

# The Allure of Particle Physics: Neutrinos, Quarks, and the Higgs Boson

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## 1 Preface

These notes were prepared for a seminar on particle physics as part of *SPLASH!*, which is sponsored by Stanford University’s Educational Studies Program. For more information regarding *SPLASH!*, see reference [1].

## 1.1 References

On the subject of references, I will try to include a lot of them! This is for two reasons: (1) responsible academic writing requires careful citation of works to give credit where it is due, and (2) so that you, as students, can read further into topics that interest you. This is important! The goal of this *SPLASH!* seminar is to get your feet wet and to introduce a new subject to you. It's up to *you* as the student to go nurture any interest in what is presented to you. For your convenience, references are included at the end of each section. Many of the references are books that you can find at public libraries or in a university physics library, such as Stanford's (which is open to the public during regular hours).

## 1.2 Prerequisites

This course was designed for intermediate high school students who have had courses in algebra and, ideally, some physics<sup>1</sup>. The most important requirement, however, is an open mind and an interest in the subject. Don't be afraid if a lot of this is unfamiliar, if it gets you interested try to take in as much as you can and then go to the references to dig deeper.

## 1.3 Errata

I have done my best to make sure that this document is free of errors. While some level of metaphor (physicists call this "hand waving") is required to bridge the years between a first course in physics and a formal course in particle physics, it is my intent to keep this document scientifically honest and true to the spirit of the physics herein discussed. I will do my best to keep an updated version of this document on my personal webspace to account for any errors or partial-errors brought to my attention post-printing [2].

## 1.4 Typesetting

For those who are considering a future in science or mathematics, this document was typeset using the  $\text{\LaTeX}$ typesetting system. In physics and mathematics nearly all professional papers are published using this system and students are expected to 'pick it up' via osmosis of some sort. Tech savvy students might be interested in getting a head start and familiarizing yourself with the system—I suggest starting at the official  $\text{\LaTeX}$ project webpage [3] and looking for a good set of instructions for Windows installation [4].

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<sup>1</sup>It is unfortunate that many schools have relegated physics to the end of the high school curriculum. By the way, I'll use footnotes for parenthetical statements that aren't central to the text.

## References

- [1] <http://www.stanford.edu/group/esp/splash/>.
- [2] <http://www.stanford.edu/~flipt/>, however this webspace will become obsolete in June 2006. Google or contact me directly to determine its URL afterwards.
- [3] <http://www.latex-project.org>.
- [4] <http://www-cryst.bioc.cam.ac.uk/~paul/latex.html>, January 21, 2006.

## 2 Setting the Stage

Let's start with a definition.

**Physics.** In current usage, restricted to The science, or group of sciences, treating of the properties of matter and energy, or of the action of the different forms of energy on matter in general (excluding Chemistry, which deals specifically with the different forms of matter, and Biology, which deals with vital energy)[1].

*Particle* physics, in particular, is the study of the properties and actions of the elementary building blocks of matter. To motivate this field of science, we should consider a brief and necessarily incomplete history of particles<sup>2</sup>.

### 2.1 Democritus and Atomos

Depending on who you ask, particle physics began with the ancient Greek philosopher Democritus<sup>3</sup> [2]. Democritus had the great idea that all matter—sand, water, people—was made up of identical, indivisible pieces that he called *atomos*, meaning *indivisible*<sup>4</sup>. Why was this important? For the first time in documented human history, somebody thought that all matter was made of the same fundamental stuff—*atomos*. The *atomos* that made up water were the same

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<sup>2</sup>I'm not sure, but I suspect that the phrase “a brief history of...” which has become very popular in recent book titles originated with Stephen Hawking's bestselling popular physics book, *A Brief History of Time*. This book is said to be one of the most purchased books that few people actually ever read I am guilty of this, and would suggest that young aspiring theoretical physicists instead “read” the illustrated version. At any rate, Hawking's title is a play on words, but since there are far too many books titled “An Introduction To...,” I imagine that authors began to use “A Brief History of...” instead, even without the Hawking's wordplay.

<sup>3</sup>These days physics and philosophy don't always see eye to eye, but in ancient Greece, science and philosophy were one in the same, perhaps because they were both searching for ‘truth’ in some sense. Ever wonder why scientists have Ph.D's—doctorates in *philosophy*?

<sup>4</sup>For you SAT junkies, the roots of this word are *a-*, meaning not, and *tomos*, meaning cut. For example, PET (a way doctors image the brain) stands for Positron Emission *Tomography*—tomography means taking cross sections, or slices (cuts) to form a picture (*graph*) of an object.

atomos that made up wine. The *atomos* that made up lead were the same as the *atomos* that made up gold. While this certainly has philosophical implications, physicists can look back and say that this helped nurture the development of the natural sciences because now all of nature was made up of the same *atomos*, which means that if we understood these *atomos*, then we'd understand nature at a fundamental level and we'd be able to reconstruct (hopefully) all of the complicated and varied behavior that we observe around us. (After all, how could fire and water be made of the same *atomos* but behave so differently?) Now people had a reason to study particle physics. The first step (and perhaps the most important) was figuring out what these elementary building blocks were.

## 2.2 Atoms and Beyond

Fast forwarding in scientific history [3], Western scholars developed a scheme for classifying matter based on Mendeleev's periodic table, in which a handful of *atoms* constitute everything. But as science progressed, more and more atoms were discovered. Now we have over one hundred atoms on our periodic table. Hum! In hindsight<sup>5</sup>, things were starting to look a little ugly. Democritus' idea of a single indivisible constituent material turned into a plethora of many indivisible constituent materials! Why should there be so many? Perhaps each of the atoms were made up of different configurations of even more fundamental particles?

Around the turn of the century, an English physicist at Trinity College at the University of Cambridge named Joseph John Thompson, discovered the **electron**, a particle smaller than an atom that seemed to be an even more fundamental and that made up atoms. One of Thompson's students, Ernest Rutherford, later discovered that most of an atom's mass was concentrated in a nucleus which was composed of positively charged **protons** (Hydrogen nuclei) and, he hypothesized, neutral particles called **neutrons**. And so we have the picture of three subatomic particles—the electron, proton, and neutron—which come together and form atoms, which, in turn, form molecules, which, in turn, form all the more complicated things in the universe. But are each of those subatomic particles fundamental, that is, indivisible in the sense of Democritus' *atomos*?

## 2.3 The Twentieth Century

The twentieth century was revolutionary for physics. On the one hand, it brought forth Albert Einstein's theory of relativity. On the other, it ushered the era of quantum physics. I will introduce these topics below.

After the Manhattan Project that developed the nuclear bomb, the United States government poured a lot of money into **high energy physics**. This is just another name for particle physics. To see why, consider the one thing

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<sup>5</sup>And this is something we can only say in hindsight, since nature is the way it is. Science tries to understand it objectively, even if it doesn't conform to what we originally expect.

anybody knows about Einstein:  $E = mc^2$ . This means energy,  $E$ , and mass,  $m$ , are equivalent up to a factor of the speed of light,  $c$ , squared. To study fundamental particles, physicists collide protons or electrons at high energies, causing them to produce new particles. In order to produce heavy, exotic particles, one needs to collide the protons or electrons at very high energies. Hence the name ‘high energy physics.’ In the latter half of the twentieth century, national laboratories such as Fermilab and the Stanford Linear Accelerator Center pioneered the search for even more elementary particles. The particles and patterns of interactions they discovered have laid the groundwork for what is now called the Standard Model of particle physics.

Quantum physics, further, has challenged our ideas of what ‘particles’ really are. This is something we’ll discuss later on. I will briefly talk about string theory and the physics of the twenty-first century towards the end of this document.

## 2.4 The Twenty-First Century

By the 1980s, particle physicists had developed a theory of elementary particles that was so successful and accurate in its predictions that they called it the **Standard Model** of particle physics. The Standard Model accurately described the behavior of the weird *quarks* that made up protons and neutrons, the elusive neutrinos that pass through your body at a rate of about ten *trillion* per second, and the exotic composite particles discovered in accelerators. By the end of the century, nearly all of the particles that the theory predicted had been discovered exactly as the theory had predicted. The two notable exceptions are the discovery that neutrinos have mass in the 1990s and the expected discovery of the **Higgs boson** at the **Large Hadron Collider**—the largest particle collider to date (expected to turn on in 2007).

## 2.5 Modern Physics and Eastern Philosophy?

I should note one more thing. I said above that *Western* scholars developed the framework for modern science. By and large this is true beginning, perhaps, around the Renaissance. However, this is not to say that Eastern thought has not contributed significantly to physics. In fact, many of the foundations of mathematics were developed in the Middle East. In more recent times, many have tried to draw parallels between Eastern mysticism and modern physics. The quintessential book for this is the *Tao of Physics* [4]. However, books such as this and more speculative works such as the recent movie *What the Bleep Do We Know* tend not to be taken very seriously by most scientists and tend to discuss qualitative and metaphorical connections rather than scientific fact.

