THE RENORMALIZATION GROUP IS ONE OF THE MOST IMPORTANT IDEAS IN THEORETICAL PHYSICS. HERE I WILL ONLY ATTEMPT TO GIVE A FLAVOR OF THE BIG IDEA — YOU WILL HAVE AMPLE OPPORTUNITIES TO EXPLORE THIS FURTHER DURING THE REST OF YOUR GRADUATE CAREERS.

WHERE WE WERE LAST TIME & IN CLASS

∞'s ARE POPPING UP EVERYWHERE.
WE'RE CAUTIOUSLY OPTIMISTIC, THOUGH, SINCE WE SEEM TO BE ABLE TO PACKAGE THEM ALL INTO COUNTER TERMS.

Where did the counter-terms come from?
WHEN WE WROTE ORIGINAL $\mathcal{L}$ ("BARE" $\mathcal{L}$ ... but this is a forbidden phrase!), WE WROTE IT IN TERMS of BARE PARAMETERS.

IN DOING SO, WE ASSUMED WE COULD TREAT ASYMPTOTIC STATES LIKE FREE PARTICLES & INTERACTION VERTICES AS IF THEY "LITERALLY" REPRESENTED THE PHYSICAL INTERACTION.

↓

THE COST of TRYING TO CRAM AN INTERACTING THEORY'S PROPAGATING STATES INTO A FREE THEORY LAGRANGIAN ⇒ INFINITIES.
BARE MASS IS NOT A SENSIBLE OR "FAMILIAR" OBJECT — IT IS CERTAINLY NOT CLASSICAL. IT IS A FREE $\mathcal{L}$ PARAM. THAT IS SUPPOSED TO ACCOUNT FOR A QUANTUM, [self] INTERACTING STATE.

WE FOUND THAT WE COULDN'T KEEP OUR FREE $\mathcal{L}$ NORMALIZED WHILE SIMULTANEOUSLY SATISFYING THE REN. CONDITIONS:

1. PROP. HAS POLE ~~RESIDUE~~ @ [PHYS] MASS
2. PROP HAS RESIDUE = 1 THERE

} analytic requirements → REN. COND.

⇒ HAD TO SPLIT OUR $\mathcal{L}$ INTO A "PHYSICAL" PART AND COUNTER-TERMS

"PHYSICAL" PART ↑ CORRECTLY NORMALIZED. THE $\mathcal{L}$ THAT WE THOUGHT WE WERE WRITING ORIGINALLY

COUNTER-TERMS ↑ LEFT-OVER PIECES

↑ turns out they're really handy for eating divergences ... but of course, they HAD to.

STILL SOME LINGERING UNCERTAINTIES

1. WHAT IS THE MEANING & SIGNIFICANCE of Λ?

cutoff of our theory. ... but why?

what about dim-reg?

still feels like we just magically got rid of a 'problem' and there were no consequences

2. WHAT'S UP W/ NON-RENORMALIZABLE THEORIES?

↳ Why does csaba say that physicists in the 60's didn't know what they were doing?

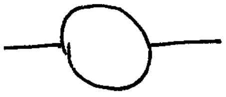
→ what is an EFT & what does it have to do with RENORMALIZATION?

→ why are all of our theories renormalizable? (if NON-REN THEORIES ARE 'NOT SO BAD')

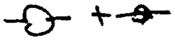
claim in class: non-renormalizable couplings are dimensionful and one should expect them to go like g/Λ^n, ie SUPPRESSED BY THE THEORY'S CUTOFF. → why?

3. A MORE PRACTICAL QUESTION THAT CAME UP ON THE HW:

↳ EVEN IF THE c.t.'s REMOVE ∞'s WE ARE LEFT WITH THINGS LIKE LOGARITHMS

eg:  $\sim \lambda^2 \log \Lambda^2/p^2$

 $\sim -\lambda^2 \log \Lambda^2/m^2$

$\Rightarrow$  $\sim \lambda^2 \log m^2/p^2$

DEPENDS ON SCALE @ WHICH THEORY IS PROBED.

