Problem Set 4
(due: 5:00 pm on Wednesday, June 7, 2006)

Do the following problems from *Introduction to Cosmology*, Barbara Ryden, Addison-Wesley:

Chapter 11: Problems 11.1, 11.2, 11.3

Do the following problem:

In lecture, we assumed that inflation occurred during the GUT phase transition in the early universe ($\epsilon_{\text{GUT}} \sim 10^{14} \text{ GeV}$). Some have speculated that instead of occurring during the GUT era, inflation may have occurred at the lower energy of $10^{12} \text{ eV}$ that is associated with the symmetry breaking of the electro-weak force.

(a) In this scenario, calculate the age of the universe $t_w$, when the inflation begins. Assume the universe is radiation dominated before $t_w$.

Next assume that the inflation potential depth $V_0 = \epsilon^4 / (\hbar^3 c^3)$, where $\epsilon$ is the energy at which the phase transition occurs. This leads to a de-Sitter universe $a \propto \exp(Ht)$.

(b) Calculate $H$ in $\text{s}^{-1}$ for this lower energy inflation.

(c) Show that the size of a causally connected region at the end of inflation is $ce^N \left[2t_w \exp(Ht_w) + H^{-1}\right]$, where $N$ is the number of e-foldings that take place during inflation.

(d) Calculate the size of this region in meters at the end of inflation if $N = 30$. 