## References

- [1] “**physics, 2**” The Oxford English Dictionary. 2nd ed. 1989. OED Online. Oxford University Press. 13 Jan. 2006 <http://dictionary.oed.com/>.
- [2] For a fantastic introduction to philosophy, I recommend Jostein Gaarder’s book *Sophie’s World*. Democritus is described in the fifth chapter. For more formal descriptions, I refer you to any beginning philosophy text.
- [3] And here I’m really skipping over some fascinating science and history that I encourage you to look into. Your high school physics and chemistry books might be a good place to start. Just *how* did these clever scientists discover these things, anyway? In this section I paraphrase the relevant articles from Wikipedia, <http://en.wikipedia.org> as of January 21, 2006. Please note that this is not a rigorous nor comprehensive source, and I recommend independent encyclopedic research on your own part if you intend to use this information in your own scholarly work.
- [4] Capra, Fritjof. *The Tao of Physics*. Fritjof Capra also cowrote the screenplay for a film called *Mindwalk*, which I recommend to any young scholar.

## 3 Scale

In the previous section, we discussed how physicists studying ‘fundamental’ physics kept looking for ‘elementary’ particles: the “stuff” that ultimately makes up all other stuff. Implicit in this is the idea that ‘elementary’ particles are *smaller* than the non-elementary stuff it makes up. This is just like saying that individual Lego blocks are smaller than the model of the Death Star that you made out of them.

### 3.1 The Importance of Scale

Scale turns out to be an incredibly important idea in physics for detailed reasons that are generally outside the scope of this seminar[1]. The gist of the idea, however, is that *physics is different at different scales!* That is to say that *nature* behaves differently depending on how ‘big’ or how ‘small’ you’re examining it. *This is both very surprising and very obvious.* This is surprising because many of us have been trained to read ‘facts’ out of textbooks and accept them as ‘truths’ (this is *not* science, even though one might be learning *about* science<sup>6</sup>). Why should these ‘truths’ be any different depending on whether you’re looking at a micrometer or a kilometer? Who is to say what ‘big’ or ‘small’ is? (Certainly atoms and galaxies would have different opinions on this, if they have opinions at all.)

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<sup>6</sup>The difference is the scientific method. I point this out only because recent events dealing with science education have made it clear that a distressingly large number of American citizens do not have a clear understanding of what *science* is and why it is important.

However, it is actually very obvious that physics should be different at different scales because we deal with situations every day where big things behave differently from small things. We know that giant ants from cheesy 1950s horror films cannot exist because their legs wouldn't be able to support their body weight[2]. We know that little water beetles can float on a pond through surface tension alone, but a battleship floats because of buoyancy (so they both float, but for very different reasons—i.e. because of different physics). More extremely, we know that atoms behave very differently from anything on a human scale (though curiously enough we believed for a long time that atoms looked very much like the solar system).

The best example, perhaps, is Newton's theory of gravity. Every high school physics student knows how to calculate the gravitational pull of the moon by the Earth. But every college physics student knows that Newton's theory is only an approximation to a more accurate theory, namely Einstein's theory of gravity (general relativity). Why, then, do we 'waste time' teaching high school students theories that are 'incorrect'? Because at the scales that high school physics cares about (something between the scale of the solar system and small ball bearings) the predictions of Newton's theory is *pretty darn close* to those of the 'correct' theory. In fact, the difference is imperceptibly small. For a student, however, the difference is significant: the Newtonian calculation is much easier and shorter than that the complicated general relativity calculation. The lesson of this, then, is that at different scales there are different *effective* theories, which we can think of as "really, really good approximations." Of course, if we were to do a calculation of the gravitational effects of a very massive galaxy, Einstein's approach is significantly more accurate than Newton's—so the Newtonian approximation is not useful outside of its regime of effectiveness.

It turns out that there's a little more to this than just the fact that one theory is a simpler "approximation" of another, and this leads to the idea of *effective field theories*, which we'll discuss very briefly in the section on quantum mechanics below.

## 3.2 Scales and Sciences

With all that in mind, I'd like to get the ball rolling with an overview of scales and associated sciences to put particle physics in proper perspective. Today modern physics deals with nature both at the largest (cosmology) and smallest known scales (particle physics). The classic introduction to scale, which I recommend highly, is Charles and Ray Eames' famous short film, *The Powers of Ten*, where the Eames discuss objects from  $10^{25}$  to  $10^{-18}$  meters[3]. Much of the material here, however, is motivated by lectures by Savas Dimopoulos<sup>7</sup>, who begins many his courses (even graduate courses) with a lecture on scale. In the following, everything is given to the nearest power of ten. When you're comparing the size of a galaxy and the size of an ant, it's ok if you're off by a

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<sup>7</sup>A professor theoretical physics at Stanford and the 2006 recipient of the American Physical Society's Sakurai Prize for lifelong contributions to theoretical physics.



factor or two of ten<sup>8</sup>.

Length	Science	Examples
$10^{-35}$ m	String Theory (?)	Approx. width of a hypothetical string. At distances smaller than this, the act of making a measurement might create a little black hole.
$10^{-15}$ m	Particle Physics, Nuclear Physics	Approx. scale of subatomic particles such as quarks. However, at this scale, the ‘width’ of a particle is ill-defined due to quantum uncertainty.
$10^{-10}$ m	Atomic and Solid State Physics	This is about the width of a hydrogen or helium atom.
$10^{-8,9}$ m	Chemistry	The width of a molecule, like DNA
$10^{-5}$ m	Microbiology	The width of a eukaryotic cell.
1 m	Biology	The scale of everyday things, like a golden retriever.
$10^{2,6}$ m	Ecology, Psychology, etc.	We have all sorts of “-ologies” at this scale because this is the scale relevant to humans and groups of humans on an every day basis.
$10^7$ m	Earth Science	The width of the Earth. (This is probably a <i>little</i> larger than what geologists actually study.)
$10^{12}$ m	Astronomy, Astrophysics	The width of the solar system, though this is somewhat of an ill-defined quantity depending on what you include in the solar system <sup>9</sup> .
$10^{18}$ m	Cosmology	The width of a globular cluster.
$10^{26}$ m	Cosmology	The size of the observable universe. We don’t know if the universe is infinite beyond this.

### 3.3 Scales and Physics

Much of science (and especially particle physics) has been driven by a paradigm of *reductionism*, the idea that in order to really understand something, one needs to understand its separate parts[5]. Implicit in this is the idea that there is some fundamental part that makes up everything, which is why many physicists are drawn to the study of elementary particles.

The lesson of all of this talk about scales can be summarized in the following statement:

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<sup>8</sup>This is related to many jokes about cosmologists, who, if they can predict something to within a factor of 100, are very excited that they’ve proved something.

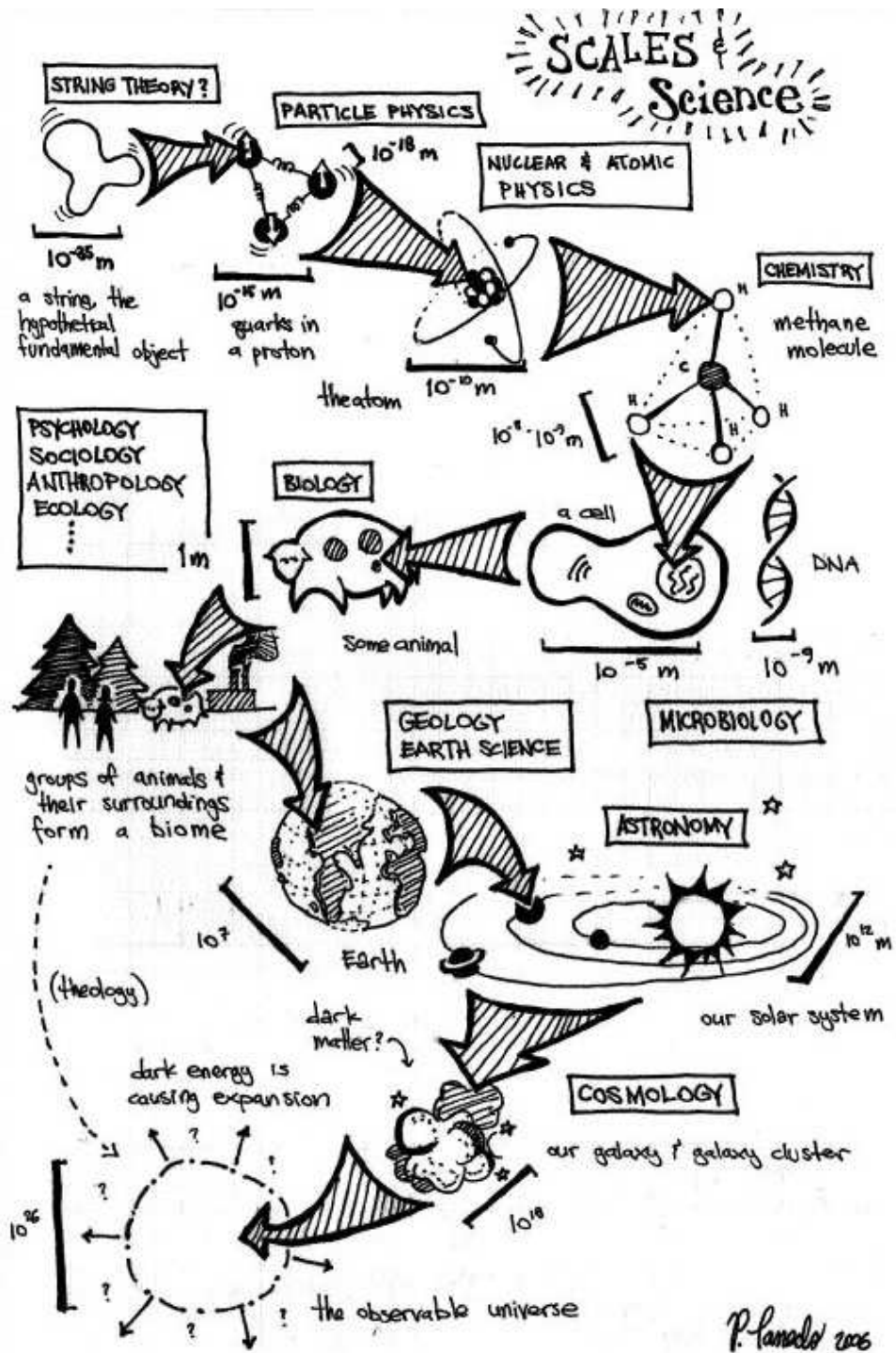


Figure 1: My own sketch of the scales of science.

