Anomalous Mesonic Interactions near a Chiral Phase Transition

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Using constituent quarks coupled to a linear sigma model at nonzero temperature, I show that many anomalous mesonic amplitudes, such as $\pi^0 \to \gamma\gamma$, vanish in a chirally symmetric phase. Processes which are allowed, such as $\pi^0\sigma \to \gamma\gamma$, are computed to leading order in a loop expansion.

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The axial anomaly is the observation that for fermions coupled to a gauge field the divergence of the current for axial fermion number is not just the standard contribution from the classical equations of motion. In addition, at one loop order the divergence also contains a new term from quantum effects [1]. Because of deep geometrical reasons, there are no further corrections to this new term beyond one loop order [2]. For similar reasons, the axial anomaly is not altered by the presence of a medium, such as a thermal bath, in either two [3] or four [4] spacetime dimensions.

Besides the axial anomaly for fermions, there are also "anomalous" mesonic interactions [1,5]. These are like the fermion anomaly in that they arise from fermion loop graphs involving an odd number of the Dirac matrix γ^5, and so in the end are proportional to the antisymmetric tensor $\epsilon^{\alpha\beta\delta\gamma}$. Despite this superficial similarity, in this Letter I show that while the axial anomaly for fermions is completely unaffected by the presence of a medium, near a chiral phase transition of second order, anomalous mesonic interactions change dramatically. As a medium I consider a thermal bath [6], which may be produced in the central region of heavy ion collisions at ultrarelativistic energies.

I begin by considering the prototypical anomalous mesonic interaction, the decay of $\pi^0 \to \gamma\gamma$ [1,5]. Because the lifetime of the π^0 is electromagnetic, and so much longer than hadronic time scales, this decay is not of experimental interest for heavy ion collisions. The essential physics, however, applies to the (anomalous) decay of the ω meson, which does decay over hadronic time scales.

I work in a constituent quark model, with two flavors and $N_c = 3$ colors of a quark field ψ. I couple quarks to photons, A_α, and to mesons, Φ, ignoring their coupling to gluons. I assume that the (quantum mechanical) breaking of the axial U(1) chiral symmetry is large at all temperatures [7], so that the relevant chiral symmetry is $SU(2)_\ell \times SU(2)_r$. Then the only [8] meson fields required are $\Phi = \sigma t_0 + i\vec{\pi}\cdot\vec{t}$, with σ a $J^P = 0^+$ meson, and $\vec{\pi}$ the 0^- pions; the flavor matrices are $t_0 = \mathbf{1}/2$ and $\mathrm{tr}(t^a t^b) = \delta^{ab}/2$.

I take a positive definite Euclidean metric, with $(\gamma^5)^2 = +1$. Left- and right-handed quark fields are constructed by using the projectors $P_{\ell,r} = (1 \mp \gamma^5)/2$, $\psi_{\ell,r} = P_{\ell,r}\psi$. The Lagrangian density for quarks is

$$\mathcal{L} = \overline{\psi}_\ell \not{D}\psi_\ell + \overline{\psi}_r \not{D}\psi_r + 2\tilde{g}(\overline{\psi}_\ell \Phi\psi_r + \overline{\psi}_r\Phi^\dagger\psi_\ell)\,. \tag{1}$$

$\not{D} = (\not{\partial} - iq\not{A})$, where q is a matrix for the electric charge of the up and down quarks, $q = e(t_3 + t_0/3)$. Excluding the electromagnetic coupling, this Lagrangian is invariant under global $SU(2)_\ell \times SU(2)_r$ chiral rotations Ω_ℓ and Ω_r, where $\psi_{\ell,r} \to \Omega^\dagger_{\ell,r}\psi_{\ell,r}$ and $\Phi \to \Omega^\dagger_\ell \Phi \Omega_r$. With electromagnetism, $e \neq 0$, the Lagrangian is invariant under rotations in the isospin-3 direction. Explicitly,

$$\mathcal{L} = \overline{\psi}[\not{D} + 2\tilde{g}(\sigma t_0 + i\vec{\pi}\cdot\vec{t}\gamma^5)]\psi\,. \tag{2}$$

If chiral symmetry breaking occurs, so $\langle\sigma\rangle = \sigma_0$, I shift $\sigma \to \sigma_0 + \sigma$, and the constituent quark mass is $m = \tilde{g}\sigma_0$. At tree level, $\sigma_0 = f_\pi = 93$ MeV is the pion decay constant.