FOR $p \ll m^2$, LOG IS LARGE! $\Rightarrow$ NONPERTURBATIVE?!

REMARKABLY (or not – if you already know statistical physics!), ALL of THESE ISSUES ARE INTIMATELY RELATED UNDER THE RENORMALIZATION GROUP (RG)

- not a GROUP except in the most trivial, stupid sense
- OPERATION ON THE THEORY THAT DETERMINES ~~TAKES IT~~ A TRAJECTORY IN THEORY SPACE; 1-PARAMETER FAMILY of THEORIES THAT ARE IDENTIFIED (in a loose sense!)

Refs: → GOOD WINTER BREAK READING

"BABY EXAMPLES" : 0812.3578 : RG + FRESHMAN E&M
"RENORMALIZATION for 1st GRADERS" – YUVAL

PEDAGOGICAL : Ann. Phys. 132 383 (1981) "DIM ANALYSIS IN A SED WAY"
WTY's LECTURE NOTES ON RG
+ hep-th/0212049
TIM HOLLOWOOD'S LECTURE NOTES — hard to find

TEXTBOOKS : Cheng & Li — A BIT OUTDATED
Weinberg Vol I — SEE LATER CHAPTERS & EFT IN VOL II
Peskin – 10, 12, 13 (not my favorite ref)
+ COLLINS
Zee – INTUITIVE, BUT LIGHT

2→ SEE ALSO STATISTICAL PHYSICS TEXTBOOKS
(I think Kardar is good)

CLASSIC ARTICLES: Wilson & Kogut : RG & THE ϵ EXPANSION
Les Houches : METHODS IN FIELD THEORY
Polchinski : Nucl. Phys. B231 269 (1984)

COLEMAN : WHY DILATATION GENERATORS DON'T GENERATE DILATATIONS, & Aspects of Symmetry.

Main Idea

PHYSICS IS INDEPENDENT OF THE REGULATOR

↳ have to say this carefully. In the HARD CUTOFF REGULARIZATION SCHEME, the REGULATOR IS THE SCALE @ WHICH THY BREAKS DOWN, eg SCALE OF NEW PHYSICS.

↑ btw: needn't be the scale of new particles
eg. ~~SSB~~ STRONG COUPLING. PION & NUCLEON THEORY BREAKS DOWN AROUND Λ_{QCD}

[WTY'S APPROACH]

$\frac{d}{d\Lambda}$ () = 0

↑ eg $\frac{d}{d\Lambda}$ ()$^{-1}$ = 0

$\frac{d}{d\Lambda}$ () = 0

WHY ()$^{-1}$? WHY 1PI?
DETAILS. WE WON'T WORRY ABOUT THEM NOW.

[[BUT: ()$^{-1}$ is THE  IN THE $\mathcal{L}$
1PI COUPLING IS THE AMP. GREEN'S FUNK,
SEE LECTURES NEXT WEEK.]]

$\left(\right.$  $\left. \right)^{-1} = p^2 - m_0^2 - \underline{\Pi(p^2)}$ ←  +  + ⋯

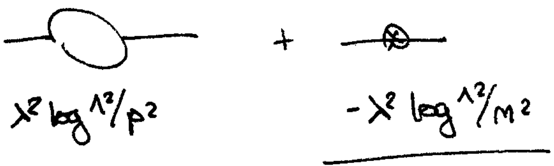
↑ DEPENDS ON λ_0, M_0, Λ
FROM THE NON-FAMILIAR LOOP CALCS YOU'VE DONE.

In fact,  $\sim \frac{1}{2} \frac{\lambda}{16\pi^2} \left(\Lambda^2 - m_0^2 \ln \frac{\Lambda^2}{m_0^2} + \cdots \right)$

IF $\frac{d}{d\Lambda}$ ()$^{-1}$ = 0, THEN THE BARE COUPLINGS MUST DEPEND ON Λ IN SUCH A WAY THAT THEY CANCEL THE EXPLICIT Λ-DEPENDENCE OF THE PHYSICAL OBJECT.