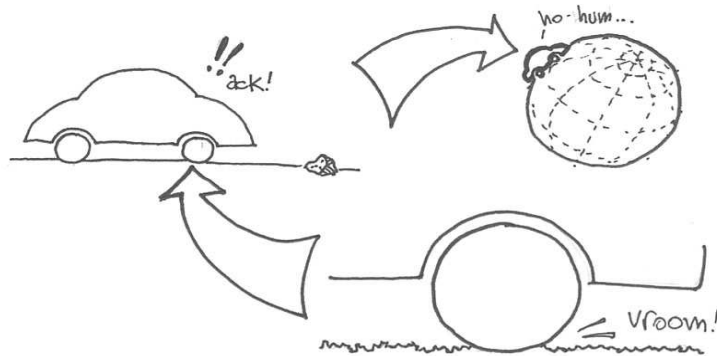


Figure 2: An example of insensitivity to different scales: a car driving on a long, straight road feels a smooth ride. The relevant scale is the width of the car’s tire; so, for example, when the driver sees a rock that is about half the size of his tires on the road, he has to swerve to avoid it. At scales much smaller than this, say the scale of a pebble, the road is actually very bumpy (take a close look at asphalt sometime). At scales much larger, such as the scale of a planet, the Earth itself is just one big bump. Despite these ‘bumps’ at different scales, the driver feels a smooth ride because he’s only sensitive to his own scale.

*A cook does not need to know string theory<sup>10</sup>.*

This means that for the most part, you only care about what nature is doing at the scale you’re considering. A cook does not need to know how the atoms that make up the molecules that make up the cells that make up the eggs that (s)he is cooking for breakfast—(s)he already has an understanding and a set of rules for how to make a meal (namely a recipe and whatever you learn at culinary school). The cook *effectively* has an understanding of what’s going on: when one puts eggs on a hot pan, they eventually coagulate into sunny side up eggs that one can eat. Of course, there’s a lot of good science in understanding what is actually going on there[6], but as far as what the cook *needs* to know, it doesn’t matter what’s going on at an atomic scale (it can be “magic” for all the cook cares). Another example is seen in figure 2

For physicists, this means that the results of particle physics 30, 60, 90 years ago are (mostly) still valid today—even if we didn’t quite understand them correctly the first time around. In fact, we know that the Standard Model (which we’ll get to) is not a complete theory—for one, it does not explain gravity. Gravity, however, is an extremely weak force, which means—for reasons we won’t get into—it becomes “relevant” (in some sense) at very small scales around  $10^{-30}m$ . From the argument above, whatever the form of the quantum theory of gravity takes (perhaps string theory), it won’t make the predictions of the Standard Model irrelevant.

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<sup>10</sup>Savas Dimopoulos

In fancy language, this concept is referred to as an **effective theory**. Newton's theory of gravity isn't *wrong* just because Einstein's theory of gravity appears to be correct. It's just a theory that is *effectively* correct (very, very, very close to the result of Einstein's theory) on the scale of everyday things (between about  $10^{-2}$  m and  $10^{10}$  m). In fact, we believe that even Einstein's theory is only an effective theory of a more fundamental theory (perhaps string theory) that unifies gravity and the Standard Model.

## References

- [1] This is formally an idea called the *renormalization group* and is best described in the paper *The renormalization group and the  $\epsilon$  expansion* by Wilson and Kogut (Phys. Rep, 1974). This is, however, a graduate-level topic.
- [2] I wrote a short “just for fun” essay on this a few years ago, it is currently available at <http://www.stanford.edu/~flipt/files/SoCo/them.pdf>
- [3] The Films of Charles and Ray Eames – The Powers of 10 (Vol. 1), Image Entertainment (1968). There is also a book by the same name.
- [4] In particular, this is motivated by his course Physics 351 given Autumn 2005. Additional information came from a section from the website of John Baez (<http://math.ucr.edu/home/baez/distances.html>), a mathematical physicist at UC Riverside.
- [5] There are some eccentrics in physics who are challenging this idea of reductionism, the main stalwart being Nobel Prize winning physicist Robert Laughlin, who proposes a new paradigm in *emergence*—that many physical phenomena are only explainable in terms of the joint behavior of a large number of parts that would otherwise be unexpected by studying only a single part. That is, “the sum is greater than its parts.” Many of Laughlin's comments haven't been received particularly well by the particle physics community (though is better received in the condensed matter physics community, in which his points of view are just facts of life). He has written a popular-level book explaining his insights: *A Different Universe*, Basic Books (2005). Check out the comments on Amazon.com for a sample of the debate. Additionally, he prepared a more technical manifesto for the National Academy of Sciences (Proceedings of..., 97, 28 (2000)), which is available at: <http://large.stanford.edu/rbl/articles/p01apr99.htm>.
- [6] See, for example: <http://www.exploratorium.edu/cooking/>, <http://www.bbc.co.uk/science/hottopics/cooking/>, and a book by Peter Barham, *The Science of Cooking* (Springer, 2001).

## 4 Special Relativity

The United Nations designated 2005 as the World Year of Physics in commemoration of Albert Einstein’s “Miraculous Year” in which he published groundbreaking papers on special relativity, Brownian Motion, and the photoelectric effect. Over the course of his lifetime, he became an international pop icon known for his famous equation,  $E = mc^2$ . Here I’ll very briefly discuss the ideas of Special Relativity, but I’ll leave most of the details to your own reading.

### 4.1 A Note: Special versus General Relativity

First a short note on nomenclature. **Special relativity** refers to Einstein’s theory of light objects that travel at very fast velocities. Special relativity is one of the standard pillars of modern physics that every undergraduate physics student learns in college and, in its most minimal form, requires nothing more than algebra to learn[1]. **General relativity** refers to Einstein’s theory of very heavy objects and how they bend “the fabric of spacetime.” This was developed about ten years after special relativity (and includes special relativity) and is more mathematically involved[2]. It is usually a graduate-level subject that is closely linked with a branch of mathematics called differential geometry. Here we will talk about special relativity. Note that you’ll hear lots of people talk about ‘grand unified theories’ that unite quantum mechanics and relativity. Statements like these refer to general relativity, as quantum mechanics and special relativity have already been successfully unified (this is called **quantum field theory**).

### 4.2 The Idea of Relativity

There are two major pillars to the theory of special relativity. If you intend to understand *anything* about relativity, you have to understand these first.

#### 4.2.1 The Principle of Relativity

The Principle of Relativity is, at the end of the day, an unremarkably obvious statement.

**The Principle of Relativity.** The laws of physics (nature) are the same in every **inertial frame of reference**.

What is this saying? An **inertial frame of reference** is one that is at constant velocity. That is to say, you are in an inertial frame of reference (or “inertial frame” for short) if you can close your eyes and you can’t tell whether or not you’re moving. In fact, this is exactly the statement of the Principle of Relativity. Imagine you were in a closed box floating in space. In your inertial frame, you think that you and your box is stationary. However, suppose your best friend is also in a box floating in space, say at some fixed velocity. As he floats by you, you think, “oh, poor bloke... he’s drifting away while I’m sitting

here not moving.” Your friend, however, sees things slightly differently. To *him/her*, it is *you* that are drifting away while *he/she* is stationary.

Who is ‘right?’ Both of course. This is because the laws of nature are the same for each of you. Each of you can run little physics experiments in your boxes and determine that the laws of physics are exactly the same as those in your physics textbooks, so long as you remain in the inertial frame.

What is *not* an inertial frame? One where the reference point is accelerating relative to another inertial frame. So a car that is speeding up is not an inertial frame. This is obvious—when we’re sitting in the car with our eyes closed, we feel a backwards force when the car accelerates. The laws of physics are *different* from when you’re just sitting in your living room.

#### 4.2.2 The Speed of Light is Constant

The second pillar of special relativity is the observation that the speed of light is constant. This is something that has been confirmed experimentally, to the extent that anything is confirmed experimentally<sup>11</sup>. The experiments that measured the speed of light, something that many people thought could be infinite until 1727 (see reference [3])—and was a very large number at any rate, are worth exploring if you have the chance[4].