I neglect the dynamics of the scalar and quark fields to derive the effective Lagrangian between the scalar and photon fields which is induced by integrating out the quarks at one loop order. Of course, in an asymptotically free theory, at very high temperatures mesons do not matter, only the quarks and gluons. Mesons are important at low and intermediate temperatures. In particular, for a chiral symmetry of $SU(2)_\ell \times SU(2)_r$, the chiral phase transition can be of second order [7] at $T = T_\chi$, which for the sake of argument I assume is the case. I work in a strict chiral limit, so that $\sigma_0(T)$ and $f_\pi(T)$ vanish as $T \to T_\chi$. The pions are massless when $T \leq T_\chi$; the σ meson, which is heavy at zero temperature, is massless at T_χ, so that at T_χ the σ and the pions form the appropriate $SU(2)_\ell \times SU(2)_r = O(4)$ multiplet. For three or more flavors the chiral phase transition is typically of first order [7]; the present analysis is then of interest if the transition is weakly first order.

Computing the amplitude for $\pi^0 \to \gamma\gamma$ is standard at zero temperature. Let the two photons be $A_\alpha(P_1)$ and $A_\beta(P_2)$, where P_1 and P_2 are the four-momenta. There are two triangle diagrams which contribute; after doing the Dirac algebra, one diagram contributes

$$-i\,\frac{4\tilde{g}e^2N_c}{3}\, mI(P_1,P_2,m)\epsilon^{\alpha\beta\delta\gamma}P_1^\delta P_2^\gamma\,, \tag{3}$$

where $\epsilon^{\alpha\beta\delta\gamma}$ is the antisymmetric tensor, and $I(P_1,P_2,m)$

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is the loop integral

$$\mathrm{tr}_K \frac{1}{(K^2 + m^2)[(K + P_1)^2 + m^2][(K - P_2)^2 + m^2]} . \tag{4}$$

$\mathrm{tr}_K = \int d^4K/(2\pi)^4$ is the integral over the loop momentum K. In the limit of small momenta the dependence on P_1 and P_2 can be neglected, with

$$\mathrm{tr}_K \frac{1}{(K^2 + m^2)^3} = \frac{1}{32\pi^2 m^2} . \tag{5}$$

The second diagram, which follows by interchanging P_1 and P_2, and α and β, contributes equally. Altogether, after using $m = \tilde{g} f_\pi$, the Lagrangian density for $\pi^0 \to \gamma\gamma$ is [1,5]

$$i \frac{e^2 N_c}{96\pi^2 f_\pi} \pi^0 \epsilon^{\alpha\beta\delta\gamma} F_{\alpha\beta} F_{\delta\gamma} . \tag{6}$$

To compute the corresponding amplitude at nonzero temperature I make several assumptions. First, I compute near the chiral phase transition, $T \sim T_\chi$. Since $\sigma_0(T) \to 0$, I can take $m(T) = \tilde{g}\sigma_0(T) \ll T$. Second, I work in the static limit, taking both external energies to vanish, $p_1^0 = p_2^0 = 0$. Because of the antisymmetric tensor, this means that implicitly I am assuming that one of the photons is a plasmon, say $\alpha = 0$, while the other is spatial, $\beta = i$. I also assume that the spatial momenta, $|p_1|$ and $|p_2|$, are much smaller than the temperature. These assumptions can be relaxed, but suffice to illustrate how anomalous mesonic interactions change near T_χ.