YOU ALREADY KNOW THIS : $M_0^2 = m^2 + \delta m^2$

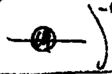
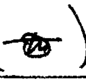
IN ϕ^3:



↑ depends on Λ, GIVES M_0^2 ITS Λ-DEPENDENCE.

CALLAN-SYMANZIK ↙

BUT WE CAN SAY THIS IN A DIFFERENT WAY

$\Lambda \frac{d}{d\Lambda}$ ()$^{-1}$ $= \left(\frac{\partial}{\partial\Lambda} + \boxed{\frac{\partial M_0^2}{\partial\Lambda}} \frac{\partial}{\partial m_0^2} + \frac{\partial\lambda_0}{\partial\Lambda}\frac{\partial}{\partial\lambda_0} \right)$ ()$^{-1}$ $= 0$

↑ WRITE $\Lambda\frac{d}{d\Lambda} = \frac{d}{d\log\Lambda}$ FOR DIMENSIONS

↑ tells us about how M_0^2 varies w/ Λ

JUST DETAILS

So RECALL:  $\sim -\frac{1}{2}\frac{\lambda_0}{16\pi^2}\left[\Lambda^2 - M_0^2 \log\frac{\Lambda^2}{M_0^2} + \cdots\right]$

$\Lambda\frac{\partial}{\partial\Lambda}$ () $\sim -\frac{\lambda_0}{16\pi^2}\left(\Lambda^2 - \frac{2m_0^2}{\Lambda}\right) + \cdots$

$\Lambda\frac{\partial M_0^2}{\partial\Lambda}\frac{\partial}{\partial m_0^2}$ ()$^{-1}$ $= -\Lambda\frac{\partial M_0^2}{\partial\Lambda} + \cdots$ from 'tree level' term

$$\Rightarrow \boxed{\Lambda\frac{\partial M_0^2}{\partial\Lambda} = -\frac{\lambda_0}{16\pi^2}\left(\Lambda^2 - m_0^2\right)} + O(\hbar^2)$$

↑ "RG EQ"

↑ check this BY RESTORING ħ's COUNTS LOOPS. (PROPAGATORS come w/ ħ's)

I'M NOT GOING TO GO INTO DETAIL ABOUT ~~[illegible]~~ WHICH TERMS ARE of THE SAME Θ IN PERT THY, NOR WILL I GO THROUGH ANY DETAIL AT ALL, REALLY...

THIS IS ALL DONE VERY WELL IN, eg. PESKIN.
WHAT IS MORE USEFUL IS TO TALK ABOUT INTERPRETATION.

WHAT'S IMPORTANT

$$\Lambda \frac{d}{d\Lambda}\left(\text{physical}\right) = 0 \Rightarrow \Lambda \frac{\partial(\text{BARE PARAM})}{\partial \Lambda} \sim \frac{\lambda^{(2)}}{16\pi^2}\left(\cdots\right)$$

$\uparrow$ as func of $M_0^2, \lambda_0^2, \Lambda$

or: $\Lambda \frac{d}{d\Lambda} "\mathcal{L}_p" = 0$

WE END UP W/ A SYSTEM of DIFFERENTIAL EQS for THE UNPHYSICAL BARE PARAMETERS:

$$\frac{\Lambda}{Z_0}\frac{\partial Z_0}{\partial \Lambda} = \gamma(\lambda_0)$$

"ANOMALOUS DIMENSION," SOME FUNC
IN STAT MECH: CRITICAL EXPONENT

$$\frac{\Lambda}{m_0^2}\frac{\partial m_0^2}{\partial \Lambda} = \gamma_m(\lambda_0)$$

$$\Lambda \frac{\partial \lambda_0}{\partial \Lambda} = \beta(\lambda_0)$$

BETA FUNCTION: GIVES "RUNNING COUPLINGS"
(cf GRAND UNIFICATION)