The conventional wisdom before people understood electromagnetism was that if light is a wave<sup>12</sup>, then it must propagate through some medium. Water waves are disturbances in water. Sound waves are disturbances in air. What did light waves propagate in? Physicists called this mysterious substance **aether**<sup>13</sup>, though they had no idea what it was or what properties it had. Since the velocity of water and sound waves depends on the properties of the particular medium they are propagating through (depending on things like the density and the elasticity of the substance), understanding the aether would hopefully lead to an understanding of the speed of light.

While people devised clever ways to measure the speed of light, they realized that since Earth was moving *through* the aether, the speed of light should be different in the direction of the motion of the surface of the Earth. The 1887 Michelson-Moreley experiment sought to measure this path difference us-

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<sup>11</sup>This is again an important note: scientists don’t really ‘confirm’ theories in the sense that they prove them right—they just show that under rigorous testing, they haven’t been proven wrong. This is all part of the scientific method, and it’s absolutely important that young scientists understand this.

<sup>12</sup>Let’s avoid any confusion about wave-particle duality. Before Einstein’s explanation of the photoelectric effect, people were happy thinking of light as a wave. We’ll discuss this duality in the quantum mechanics section below.

<sup>13</sup>The etymology of this word is somewhat interesting. According to Wikipedia, it was originally an idea from ancient Greek philosophy that referred to the essence of the gods. Aristotle later referred to it as the “stuff” that filled empty space—since the very idea of “empty space” seemed unnatural to him. In *Relativity Visualized*, Lewis Epstein notes that ‘aether’ is derived from the greek word *αιθερω*, which means something like “I burn” or “I am in eternal motion.” To distinguish between these somewhat archaic ideas and the aether that light was proposed to propagate through, we sometimes call this latter instance **luminiferous aether**.

ing a device called an interferometer (again, the is another marvelous bit of experimental physics that is worth reading about) but the result was surprising: there didn't appear to be any measurable difference in the speed of light! All subsequent experiments measuring this *aether wind* yielded null results.

Maxwell's theory of electromagnetism was able to explain the constancy of the speed of light without invoking the aether, and now the aether has been banished from physics.

### 4.2.3 Beyond Galilean Relativity

Let me now note why this is very important. Before special relativity, textbooks will say that physicists believed something called **Galilean relativity**. I am willing to bet, however, that before Einstein this term had never existed. The reason for this is that mathematically and intuitively Galilean relativity is, as physicist Tony Zee<sup>14</sup> would say, "more obvious than obvious." (The only reason it even got a name was so that physicists could refer to the 'old way' of doing things.) It is the notion that if you are driving a hybrid vehicle on the freeway at 65 miles per hour and you pass an SUV going 60 miles per hour, the person driving the SUV sees you passing at 5 miles per hour. This all makes perfect sense (or at least it should—otherwise, I'm an awful teacher).

Let's be a little fancy-schmancy and write this down in equations. Let me define an inertial reference frame  $\mathcal{O}$ . This just means that I can set up a coordinate system in this frame and, by the principle of relativity, my physics textbooks should be valid. Further, let me define two more inertial reference frames:  $\mathcal{O}'$  moving at velocity  $v_1$  relative to frame  $\mathcal{O}$  and  $\mathcal{O}''$  moving at velocity  $v_2$  relative to frame  $\mathcal{O}$ . If all of this is a little abstract, then think of it this way: I'm standing on a sidewalk; everything I see and measure will be in frame  $\mathcal{O}$ . A cars driving on the street defines frame  $\mathcal{O}'$ . I see the car<sup>15</sup> and everything inside of it moving at velocity  $v$ . A person inside one of the cars (say, the one defining frame  $\mathcal{O}'$ ), the driver, the steering wheel, and any luggage in the trunk are all stationary within that reference frame. In fact, In the  $\mathcal{O}'$  frame, the driver observes *me* travelling at velocity  $-v$ .

Anyway, Galilean relativity just says that if there is an object moving with velocity  $u$  relative to the  $\mathcal{O}$  frame, then it is moving with velocity  $u' = u - v$  in the  $\mathcal{O}'$  frame<sup>16</sup>. Make sure that the minus sign makes sense.

Now it shouldn't be too surprising to see why this doesn't jive with the constant speed of light. Instead of 'an object moving with velocity  $u$ ,' let that object be a photon, a particle of light (we won't get into wave-particle duality yet, so just trust me that it's a particle). If I'm in inertial reference frame  $\mathcal{O}$ , I measure the speed of light to be the constant value  $c$ . This is just a fact that

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<sup>14</sup>Professor Zee is a theoretical physicist at UCSB who wrote an *excellent* text on Quantum Field Theory that I highly recommend a few years down the line if you continue on in Physics.

<sup>15</sup>By the way, the car only defines an inertial reference frame while is moving at constant velocity.

<sup>16</sup>If you are more comfortable with explicit vector notation, it is more accurate to replace all the velocities  $u$  and  $v$  with vector velocities  $\mathbf{v}$  and  $\mathbf{u}$ .

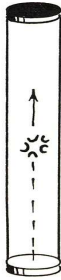


Figure 3: A schematic of a light clock from Lewis Epstein’s book *Relativity Visualized*. A photon bounces from the bottom mirror to the top and back. Every round trip is one unit of time.

is apparently one of the laws of physics. However, by the principle of relativity, the laws of physics are the same in every inertial reference frame. That means that someone else in inertial reference frame  $\mathcal{O}'$  will also observe the photon to be moving at speed  $c' = c$ . That is to say that the speed that this observer measures has the same numerical value as the speed that I observed in frame  $\mathcal{O}$ . This is a little fishy, because we just said in the paragraph above that the speed of an object (a photon) in the  $\mathcal{O}'$  frame should be  $c' = c - v$ . There’s a contradiction! Hence, if the principle of relativity and the speed of light being constant are true, then Galilean relativity doesn’t hold. At this point you might be reconsidering our postulates—but recall that (a) experiments have shown that the speed of light is constant, and (b) the principle of relativity is a very reasonable thing to expect<sup>17</sup>.

### 4.3 Spacetime Gone Wild

From these two postulates of special relativity we can actually derive some very surprising results. What we are about to do is what Einstein called a *gedanken* experiment, or ‘thought’ experiment.

#### 4.3.1 Time Dilation

Imagine you had a light clock like the one in figure 3. Such a device uses the constancy of the speed of light to measure time. The clock has some fixed length and every time the photon bounces from one end to the other and back, we count that as a unit of time. (In some sense this is the most natural way to count time.)

Now consider two reference frames,  $\mathcal{O}$  which is at rest relative to a train station, and  $\mathcal{O}'$  which is at rest relative to a train that is just passing the

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<sup>17</sup>If this isn’t satisfactory reason to believe what I’m about to tell you, then good for and you can rest assured that the theory of special relativity as a whole has also been rigorously tested



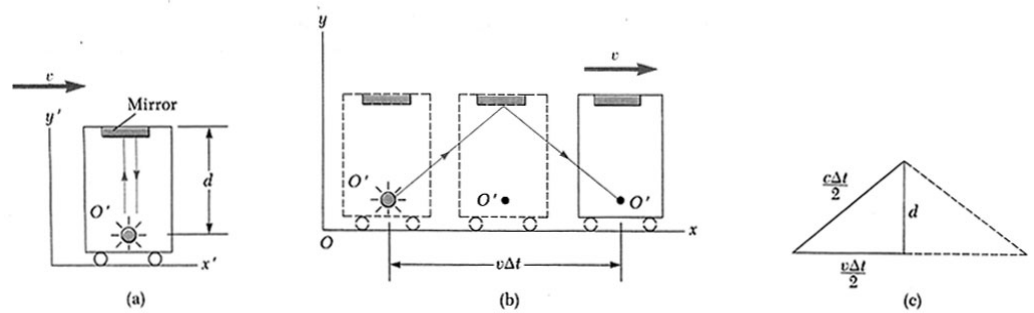


Figure 4: (a) The light clock in the  $\mathcal{O}'$  frame. (b) The light clock in the  $\mathcal{O}$  frame. (c) The triangle formed by the path of the photon in  $\mathcal{O}$ . Image from *Modern Physics*, Serway and Moses (Brooks Cole 2004).

station at constant velocity  $v$ . A passenger in the train is holding up a light clock as in figure 4(a). In one unit of time, the photon traverses a distance  $2d$ . Let us call this amount of time, as measured in the  $\mathcal{O}'$  frame,  $\Delta t'$ . Note that this means  $2d = c\Delta t'$ .

But now consider the path in the  $\mathcal{O}$  frame, figure 4(b). The photon is no longer travelling back and forth, but in a triangular path with side lengths given by figure 4(c). This is because in the  $\mathcal{O}$  frame, the train car is displaced by an amount  $v\Delta t$  over the amount of time ( $\Delta t$ ) that the photon hits the top of the light clock and comes back.

Now, the distance  $d$  has the same value in both frames<sup>18</sup>, so we can replace  $d$  in figure 4(c) by  $c\Delta t'/2$  from figure 4(a). We can then use the Pythagorean theorem to state:

$$\left(\frac{v\Delta t}{2}\right)^2 + \left(\frac{c\Delta t'}{2}\right)^2 = \left(\frac{c\Delta t}{2}\right)^2$$

Simplifying we get:

$$(c\Delta t')^2 = (c^2 - v^2)\Delta t^2 \quad (1)$$

or:

$$\Delta t = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\Delta t' \quad (2)$$

But read over equation (2) again and think about what it means! Note that the value of the fraction in this equation is greater than one. The unit of

<sup>18</sup>This is somewhat of a postulate as well. It should seem reasonable that nothing strange happens in directions perpendicular to the relative motion.

