AGENDA

SPECIAL RELATIVITY
- MOTIVATION
- LORENTZ TRANSFORMS
- VELOCITY TR
-
-

note to self:
ASK FOR FEEDBACK

SECTION NEXT WK?

CAN SKIP THIS

Why we should expect relativity (AS GOOD E & M STUDENTS)

GRIFFITHS DOES EXAMPLE OF LOOP ON A TRAIN PASSING MAGNET

$\mathcal{E} = -\dot{\Phi}$

→ B/C MOTIONAL EMF IN MAGNET'S FRAME
→ B/C FARADAY'S LAW IN LOOP'S FRAME
} different interpretation

A MORE EXPLICIT EXAMPLE : Relativity PREDICTS MAGNETISM!

done more formally in Griffiths

2 WIRES W/ CURRENTS
+ CHARGES MOVE ONE WAY
− CHARGES MOVE OTHER WAY

$+ \quad - \quad + \quad - \quad + \quad -$ $\qquad \overset{v_+}{\leftarrow} \; \overset{v_-}{\rightarrow}$

$+ \quad - \quad + \quad - \quad + \quad -$

in rest frame of a $\ominus$ (all (−) in rest frame)

$\ominus$

$+ + \ominus + + + \ominus + + + \ominus$

w

+ CHARGES ARE DENSER! → ATTRACTIVE ELECTRIC FORCE

in our frame (LAB) WE CALL THIS A "MAGNETIC" FORCE.

Why we should be skeptical

$\vec{F}_{LORENTZ} = q(\vec{E} + \vec{v}\times\vec{B})$

VELOCITY DEPENDENCE??

THE HEART OF STR

$\boxed{\begin{array}{l}\text{1. PRINCIPLE OF RELATIVITY} \\ \text{2. UNIVERSAL SPEED OF LIGHT}\end{array}}$

"LAWS OF PHYSICS" SAME IN ALL FRAMES
INERTIAL
not "YOU GET THE SAME #'s"
but "YOU GET SAME RULES"

c = const.

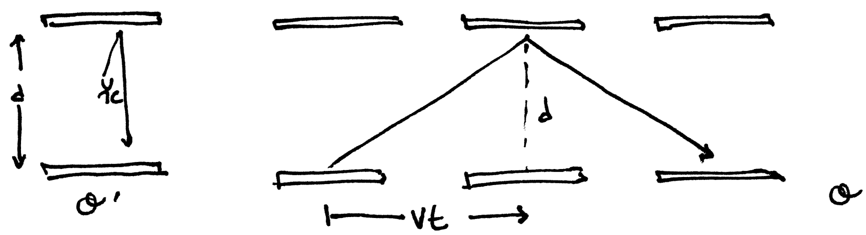
LORENTZ TRANSFORMS

ANY GOOD PHYSICIST CAN DERIVE THE LORENTZ TRANSFORMATIONS EXTEMPORANEOUSLY w/o NOTES OR A BOOK!

AS SUCH, I WILL NOT GO THROUGH THE DERIVATION IN DETAIL
BUT IT IS <u>VERY</u> IMPORTANT THAT YOU DO THIS!
REQUIRES NOTHING BUT ALGEBRA & COMMON SENSE
TRAINS YOUR BRAIN TO THINK THROUGH PROBLEMS LOGICALLY

STEP 1a) TIME DILATION

how: light clocks on a train



notice: O' : PHOTON TRAVELS DIST $2d$
O : —— " —— DIST $2\sqrt{v^2t^2+d^2}$

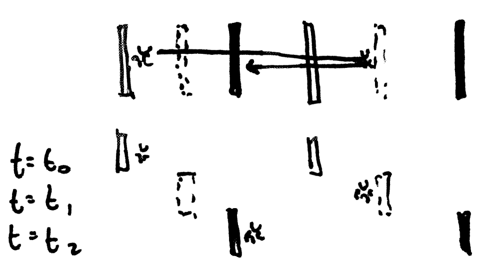
but SPEED OF PHOTON IS CONSTANT
$\Rightarrow$ TIME DILATION

$$\boxed{\gamma = \frac{1}{\sqrt{1-v^2/c^2}}} \qquad \gamma > 1$$

$$\boxed{t = \gamma t_0}$$

t_0 = time in rest frame of the process
"<u>PROPER TIME</u>"