Under these assumptions, the computation of $\pi^0 \to \gamma\gamma$ at nonzero temperature is utterly trivial. To leading order in m, all I have to do is compute the integral in (4) for $m = 0$ and $T \neq 0$. At $T \neq 0$, the fermionic loop momentum $k^0 = (2n + 1)\pi T$, summing over all integers n,

$$\mathrm{tr}_K \frac{1}{(K^2)^3} = T \sum_{n=-\infty}^{+\infty} \int \frac{d^3k}{(2\pi)^3} \frac{1}{[k^2 + (k^0)^2]^3} = \frac{1}{16\pi^4 T^2} \sum_{n=1}^{\infty} \frac{1}{(2n - 1)^3} = \frac{7}{128} \frac{\zeta(3)}{\pi^4 T^2} . \tag{7}$$

In doing the integral it is most convenient to first integrate over the spatial momenta, and then do the sum over the integers n. This sum generates a zeta function, $\zeta(r) = \sum_{n=1}^{\infty} 1/n^r$; in (7), $\zeta(3) = 1.202\,06, \ldots$ enters. Consequently, in the static limit about T_χ, after integration by parts the Lagrangian density for $\pi^0 \to \gamma\gamma$ is

$$i \frac{7\zeta(3) e^2 \tilde{g}^2 N_c}{96\pi^4 T^2} \epsilon^{ijk} (\sigma_0 \partial_i \pi^0) A_0 \partial_j A_k . \tag{8}$$

I emphasize that (8) holds only under the given approximations [9,10], in particular, in the chiral limit *near* T_χ. At zero temperature (6) can be derived using the partial conservation of the axial current (PCAC) and the standard axial anomaly for fermions. At low temperatures, presumably something like (6) can be derived by using PCAC at $T \neq 0$; at the very least, the zero temperature f_π should be replaced by $f_\pi(T)$ [10]. In contrast, (8) is valid solely about T_χ, where $f_\pi(T)$ is small, and PCAC breaks down.

It is natural for the constants in (8) to differ from those in (6). If they did not, and the amplitude for $\pi^0 \to \gamma\gamma$ were proportional to $1/f_\pi(T)$ at all temperatures, then the amplitude would diverge as $T \to T_\chi$, when $f_\pi(T) \to 0$. Instead, the constants change from $\sim 1/f_\pi$ at zero temperature to $\sim \tilde{g}^2 \sigma_0(T)/T^2$ near T_χ, so that instead of diverging, the amplitude vanishes as $T \to T_\chi^-$ [11].

I have written (8) in a suggestive manner. The axial current for the scalar fields is $\vec{J}_{A,i} = (\sigma_0 + \sigma)\partial_i \vec{\pi} - \vec{\pi}\partial_i \sigma$. Thus (8) is part of the Lagrangian density

$$i \frac{7\zeta(3) e^2 \tilde{g}^2 N_c}{96\pi^4 T^2} \epsilon^{ijk} J^3_{A,i} A_0 \partial_j A_k . \tag{9}$$

To demonstrate that (9) is correct, observe that it predicts $\pi^0\sigma \to \gamma\gamma$ even when $\sigma_0 = 0$. This process is given by six box diagrams, and can be checked directly. It is easiest to calculate the box diagrams in two limits: first, when the σ has zero momentum, and then, when the π has zero momentum. The two coefficients have the value given by (9), with the appropriate change in sign.

The change in anomalous mesonic interactions near T_χ can be understood generally. The restoration of chiral symmetry at $T \geq T_\chi$ requires all couplings to be manifestly chirally symmetric. Hence electromagnetic amplitudes must commute not just with the third component of the vector charge (which is just isospin symmetry), but with the third component of the axial vector charge as well. This implies that $\pi^0 \to \gamma\gamma$ vanishes, and constrains the amplitude for $\pi^0\sigma \to \gamma\gamma$ to have the form in (9), with a coupling directly to the third component of the axial current, $J^3_{A,i}$. While the structure of the operator for $\pi^0\sigma \to \gamma\gamma$ is dictated by the chiral symmetry, the coefficient in front is not; since it is proportional to the coupling constant $\tilde{g}^2$, whose value is arbitrary, it is not universal, and likely receives corrections in $\tilde{g}^2$ from graphs to higher loop order.

Because I have computed in the static limit, the expression in (9) is gauge invariant: both $\epsilon^{ijk}\partial_j A_k$ and A_0 are each unchanged under static gauge transformations. The amplitudes can also be computed away from the static limit: then each external energy p_0 is analytically continued as $p_0 = -i\omega + 0^+$, where ω is a continuous, Minkowski energy. For nonzero ω and p the results are more involved, and are nonlocal in coordinate space. In the limit of small momenta, where each ω and $p \ll T$, and to leading order in $\tilde{g}$ near T_χ, the resulting amplitudes are similar to those which arise for hard thermal loops [12]. This is because the diagrams are at one loop, with discontinuities due to massless (fermion) fields at nonzero temperature. In the end, the result for $\pi^0\sigma \to \gamma\gamma$ must be gauge invariant, but in a more involved fashion.