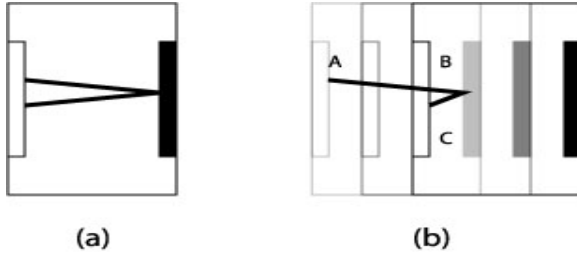


Figure 5: (a) The light clock on its side in the  $\mathcal{O}'$  frame. (b) The light clock on its side in the  $\mathcal{O}$  frame.

time in one frame is *longer* than the unit of time in the other frame! This is a phenomenon called **time dilation**. And just like that, we've begun to mess with the nature of space and time.

### 4.3.2 Length Contraction

We'll play this game once more for good measure.

Let's tip over our our light clock and repeat the experiment. Suppose our box has length  $L$ . In  $\mathcal{O}'$ , our photon travels distance  $2L'$  in some time  $\Delta t'$ . That is,  $2L' = \Delta t'c$ . We've already figured out the relation between  $\Delta t'$  and  $\Delta t$ . Why are we labeling  $L'$  differently from  $L$  when we didn't do this with  $d$  above? Because  $L$  is in the direction of motion in the train, and if something funky is going on with relative motion, then it is reasonable to suppose it would occur in the direction of motion.

In the case of the  $\mathcal{O}$  frame, there are two steps, as sketched in figure 5. From A to B, the train has moved a distance  $v\Delta t_1$ , so to hit the far mirror, the photon travels distance  $L + v\Delta t_1$  in time  $\Delta t_1$ . On the trip back from B to C, the train advances a distance  $v\Delta t_2$ , so on the return trip, the photon travels distance  $L - v\Delta t_2$  in time  $\Delta t_2$ . We then have the following equations:

$$L + v\Delta t_1 = c\Delta t_1 \quad (3)$$

$$L - v\Delta t_2 = c\Delta t_2 \quad (4)$$

From these we can write:

$$\Delta t_1 + \Delta t_2 = \frac{L}{c - v} + \frac{L}{c + v} \quad (5)$$

But the left hand side of this equation is just the total time  $\Delta t$ , which we know from above is equal to  $\frac{1}{\sqrt{1-v^2/c^2}}\Delta t'$ . Thus, substituting this along with our relation for  $L'$  and  $\Delta t'$  in the  $\mathcal{O}'$  frame:

$$\begin{aligned}
\frac{1}{\sqrt{1-v^2/c^2}}\Delta t' &= \frac{2Lc}{c^2-v^2} \\
\frac{1}{\sqrt{1-v^2/c^2}}\frac{2L'}{c} &= \frac{2Lc}{c^2-v^2} \\
L &= \sqrt{1-v^2/c^2}L'
\end{aligned}
\tag{6}$$

Once again, equation (6) requires a second look. The quantity in the square root is less than one, so the light clock appears to have shrunk compared to what it would have looked like if it were at rest ( $L'$ ).

#### 4.4 Energy and Mass

So we've already derived the main results of special relativity! As an object moves faster, it appears to shrink (length contraction) and processes within the object appear to slow down (time dilation). Of course, observers who are moving in the same reference frame as the object don't notice this shrinking. In fact, they see the rest of the world moving in the opposite direction at some velocity—and so they see everything else exhibiting length contraction and time dilation. But isn't this contradictory? If a person at the train station sees a fast train shrinking, shouldn't the person on the train see the train station expanding? No! And this (depending on how uneasy you are about it) is one of the many paradoxes of special relativity. Think about it—that would have violated the principle of relativity.

There are a few other effects that one can derive—for example, the concept of 'simultaneous events' becomes somewhat ill-defined—but unfortunately we've already gone far off topic of our main goal of talking about particle physics. If this has piqued your interest, I highly recommend *Relativity Visualized* by Epstein (a beginners book) or *Spacetime Physics* by Taylor (for students who have had a year of high school physics)[1].

One more note, in equations (2) and (6), we had the same quantity  $\sqrt{1-v^2/c^2}$  showing up. This is such an important quantity in special relativity that it has a name:  $1/\gamma$ . Or, in other words:

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

$\gamma$  measure the extent to which special relativity is different from Galilean relativity (i.e. everyday relativity). The speed of light,  $c$ , has a numerical value of 300,000,000 meters per second. That's much, *much* faster than anything we experience on a day to day basis. Thus, typical velocities  $v$  that we'd care about are much smaller than  $c$ , and so  $v/c$  is a number smaller than one. That means  $v^2/c^2$  is an even smaller number, and  $1-v^2/c^2$  is pretty darn close to 1. Thus  $\gamma$ , for everyday motion, is very close to 1, and the effects of relativity aren't noticed. Lengths are pretty much the same and time doesn't really dilate. As

you start looking at frames that are moving at velocities that are almost as big as  $c$ , however, all of this changes and  $\gamma$  becomes a number bigger than one that significantly affects lengths and times as described above.

Let me, however, describe one famous result without giving a rigorous derivation. The famous equation  $E = mc^2$  is what is what we need to understand particle physics. This states the equivalence of mass and energy–matter is just a (very compact) form of energy, and so it might be reasonable to believe that matter can be converted into energy and vice versa. In this way, one type of matter can turn into energy, which then turns into another type of matter. This is the main idea behind particle colliders.

But where did  $E = mc^2$  come from? I'll give two motivations. (1) What we've done in the previous subsections is show how the postulates of special relativity lead to the phenomena of time dilation and length contraction. All of Newtonian mechanics, however, was based on measures of distance, time, and mass<sup>19</sup>. We've already modified distance and time, so we might expect the concept of mass (and energy) to change also. Actually, I should warn you—old textbooks like to talk about mass changing, but this is an old-fashioned way of looking at things. You'll see what I mean shortly. (2) Slightly more true to the mathematical formalism (that I haven't really gotten into) but also slightly more abstract is the idea that physicists like 'invariants' – things that don't change. Examples in classical physics are energy ( $E$ ) and momentum ( $p$ ). In special relativity, it's very clear that our Newtonian concepts of energy and momentum are not invariants because space and time aren't invariant. However, it turns out that together energy and momentum are *covariant*, meaning that they change into each other in a well defined way when you go into a different reference frame. Length and time, for example, are together *covariant* because of how they transform<sup>20</sup>.

It turns out that in the case of length and time, there *is* an invariant quantity, and that is:  $\Delta x^2 - c^2\Delta t^2$ . No matter what frame you're in, this quantity has the same numerical value. It turns out that for energy and momentum, there is another invariant of the form  $E^2 - p^2c^2$ . This invariant turns out to be equivalent to  $(mc^2)^2$ . The  $m$  here is the invariant mass of the particle—i.e. it doesn't change in different reference frames. (This might be 'obvious,' but old textbooks like to talk about masses changing—this is a valid though less-enlightening point of view. Rather, we prefer to think about the energy of the particle changing. This is better because the transformation between energy and momentum is then exactly analogous to that of time and length.) Rearranging this equation, we get  $E^2 = (mc^2)^2 + p^2c^2$ . To get  $E = mc^2$ , just consider the case when the momentum of a particle is very small (say, the rest frame of the particle where it is not moving at all).

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<sup>19</sup>Consider, for example,  $F = ma$ , and check what is actually being measured in each quantity.

<sup>20</sup>This isn't immediately clear from the above analysis, I'm afraid, so I suggest looking at the references below if you'd like to study this further.

## References

- [1] In fact, I highly encourage you to do your own readings in special relativity. Most university general physics books now have chapters on “modern physics” that include special relativity. In addition, there are a few books that I think are especially good. At a beginner’s level (no formal background in physics required), I recommend Lewis C. Epstein’s *Relativity Visualized* (Insight Press, 1985). For more formal study—i.e. perhaps for someone who has completed a year of high school physics, consider *Spacetime Physics* by Taylor and Wheeler (Freeman, 1992). You can probably get pretty far by searching on the web, as well; one good site is <http://gregegan.customer.netspace.net.au/FOUNDATIONS/>.
- [2] General relativity is a little tougher to jump into, but after you’ve successfully navigated through special relativity, you can start diving in. A good popular level book is Anthony Zee’s *Einstein’s Universe: Gravity at Work and Play* (Oxford University Press, 2001). I have been told that the book *Exploring Black holes: Introduction to General Relativity* by Taylor and Wheeler (Addison Wesley, 2000) is particularly good at an introductory level. For those of you who wish for a more formal, if idiosyncratic, study of the mathematics from the ground up, consider reading *Gravitation* by Thorne (Freeman, 1973). It’s a huge, fat book that teaches differential geometry and general relativity using various ‘cartoons’ of the mathematics.
- [3] [http://en.wikipedia.org/wiki/Timeline\\_of\\_luminiferous\\_aether](http://en.wikipedia.org/wiki/Timeline_of_luminiferous_aether)
- [4] A quick search online gave this nice summary from Wikipedia: [http://en.wikipedia.org/wiki/Speed\\_of\\_light](http://en.wikipedia.org/wiki/Speed_of_light)

## 5 Quantum Mechanics

It is very possible that quantum mechanics has inspired more wonder and mystery in the general public than any other field in physics. If this is so, then it would be for very good reason. While the theory of special relativity is grounded in experimental observation (the speed of light being constant) and reasonable assumptions (the principle of relativity), quantum mechanics is based on experimental observation and *radical* assumptions. Because the theory of quantum mechanics requires a little more formalism than we can include here, we will not go into the same detail that we did in special relativity.