STEP 1b) LENGTH CONTRACTION



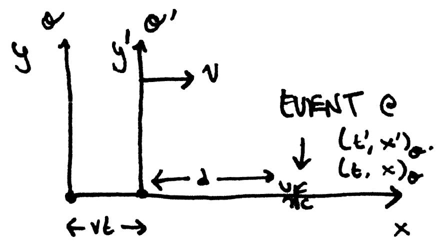
same idea, turn light clock over & use $t = \gamma t_0$

$$\Rightarrow \boxed{L = \frac{1}{\gamma} L_0}$$

<u>BTW</u>: DIR. ⊥ to motion are unaffected
(CAN REASON USING PRINCIPLE OF RELATIVITY)

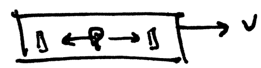
STEP 2 now we know how Δt's & Δx's transform, but let's actually transform btwn coordinate systems

(THE DIFFERENCE: COORD SYSTEMS HAVE ORIGINS!)



select coordinates s.t. they agree at $(t, \vec{x}) = (t', \vec{x}') = (0,0)$

(note they only agree @ (0,0) since simultaneity is ill defined)

gedanken: 

IN $\mathcal{O}$, WE OBSERVE $d = \frac{1}{\gamma} \cdot x'$

$\Rightarrow \quad x = vt + \frac{1}{\gamma} x'$

$$\boxed{\begin{aligned} x' &= \gamma(x - vt) \\ x &= \gamma(x' + vt') \end{aligned}}$$

← RHS IN $\mathcal{O}$, LHS IN $\mathcal{O}'$

← BY SYMMETRY

then invert to get t's

$$\boxed{\begin{aligned} t' &= \gamma(1 - v/c^2\, x) \\ t &= \gamma(1 + v/c^2\, x) \end{aligned}}$$

STEP 3 USE GROWN UP UNITS : $x^0 = ct$, $\beta = v/c$ ($= v$ IN NAT. UNITS)

$$\boxed{\boxed{\begin{aligned} x^{0\prime} &= \gamma(x^0 - \beta x^1) \\ x^{1\prime} &= \gamma(x^1 - \beta x^0) \end{aligned}}}$$

4-VECTOR NOTATION

PARADOXES

- LADDER-IN-BARN → SIMULTANEITY (IMPORTANT)
- TWIN

4-VECTORS

NOW WE ADD SOME FORMALISM (not "just math" - UNIFIED TREATMENT!)
WE TALKED ABOUT THIS BEFORE

• METRIC : "BILINEAR QUADRATIC FORM" (SYMMETRIC), $g_{\mu\nu}$

PRODUCES A SCALAR THAT IS QUAD. IN ARGS.

TAKES 2 ARGUMENTS, LINEAR IN EACH

EUCLIDEAN : DIAG (1,1,1)
LORENTZ : DIAG (-1,1,1,1) OR DIAG (1,-1,-1,-1)

METRIC TAKES TWO VECTORS V^μ, W^μ (CONTRAVARIANT)
PRODUCES A SCALAR $g_{\mu\nu} V^\mu W^\nu$

COLUMN

SUMMATION CONVENTION : "CONTRACT" ALONG UPPER/LOWER INDICES

• METRIC IS USED TO LOWER INDICES
→ TURNS CONTRAVARIANT VECTOR INTO COVARIANT

$$g_{\mu\nu} V^\nu = V_\mu$$

SO CAN WRITE $g_{\mu\nu} V^\mu W^\nu = V^\mu W_\mu = V_\nu W^\nu = V \cdot W$

just nomenclature

BUT ALL THIS METRIC STUFF IS KINDA HIGH-BROW

EFFECTIVELY, IF $V^\mu = (V^0, V^1, V^2, V^3)$

$$\Rightarrow V_\mu = (-V^0, V^1, V^2, V^3)$$

WHY ARE WE DOING THIS?
· 4-VECTORS ARE NICE BECAUSE THEY ARE A REPRES. OF THE LORENTZ GROUP

i.e. THEY ARE "COMPLETE" UNDER LORENTZ TR.
→ TRANSFORM INTO THEMSELVES
→ UNLIKE POSITION, WHICH MIXES w/ TIME

WE CAN WRITE

$$\bar{V}^\mu = \Lambda^\mu{}_\nu V^\nu \qquad \Lambda = \begin{pmatrix} \gamma & -\gamma\beta & & \\ -\gamma\beta & \gamma & & 0 \\ & & 1 & \\ & 0 & & 1 \end{pmatrix}$$

FOR $\bar{V}^\mu$ IN $\bar{O}$ FRAME.