BUT THESE ARE BARE COUPLINGS, SO WE DON'T REALLY CARE WHAT THEY DO.
WHAT DOES THIS MEAN FOR THE <u>RENORMALIZED COUPLINGS</u>?
(physical

<u>naive</u>: nothing. PHYSICAL PARAMS ARE PHYSICAL ... DON'T CHANGE ... RIGHT?

no: RECALL :  $\sim \lambda^2 \log M^2/p^2$

PHYSICAL 2-PT FUNC DEPENDS ON ENERGY @ WHICH PROBED

WE CAN REPACKAGE THE RG EQS INTO DIFF EQ ON PHYSICAL COUPLINGS

W/O GOING INTO DETAILS:

$$\frac{\mu}{Z}\frac{\partial Z}{\partial \mu} = \gamma(\lambda)$$

$$\frac{\mu}{m^2}\frac{\partial m^2}{\partial \mu} = \gamma_M(\lambda)$$

$$\mu\frac{\partial \lambda_i}{\partial \mu} = \beta_i(\lambda)$$

← GEN. TO MANY COUPLINGS.

I'LL LEAVE DETAILS + PHILOSOPHIZING TO YOU & TO NEXT SEMESTER.

BUT: THIS IS A SYS OF DIFF EQ, BC SET BY RSN. COND.

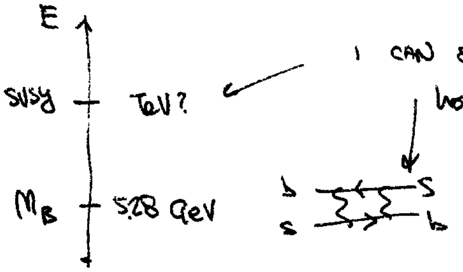
μ: COULD BE Λ. BUT MORE GENERALLY WE USE A MASS-INDEP. SCHEME, eg DIM REG. THE SCALE μ NEEDN'T BE TIED TO PHYSICAL SCALES: M^2 OR Λ^2.

SOLUTION TO THESE EQNS IS A FLOW IN $\underbrace{(Z, m, \lambda)}_{\text{theory space}}$ SPACE

theory space.

WHAT YOU SEE IN DIM REG HW: CHOOSING $\mu^2 \sim p^2$ MAKES THE LOOP LOGS SMALL AND PERT THY WELL BEHAVED.

BUT SOMETIMES THIS ISN'T GOOD ENOUGH.
eg. FLAVOR PHYSICS.



postulate

I CAN SPECIFY PHYS HERE

how does it affect phys here?

logs REAPPEAR.

SAME EQ AS ALWAYS.

$$g(\mu') \sim \frac{g(\mu)}{1+\frac{g}{16\pi^2}\log \mu'/\mu}$$

TURNS OUT THAT THE RG EQS SOLVE THIS BY RESUMMING THESE LOGARITHMS!

INTUITION: WE DO RG TRANSF INFINITESIMALLY; RENORMALIZE @ EACH STEP S.t. SMALL LOGS ARE REABSORBED INTO RUNNING COUPLING WHILE PERTURBATIVE.

TECHNICAL: <u>REALLY</u> A RESUMMATION of LOGS TO ALL ORDERS IN PERT THY!! SEE MY EFT NOTES IN THE COURSE FOLDER.