What we will do, however, is talk about the important principles of quantum mechanics in a way that is *true* to the subject itself. It is unfortunate that many popular science books do not present quantum physics in a way that I feel captures the essence of quantum theory.

The word ‘quantum’ means discrete, as opposed to continuous. The usual Californian analogy is the beach: the sand on the beach is discrete, with one

quantum of sand being a single grain, while the water is continuous as a fluid<sup>21</sup>. Unfortunately, it's not initially very clear what is being quantized when we first talk about quantum mechanics. As it turns out, things like the energy of a confined particle (like electrons in an atom) and angular momentum are quantized. We'll leave these topics for a more proper discussion of quantum theory[1].

What actually happens in quantum mechanics is that we question the nature of reality when it's not being observed and the concrete classical theory of physics is replaced by a probabilistic theory that doesn't make strict predictions that certain events will happen, rather it makes strict predictions about the *probabilities* that different events *might* happen<sup>22</sup>.

## 5.1 If a tree falls and nobody is around...

You've probably heard the old riddle of a tree falling in the forest with nobody around to hear it. The question is whether or not a the tree makes a sound. The riddle addresses the idea that perception of reality may, in some sense, create that reality—how do we know that falling trees make sounds when we're listening? We can't run an experiment to test this, since doing the experiment would ruin the conditions (nobody listening) that you want to study. So how do we know anything about what happens when we're not observing the world?

This idea is actually central to quantum physics, and represents the paradigm shift that physicists make as they shift from classical to quantum models of nature. In quantum mechanics something very special (and supremely strange) happens when we make observations. The gist of it is that when we're not directly observing something we don't know what it's doing. But while we don't know what it's doing, it's actually in an ill-defined 'superposition' of states where it's doing everything. (Huh?) What I mean by that is that when we observe something we see it in some definite state, this is what we *mean* by observation. However, immediately before we observed it, it was 'in-between' all possible states. I'll try to make this more clear in the following sub-sections.

The chances are that you didn't understand that last paragraph. The reason why, I think, is because your brain won't let you. Quantum physics presents such a radical, non-sensical way of looking at things that our brains just aren't used to thinking about reality this way<sup>23</sup>. This is probably why most people "learn" quantum mechanics by working through the abstract math first—because that

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<sup>21</sup>Haha! Did I fool you? The water, of course, is *not* continuous, it's made up of water molecules—which are discrete. However, at the scale that human beings care about, it is *effectively* fluid. Remember the section on scales? This reiterates the very important lesson that physics at one scale is insensitive to physics at very different scales.

<sup>22</sup>This is like saying instead of predicting whether the Stanford basketball team will win a home game game, you predict the odds of winning the next game. This is something that can be tested, for example, by considering a

<sup>23</sup>Developmental psychologists coined the phrase **object permanence** for the realization that babies have that when you cover something with a blanket, it doesn't stop existing, but is only hidden from view. Quantum physicists must try to 'unlearn' this fact that one learns when one is something like 6 months old.

way they don't always realize how strange things are under the abstraction. You should be concerned that we're futzing of the idea of reality. You should also be concerned that the very act of observation causes the system to "collapse" into an observable state from this intermediate state (what's so special about observation?). However, you should know that quantum theory has passed every possible test that the last century has managed to throw at it, and—for now at least—it looks like the quantum way of thinking represents a genuinely new paradigm for understanding the universe.

So to address the puzzle: a tree is standing. We then cover our ears and close our eyes. When we uncover our ears and open our eyes some time later, we've discovered that the tree has now fallen. The possible intermediate states were (1) it made a sound when it fell, and (2) it didn't make a sound when it fell. However, since we didn't observe this intermediate stage, the tree *both* made a sound *and* didn't make a sound while we weren't looking or listening. What does that even *mean*? Well, it's something that you have to take at face value to this level of mathematical rigor. Otherwise you can think of it as some probability distribution of possible intermediate states.

Thus, quantum physics settles an age-old question, if in a somewhat unsatisfactory (though revolutionary) way.

## 5.2 The Double Slit Experiment

The double slit experiment is meant to illustrate this principle. Consider figure 6. Suppose you had a particle starting at some point  $S$  that is later observed at point  $O$ . In between both points is an infinite, impermeable barrier with two small slits in it,  $A_1$  and  $A_2$ . In order for the particle to go from  $S$  to  $O$ , it must have passed through  $A_1$  or  $A_2$ , as shown by the dotted lines. Since we only know that the particle started at  $S$  at the initial time and ended at  $O$  at the observation time, then quantum mechanics says that the particle took both paths.

We can make this example a little more complicated. In figure 7 we have added a second impenetrable barrier. Now there are three holes in the first barrier ( $A_1, A_2, A_3$ ) and four in the second ( $B_1, B_2, B_3, B_4$ ). The possible paths are all combinations of one of the first set of holes with all combinations of one of the second set of holes. So there are now  $3 \times 4 = 12$  different possible intermediate states. In between  $S$  and  $O$  the particle apparently smears itself out over twelve possible intermediate states.

Let's extend this to its absurd limit where we squeeze in an infinite number of barriers with an infinite number of holes. Well, this is an uncountable number of possible intermediate states—but look at what's happened. If we have an infinite number of barriers jammed together (like a stack of papers), each with an infinite number of holes, then this is the same thing as having no barriers at all! (A barrier with infinite holes is empty space.) So in figure 8 we have drawn some of the possible paths that a particle takes from point  $S$  going to point  $O$ , where it is observed. In between observation points  $S$  and  $O$ , the particle takes *all* possible paths between the two points!

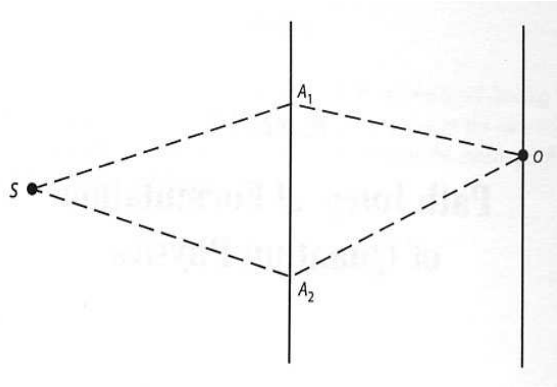


Figure 6: Double slit interference. A particle at  $S$  is observed at  $O$ , and hence must have travelled through either  $A_1$  or  $A_2$ . Quantum mechanics says that it travels through both paths. Image from *Quantum Field Theory in a Nutshell*, Zee (Princeton University Press 2003).

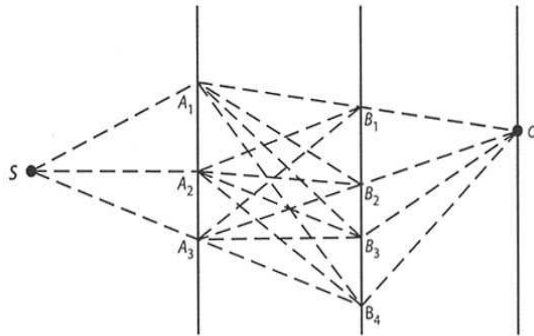


Figure 7: Multiple slit interference. Image from *Quantum Field Theory in a Nutshell*, Zee (Princeton University Press 2003).



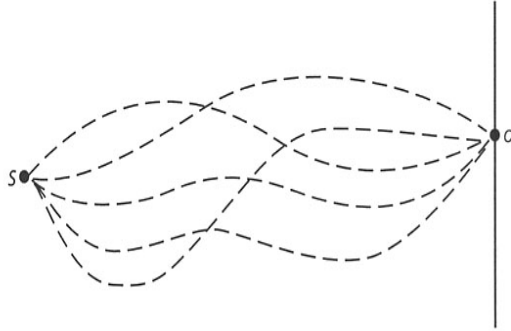


Figure 8: Infinite slit interference, which is equivalent to free propagation from  $S$  to  $O$ . Image from *Quantum Field Theory in a Nutshell*, Zee (Princeton University Press 2003).

Here's the key point: each path has an associated probability. In the case of no barriers, we would expect a particle to travel from  $S$  to  $O$  in a straight line (where we assume no external forces). This path, indeed, is the path with the largest probability. Weird squiggly paths, like the ones in figure 8, have much lower probabilities.

Does that make sense? Good. Because I 'kind of' lied to you in that last paragraph. Each path doesn't actually have a probability associated with it. To be absolutely accurate, it has a **probability amplitude**. What's the difference? The probability amplitude is a quantity that can be negative or even imaginary. The probability of a given path is given by the square of the absolute value of this amplitude. This is important because amplitudes can *interfere* with one another.

