PHYS 233A, ECE 237A, PHYS 147A

Problem Set in Probability and Random Processes

1. The pdf for N_1 selected with probability p from a total of N trials is binomial $b(N_1;N,p)$. Show that in the limit of N large, p small, but pN arbritary, we obtain the Poisson pdf $p(N_1;pN)$. Show that if N itself is a Poisson random variable (mean N_0), then N_1 is distributed as $p(N_1;pN_0)$.

2. Show that the Poisson pdf $p(N_1;pN)$ approaches the normal pdf n$(N_1;pN, pN)$ with mean pN and variance pN in the limit $pN>>1$.

For problems 1 and 2 you will need Sterling's formula $N! \sim N^N e^{-N} sqrt(2)\pi N$

3. Calculate the standard errors of the sample mean and sample variance for a normal distribution where there are N measurements. Compare your result with what you expect from the Central Limit theorem.

4. Consider a random process $w(t)$ analogous to the process $z(t)$ discussed by Barrett and Swindell, Section 3.3.3 but in which the delta function $\delta(t-t_j)$ is replaced by a shift-invariant PSF $psf(t-t_j)$. Obtain the mean and autocorrelation function of $w(t)$ is $w(t)$ stationary. If so, obtain the corresponding spectral density, $S(\omega)$. Are the w(t) at different t correlated?