With this example in hand, we can compute how many other anomalous mesonic interactions change near T_χ. An example of importance for heavy ion collisions is the ω meson. The ω meson is special in that, because it couples to the isosinglet current for quark number, it only couples through anomalous interactions [13,14]. There are two anomalous interactions of importance at zero temperature, $\omega \to \rho\pi$ and $\omega \to \pi\pi\pi$. The corresponding lifetime of the ω meson is $\sim$20 fm$/c$, which, while long, is still of hadronic time scales.

To couple Φ to vector mesons, I assume strict vector meson dominance [14]. Neglecting electromagnetism, the covariant derivative in (2) becomes

$$\not{D} = \not{\partial} - ig(\not{\phi} t_0 + \vec{\not{\rho}} \cdot \vec{t} + \gamma^5 \vec{\not{a}}_1 \cdot \vec{t}\,), \qquad (10)$$

where g is the coupling constant to vector mesons. Under chiral rotations the isotriplets (the $J^P = 1^-$ $\vec{\rho}$ and the 1^+ $\vec{a}_1$) mix with each other, while the isosinglets (the 1^- ω and the 1^+ isosinglet f_1, which I neglect) are invariant.

Up to trivial factors of isospin, the diagrams which contribute to $\omega \to \rho\pi$ are the same as for $\pi^0 \to \gamma\gamma$. To one loop order at zero temperature, about zero momentum the Lagrangian density for $\omega \to \rho\pi$ is

$$-i\,\frac{g^2 N_c}{8\pi^2 f_\pi}\,\epsilon^{\alpha\beta\delta\gamma}\omega_\alpha \partial_\beta \vec{\pi} \cdot \partial_\delta \vec{\rho}_\gamma\,. \qquad (11)$$

Near the chiral phase transition, and taking a plasmonic $\omega_\alpha = \omega_0$ to conform to the static limit, this becomes

$$-i\,\frac{7\zeta(3) g^2 \tilde{g}^2 N_c}{32\pi^4 T^2}\,\epsilon^{ijk}\omega_0(\vec{J}_{A,i} \cdot \partial_j \vec{\rho}_k - \vec{J}_{V,i} \cdot \partial_j \vec{a}_{1,k})\,, \qquad (12)$$

where the isospin vector current $\vec{J}_{V,i} = \vec{\pi} \times \partial_i \vec{\pi}$. This expression is the product of a left-handed current times a left-handed gauge field, minus the same for right-handed quantities, and so is invariant under the symmetries of chiral $SU(2)_\ell \times SU(2)_r$ and parity.

For the second process, $\omega \to \pi\pi\pi$, about zero momentum the Lagrangian density is

$$+i\,\frac{g N_c}{24\pi^2 f_\pi^3}\,\epsilon^{\alpha\beta\delta\gamma}\omega_\alpha \partial_\beta \vec{\pi} \cdot \partial_\delta \vec{\pi} \times \partial_\gamma \vec{\pi}\,, \qquad (13)$$

to one loop order at zero temperature. To compute the corresponding amplitude about the chiral phase transition requires the integral

$$\mathrm{tr}_K\,\frac{1}{(K^2)^4} = \frac{31}{1024}\,\frac{\zeta(5)}{\pi^6 T^4}\,, \qquad (14)$$

$\zeta(5) = 1.036\,93, \ldots$. In the static limit near T_χ, the Lagrangian density for a plasmonic ω_0 is

$$i\,\frac{31\zeta(5) g \tilde{g}^4 N_c}{256\pi^6 T^4}\,\epsilon^{ijk}\omega_0(\sigma_0 + \sigma)\partial_i \vec{\pi} \cdot \partial_j \vec{\pi} \times \partial_k \vec{\pi}\,. \qquad (15)$$

Remembering that ω_0 is a chiral singlet, this can be written in a form which is manifestly chirally symmetric,

$$i\,\frac{31\zeta(5) g \tilde{g}^4 N_c}{64\pi^6 T^4}\,\epsilon^{ijk}\omega_0 \times \mathrm{tr}[\Phi^\dagger \partial_i \Phi \partial_j \Phi^\dagger \partial_k \Phi - (\Phi \to \Phi^\dagger)]\,. \qquad (16)$$