To see this, let's turn the question inside out. Until now we were considering two observation points  $S$  and  $O$ , and thinking about what possible intermediate states the particle could have taken. Let's suppose that we only know the particle started at  $S$ . What is the probability that it will be at  $O$  at some given time? To figure this out, we simply add together the probability amplitudes for each path to get from  $S$  to  $O$  in the amount of allotted time and square the quantity. That is if we label the paths by  $1, 2, 3, \dots$  and the probability amplitudes for each path by  $P_1, P_2, P_3, \dots$ . Then the probability to observe the particle at  $O$  is  $|P_1 + P_2 + P_3 + \dots|^2$ . The fact that we're squaring *after* we sum together the amplitudes makes things interesting. A rough motivation for this is that we get cross terms:  $(P_1 + P_2)^2 = P_1^2 + 2P_1P_2 + P_2^2$ . The  $2P_1P_2$  term wouldn't have been there if we added probabilities:  $P_1^2 + P_2^2$ .

Now I'd like to pause and say that I'm skimming over a lot of really good physics that *you* can understand. I've mentioned lots and lots of references in this document so far, but there is one reference that I would *especially* like to emphasize: the late Richard Feynman (whom I described earlier as the greatest

American physicist of all time) has a series of video recorded lectures introducing quantum physics (properly!) to a general audience. These are available for free online[3] and I consider them required watching for future physicists and scientists.

Anyway, I have not told you *how* we figure out the probability amplitude for a given path. This is outside of our scope and is, in fact, the subject of most of a first course on quantum mechanics. You'll have to take my word for it that these probability amplitudes turn out to obey equations that mathematically look very similar to those obeyed by waves.

### 5.3 Wave-Particle Duality

The way the double slit experiment works in practice is that one places a detector on one side of the double slits and a particle source on the other side. One then shoots individual particles out of the source and measures where each particle hits. This way, by shooting several particles, you get a distribution of hits. For a large enough number of particles, this distribution reflects the probability distribution described in the previous section. Because the probability amplitudes are governed by equations that are like wave equations, the net effect is that the probability distribution looks that of the intensity of waves interfering through the slits. In some sense, this is precisely what's happening, only the waves aren't waves of water or matter, they're waves of *probability amplitude*.

In my experience the concept of wave particle duality is often misunderstood by the general public. By particle I mean a point-like object, by wave I mean something that undergoes interference and diffraction. One of the classic examples of wave-particle duality that helped motivate quantum theory was the apparent wave-like and particulate nature of light<sup>24</sup>. Particles of light are called photons. Figure 9 shows the double slit experiment for light, which is usually explained using Maxwell's theory of electromagnetism (mentioned above in the relativity section). In Maxwell's theory light is a self-propagating electromagnetic wave, and the interference effects made perfect sense. Maxwell's theory, however, is a macroscopic approximation to the theory of quantum electrodynamics—a theory based on the principles we've outlined above. Thus, at the scale of the photons (remember the section on scales and effective theories?) what is actually happening is that the probability amplitudes are interfering and producing the wave pattern seen.

Because the probability amplitudes mimic wave behavior, sometimes they are called probability waves. To reiterate, however, the main point is that there are probability amplitudes which can interfere with each other, which means the probability distributions exhibit wavelike behavior.

If all of this is true, though, how does particulate behavior ever emerge? Once again the answer is provided by scales! You didn't think that section in the beginning was actually this important, did you? When the probability

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<sup>24</sup>Again, here I skip some rich history of physics that includes such names as Isaac Newton and Albert Einstein, among many illustrious others.

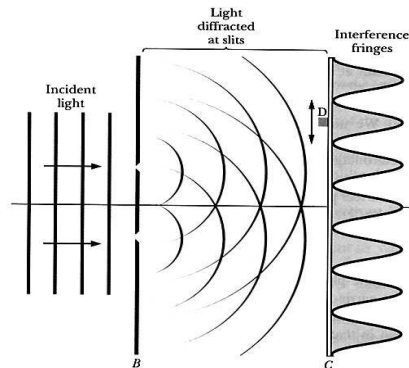


Figure 9: A schematic of double slit interference of light. Image from *Fundamental of Physics, 6th ed.*, Halliday, Resnick, and Walker (Wiley 2000).

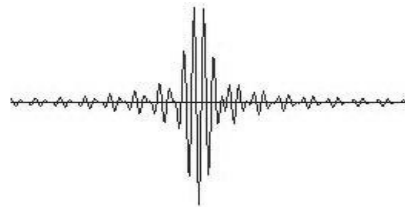


Figure 10: An example of localization of a probability amplitude. If the wavelength of an amplitude is much smaller the scale that we are making our observation, then the “wave packet” behaves like a particle.

amplitude is highly localized, as in figure 10, the probability density (which is the square of the amplitude) is also highly localized. The relevant length scale is the wavelength of the amplitude, and when our observation scale is much larger than this, then the probability density looks like a point-like particle. Such a localized “wave packet” behaves like particles, as well, for their scale makes them insensitive to the interference and diffraction effects that we would otherwise notice on our scale.

And so there it is: an honest-to-goodness explanation of wave-particle duality. It holds for photons, electrons, and all the other particles in the standard model. In fact, wave-particle duality has been observed in large molecules! (We’ll see in the next section that there is a parameter that measures what scales are affected by quantum mechanics.) There’s nothing spooky or mystical about it, despite what many popular books might claim.

## 5.4 Heisenberg Uncertainty Principle

As mentioned earlier, in quantum physics you lose some degree of predictivity. We know that in between observed states (say, the state of a particle being in position  $S$  and the state of being in position  $O$ ) particles are smeared out over all possible intermediate states. Once you observe a particle, it collapses into a definite state—instead of being smeared out over different possible positions, it manifests itself in a set position. Until you observe that particle again, it takes all possible states to get between the point of observation and the next point of observation in the future.

Now the right question to ask, however, is what are the ‘possible’ states? In the two slit experiment, the particle could take two possible paths, for example. Freely propagating particles, we saw, could take any path. However, these paths are constrained—they all have to start from the initial observation (this is a very sensible and obvious statement that doesn’t say much). Similarly, general intermediate states are constrained by their last observation. Remember that this is because the act of observation causes the probability amplitude to ‘collapse’ into a particular observable state. Said another way, when we’ve measured a particle, the probability density is such that the particle has a 100% chance of being in the state we measured, and so all paths that that particle takes must start out in that state.

But there are more than one types of observation. We can look and measure a particle’s position or its momentum, for example. One of the famous results of the mathematics of quantum mechanics is that you can’t simultaneously know a particle’s position and momentum. This is called the **Heisenberg uncertainty principle**. What it’s really saying is that when you observe a particle’s position (hence collapsing the probability amplitude so that it has a 100% chance of being in that position at that instant), there are a range of ‘possible intermediate states’ before the next measurement that have a range of different possible intermediate momenta. Now I say ‘before the next measurement’ even if the next measurement is just a microsecond afterwards. You cannot simultaneously measure both position and momentum—one measures where you are in space and the other measures how quickly you’re travelling in space<sup>25</sup>. Similarly, if you observe a particle’s momentum, there are a range of intermediate states with a range of intermediate positions. That’s actually the meat of the statement—I should mention that popular culture makes a much bigger deal of the Heisenberg uncertainty principle than physicists, for whom it is somewhat of a “cute result.”

Ah! But then you ask—wait a second, I’m a clever high school student and I can devise a clever experiment to measure a basketball’s position and momentum at a single instant simultaneously. Say, using a camera and a radar gun. Apparently this uncertainty principle idea doesn’t apply to me. Why is that? If you think about it a little more, you might guess that the answer is once again scale. There is some fundamental scale that is set in quantum mechanics

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<sup>25</sup>Note: saying that you cannot simultaneously observe position and momentum is *not* the Heisenberg uncertainty principle, which says that you cannot simultaneously *know* the position and momentum of a particle to arbitrary accuracy.

that determines when its effects are relevant. The scale is given by the wavelength of a particle's probability amplitude, and is proportional to the number  $\hbar$  (called **Planck's constant**), which actually is a measure of energy multiplied by time (don't worry about this, we'll see in the next section that it's really just a ratio).  $\hbar$  has a numerical value of roughly  $1.0546 \times 10^{-34}$  Joule seconds. In some sense,  $\hbar$  measures the degree to which measuring a particle's position will produce uncertainty in its momentum. In the language of linear algebra (which is the true mathematical language of quantum mechanics), this means that position and momentum are non-commuting operators, which you can say to impress your friends.

Mathematically, the uncertainty principle is stated as:

$$\Delta x \Delta p \geq \frac{\hbar}{2} \quad (7)$$

Here  $\Delta x$  and  $\Delta p$  measure our 'uncertainty' in how well we know the position and momentum of a particle, respectively.

Are there other observables, other than position and momentum, that have uncertainty principles? Certainly! But one particularly important 'uncertainty principle' is between energy and time:

$$\Delta E \Delta t \geq \frac{\hbar}{2} \quad (8)$$

I should warn you, however, that time is *not* an observable—so this relation doesn't really come from the same logic that equation (7) came from. However, I can give a rough (but somewhat misleading) motivation: in the special relativity section, we noted that  $x$  and  $t$  are related in a special way, namely they were *covariant*. Similarly,  $E$  and  $p$  are related in the same way. If  $x$  and  $p$  are related by the Heisenberg uncertainty principle, it's not too much of a stretch to *guess* that  $E$  and  $t$  might also. There are two things wrong with this motivation: (1) it only weakly hints that  $E$  and  $t$  might be related without addressing the fact that  $t$  is not an observable, (2) thus far we've been talking about nonrelativistic quantum mechanics, so there's no good reason to use special relativity for motivation. That being said, some mathematics (beyond our scope presently) will confirm the energy-time uncertainty principle.