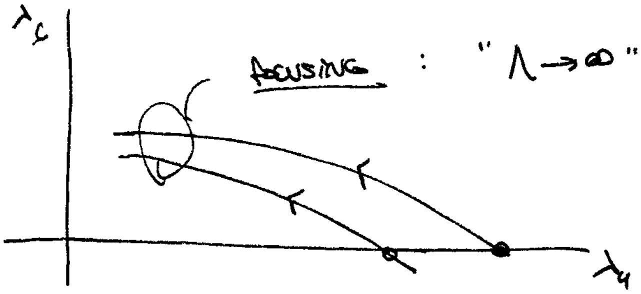
↳ not a resummation of whole PERT EXP, JUST PART of IT//

LET ME GIVE A PICTURE... SLIGHTLY DIFFERENT PERSPECTIVE (POLCHINSKI - EXACT RENORMALIZATION GROUP)

CONSIDER THE FLOW IN THE $\underline{\lambda_4, \lambda_6}$ PLANE

$\hookrightarrow \frac{\lambda}{4!}\phi^4 + \frac{\lambda}{6!}\phi^6$

<u>RG FLOW</u> :



<u>FOCUSING</u> : "$\Lambda \to \infty$" (<u>CONTINUUM</u> LIMIT)

or STATMECH

FLOWS OF TWO DIFFERENT THEORIES

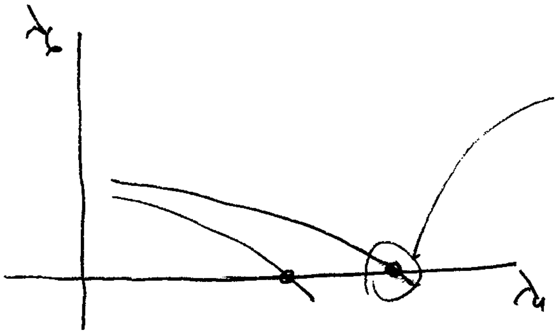
<u>MEANING OF FLOW</u>: AS I TAKE $\Lambda \to \infty$ ($\Leftrightarrow$ E SMALL BY DIM ANALYSIS THE PARAMETRIC DEPENDENCE IS E^2/Λ^2.)

<u>CRITICAL SURFACE</u> : THE RG FLOW TAKES AN @ n - DIM THEORY SPACE ($n \sim \infty$) AND PROJECTS IT DOWN TO LOWER DIMENSIONAL SPACE (FINITE).

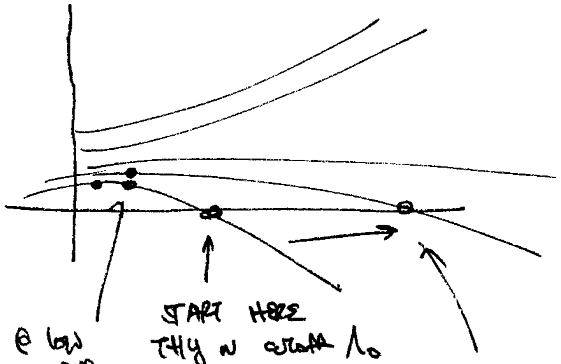
$\rightarrow$ YOU KNOW THIS: WE ONLY CARE ABOUT <u>RENORMALIZABLE</u> THEORIES. NON RENORMALIZABLE THEORIES @ HI E PROJECT DOWN TO REN. THEORIES @ LOW ENERGIES.

$\hookrightarrow$ WE CAN PARAMETERIZE DEVIATION FROM REN THY BY CONSIDERING SMALL PERT OFF THE CRITICAL SURFACE.

"<u>EXACT RG</u>":



START W/ THEORY W/ λ_4 ONLY. NICE THEORY. RENORMALIZABLE, etc.



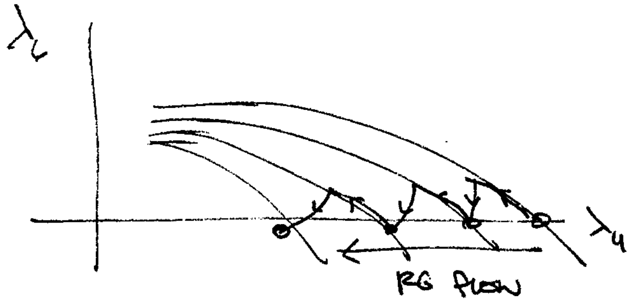
@ low E, END UP HERE

START HERE THY w/ cutoff Λ_0

this is a theory that GOES TO SAME FIXED POINT (ie VERY CLOSE TO LOW E THY) BUT w/ A LARGER CUTOFF, ie IT RUNS FOR A LONGER TIME TO GET THERE.

