?.)