What concerns us most is this result, equation (8). This says that for small amounts of time, we can have a significant variance in energy. Well wait a moment! Every half-decent course in physics will teach you that energy is conserved... so what the uncertainty principle is really telling us is that we can *violate the conservation of energy, but only for really short amounts of time!* This is like saying you can get away with breaking the law, but only so long as you do it for a short enough period of time that nobody catches you. (Clever students reading this will have already realized that the original position-momentum uncertainty principle similarly implies some violation of conservation of momentum.) I'll also note that this violation of energy conservation is only true for the

undetermined intermediate states in between observations. If I measured the energy of a closed system at two times, the energy *will* be conserved. All that the uncertainty principle says is that in between the observations the energy could have fluctuated. This idea, coupled to the idea that energy and mass are equivalent from special relativity, are central to particle physics.

Just like  $\gamma$  measured when SR is ‘on,’  $\hbar$  measures when quantum mechanics is on. As you can imagine, the scales set by  $\hbar$  are much smaller than the scales we usually care about.

## 5.5 Quantum Field Theory I – Why does every electron look alike?

One of the things that we assume about elementary particles is that they’re all elementary particles of a particular kind are identical, as if they all came from the same particle factory at the beginning of the universe. This is a sensible thing to assume; after all, every atom has exactly the same properties as every other atom of the same type (this is why we can have a periodic table) and every Lego block of a certain kind comes from a factory with some ‘standard’ mold. With Lego blocks, however, you can write your name in permanent marker on a block and you’ll be able to tell it apart from every other block of that type. We know that every elementary particle is actually indistinguishable—there is no analog to ‘marking’ one of them to distinguish it from any other.

I would now like to give a quantum explanation for why every electron looks alike. This will be the foundation of what is called **quantum field theory**, which is the marriage of quantum mechanics and special relativity.

We’ve discussed how a single particle can be represented as a probability amplitude. The next step in generalizing this is to think more about this probability amplitude over all of space. In this sense, the probability amplitude is a function over all positions. This is an example of a field—an idea you might be familiar with from concepts such as the electromagnetic field, or a gravitational field. Other examples include temperature on the surface of the Earth—every point on the Earth has a temperature which is a function of position. The *electron* field just tells us the probability amplitude of an electron existing at a certain position in the universe.

I claim that there is a single *electron* field that permeates the entire universe. A single electron is a localized disturbance on this field. This is analogous to a bed mattress with someone jumping up and down on it. That disturbance—the up and down oscillations—is exactly what’s going on in figure 10. However, we can add another person to jump on the bed mattress. Now we have two, separate, localized disturbances—which we observe as two separate particles of identical type. Why are they identical? Because they come from the same field. We can add more localized disturbances to our ‘mattress,’ but the important point is that since they are all on the *same* mattress (field, say the electron field), the resulting particles which they manifest themselves as are the same.

## References

- [1] That being said, there are still a few books that I would recommend. At a popular level, Brian Greene’s now-classic book *The Elegant Universe* (Vintage 2000) is a well-told story. The problem is that there are very few quality high-school level text that tackle the topic well. Perhaps the best is Richard Feynman’s *The Feynman Lectures on Physics* (Addison Wesley 2005, 2nd ed), which is directed at a university audience, though the three volume set is self contained. The first few chapters of the third volume should serve as a good, honest introduction to quantum theory from the greatest American physicist of all time. After some formal study of physics (and a solid handle of calculus), the standard university text is David Griffiths’ *Introduction to Quantum Mechanics* (Prentice Hall 2004).
- [2] The history of quantum theory is particularly rich given its revolutionary nature. I encourage you to read about its development and eventual acceptance by the scientific community in book such as *QED and the Men Who Made It* (by Schweber, Princeton University Press 1994). Many popular-level books, such as Hawking’s *A Brief History of Time* (Bantam, 1998) address the history of physics and modern physics in their first few chapters.
- [3] Feynman’s Douglas Robb Memorial Lectures are available for free online at: <http://www.vega.org.uk/video/subseries/8>. These lectures eventually led to the book *QED: The Strange Theory of Light and Matter* (Princeton University Press 1988), which I also strongly recommend.

## 6 Natural Units

This is just a brief introduction to absolute units, in which particle physicists express all measurements in terms of a single measurement, say, energy. The main idea is that we’ve introduced some fundamental constants of nature in the past two sections: (1) the speed of light,  $c$ , and (2) Planck’s constant,  $\hbar$ . These are essentially relations between length and time ( $c$ ) and energy and time ( $\hbar$ ).

Because these relations are ‘fundamental’ in some sense, we will set their corresponding constants to unity. That is,  $c = \hbar = 1$ . Note that we’ve also dropped units. The consequences are that we can start measuring things in odd units (as you’ll shortly see). Note that for the most part, this is just shorthand so lazy theoretical physicists don’t have to keep writing  $c$  and  $\hbar$  in their equations. (Though actually it does end up providing more physical insight—but we’ll leave that for another time.)

### 6.1 Length is Time

We can look up  $c$  to have the value  $3 \times 10^8$  m/s. Thus, setting  $c = 1$  is equivalent to setting one second equal to  $10^8$  meters. Thus everything that we measure

in meters we can now measure in seconds. This is the same idea as measuring galactic distances in ‘light-years.’

## 6.2 Energy is Time<sup>-1</sup>

Setting  $\hbar = 1$  means  $1.0546 \times 10^{-34} = 1$ . Thus one inverse second is equal to  $1.0546 \times 10^{-34}$  joules. We can measure time as inverse energy (or energy as inverse time).

## 6.3 Bonus: Mass is Energy

We have one more bonus relation:  $E = mc^2$ . Since  $c = 1$ , this means  $E = m$ . Mass is the same as energy, with a constant of proportionality given by the numerical value of  $c$ .

## References

- [1] Nobel Prize winner Frank Wilczek is currently writing a short series of articles on absolute units. The first of which, “Reference Frame: On Absolute Units, I: choices” is in the October 2005 issue of *Physics Today*.



## 7 The Standard Model

### 7.1 Quantum Field Theory II: virtual particles, antiparticles

### 7.2 Fermions: Matter Particles

#### 7.2.1 Leptons

#### 7.2.2 Quarks

### 7.3 Interactions

#### 7.3.1 Charges

#### 7.3.2 Conservation Laws

### 7.4 Bosons: Force Particles

#### 7.4.1 The photon and Quantum Electrodynamics

#### 7.4.2 The W and Z Bosons and the Weak Force

#### 7.4.3 The Gluon(s) and the Strong Force

#### 7.4.4 The Higgs Boson and the Origin of Mass

#### 7.4.5 Where's the Graviton?

### 7.5 Feynman Diagrams

#### 7.5.1 The Idea

#### 7.5.2 The Feynman Rules of the Standard Model

## References

- [1] The best *absolute* beginner's introduction to the Standard Model and particle physics is the Particle Adventure, at <http://particleadventure.org/particleadventure/>. There are some old issues of Scientific American that also have good introductions, a good compilation is *Scientific American: Particles and Fields* (Scientific American, Inc. 1953). A more recent version is *Particle and Forces, At the Heart of the Matter, Readings from Scientific American* (W.H. Freeman and Company 1990). You can view (and purchase) a copy of the Standard Model 'particle chart' through the Contemporary Physics Education Project at <http://www.cpepweb.org/>. A particularly good introductory book accessible to high school students is by R. Barnett, H. Muehry, and H. Quinn is *The Charm of Strange Quarks* (Springer Verlag 2000). Finally, a slightly more advanced book is *The Ideas of Particle Physics: An Introduction for Scientists* by Goughlan and Dodd (Cambridge University Press, 1991).

## 8 Beyond the Standard Model?

### 8.1 Why we believe in physics beyond the Standard Model

### 8.2 Supersymmetry

### 8.3 Stringy-Wingy Theory

## 9 About the Author

Philip ‘Flip’ Tanedo is in his fourth (and final) year as an undergraduate at Stanford University, majoring in Physics (B.S. Honors) and Mathematics (B.S.). During this time he has participated in research on accelerator physics, condensed matter experimental physics, and theoretical particle physics. His honors thesis is on the phenomenology of split-supersymmetric models in the Higgs resonance region. Following this year, he will spend two years of graduate study and research in the United Kingdom as a Marshall scholar. He will spend a year at the University of Cambridge pursuing a Certificate of Advanced Studies in Mathematics through their Department of Applied Mathematics and Theoretical Physics and then a year at the University of Durham conducting research with the Institute for Particle Physics Phenomenology. He plans on returning to the United States for a Ph.D. program in theoretical particle physics and, as of this writing, is waiting to hear back from the schools to which he applied. In addition to particle theory, he is very interested (and concerned) about science education in the United States. His recreational activities include triathlon training with the Stanford University Triathlon Team, steel drums with Stanford Cardinal Calypso, and reading upbeat stories (his current favorite is *Life of Pi*).