The anomalous interactions of the ω meson have dramatic consequences for a "thermal" ω. Without anomalous interactions, strict vector meson dominance implies that the mass of the thermal ω does not change with temperature [14]. Including anomalous interactions, both the mass and the width of the thermal ω *must* change. The present results demonstrate that even the form of the anomalous interactions change near T_χ. These changes may be observable by measuring a shift in the ω peak, as seen in the dilepton spectrum in heavy ion collisions at ultrarelativistic energies.

I conclude by discussing one last anomalous process. Extending the model to three flavors, $\pi = \pi^a \lambda^a$ for the SU(3) flavor matrices λ^a, at zero temperature the Lagrangian density for $KK \to \pi\pi\pi$ is [5]

$$i\,\frac{2N_c}{15\pi^2 f_\pi^5}\,\epsilon^{\alpha\beta\delta\gamma}\,\mathrm{tr}(\pi\partial_\alpha\pi\partial_\beta\pi\partial_\delta\pi\partial_\gamma\pi)\,. \qquad (17)$$

Using the integral

$$\mathrm{tr}_K\,\frac{1}{(K^2)^5} = \frac{635}{32768}\,\frac{\zeta(7)}{\pi^8 T^6}\,, \qquad (18)$$

where $\zeta(7) = 1.008\,35, \ldots$, then about the chiral phase transition, to one loop order the Lagrangian density between one σ meson with zero momentum and five π's is

$$i\,\frac{127\sqrt{2}\,\zeta(7)\tilde{g}^6 N_c}{256\sqrt{3}\,\pi^8 T^6}\,\epsilon^{ijk}(\sigma_0 + \sigma)\,\mathrm{tr}(\pi\partial_0\pi\partial_i\pi\partial_j\pi\partial_k\pi)\,. \qquad (19)$$

As before, (19) implies that while the amplitude for $KK \to \pi\pi\pi$ vanishes in a chirally symmetric phase, that for $KK \to \pi\pi\pi\sigma$ does not. While I have written (19) in a form which appears to be Lorentz invariant, it is valid *only* in the limit where $\sim\partial_0\pi$ is vanishingly small. Terms of higher order in the frequency involve expressions which are nonlocal in coordinate space.

At zero temperature (17) is the first term in the expansion of a chirally symmetric Lagrangian in five dimensions. Witten showed that the form of this Lagrangian is dictated by topology in five dimensions [5], which fixes the constant in front, up to an overall integer equal to the number of colors. Likewise, it must be possible to write (19) in chirally invariant form. Note, however, that the constant in front of (19) does not appear to be constrained by topology, since it involves a coupling constant, $\tilde{g}$, whose value is arbitrary [15].

The following picture emerges. At low temperatures, only pions are massless, with their anomalous interactions governed by the generalization of the Wess-Zumino-Witten Lagrangian [5] to nonzero temperature [10]. If the chiral transition is of second (or weakly first) order, though, then near T_χ anomalous mesonic interactions are governed by a new Lagrangian, which includes the terms in (9) and (19) as two examples. A new Lagrangian emerges because near the critical point new modes—the σ mesons [19]—become light, so the anomalous mesonic Lagrangian adjusts to include them.

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Note added.—After this work was submitted for publication, I received [20], which, following [6], obtains results similar to those herein.

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[19] For three or more flavors, $n_f \geq 3$, Φ must include all $2n_f^2$ fields [8]. Besides interactions $\sim\sigma(\pi^a)^5$ as in (19), there are many others, including $\sim\sigma(\sigma^a)^2(\pi^a)^3$, $\sim\sigma(\sigma^a)^4\pi^a$, $\sim\eta'(\sigma^a)^5$, $\sim\eta'(\sigma^a)^3(\pi^a)^2$, and $\sim\eta'\sigma^a(\pi^a)^4$. Like other anomalous mesonic interactions, these terms are all odd under Witten's [5] transformation of $(-1)^{N_\pi}$, where $\eta' \to -\eta'$, $\pi^a \to -\pi^a$.

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