<u>WHY DO THIS</u>: CAN TALK ABOUT FLOW W/IN MY ϕ^4 THEORY W/O HAVING TO CALCULATE LOOPS w/ ϕ^6 INT.

→ IT IS <u>NOT</u> TRUE THAT ϕ^6 TERM IS ZERO IN IR
→ ONLY THAT IT IS DETERMINED BY ϕ^4 TERM.



YET ANOTHER WAY of LOOKING AT IT

RG flow IS WHAT HAPPENS AS WE GO TO LOWER & LOWER ENERGIES.

$\Leftrightarrow$ Λ GOES TO ∞

Wilsonian Picture : WE ARE "INTEGRATING OUT" UV PHYSICS

BEFORE TECHNICAL REMARKS: YOU CAN SEE WHY THIS WOULD PROJECT DOWN TO A SUBSPACE (of renormalizable couplings) IN THEORY SPACE. YOU'RE "THROWING AWAY" INFO.

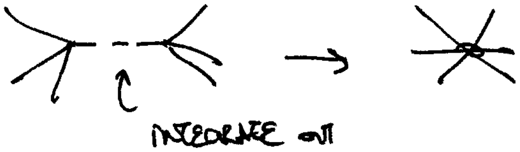
BASIC IDEA: EACH MODE (of THE CONTINUUM of MOMENTA) IS A HARMONIC OSCILLATOR.

WE WANT TO "INTEGRATE OUT" THOSE MODES.

reference to the path integral

HEURISTICALLY: EXPLICITLY SOLVE THEIR FREE THY SINCE THEY DON'T PARTICIPATE IN LOW E OBS.

BUT BASIC IDEA IS SAME AS OUR ϕ^6 THEORY:



→ this is done in ch 12 of PESKIN.

GOING FROM A THY W/ MODES UP TO Λ

→ THY W/ MODES UP TO $\Lambda' < \Lambda$

⇝ SCALE TRANSFORMATION.

$$\text{RG}: \; S[Z(\mu)^{1/2}\phi;\mu,g_i(\mu)] = S[Z(\mu')^{1/2}\phi;\mu',g_i(\mu')]$$

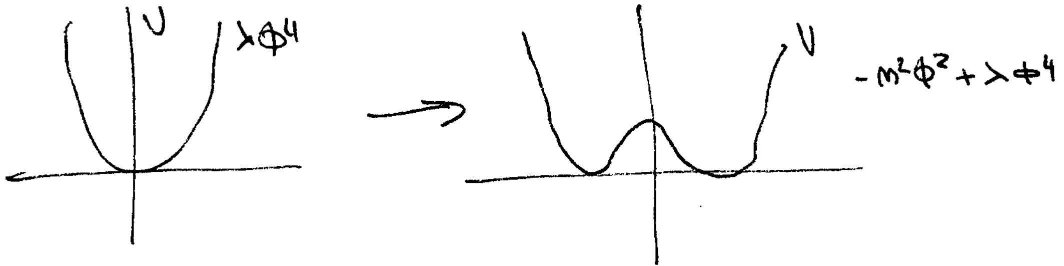
CAN "UNDO" RG TRANSF w/ SCALE TRANSF
BUT UNDER SCALE TRANSF g_i's DON'T CHANGE.

↳ SO NEW THY IS NOT THE SAME

SO WHAT:

- COUPLINGS CHANGE → they can go from strong ⟷ weak
  ↳ DIFFERENT PHASES.

- PARAMETERS CHANGE: CAN GENERATE NONTRIVIAL POTENTIAL



"PHASE TRANSITION"