Problem Set for Lectures 1-2

Problem 1

(a) As given in the Data , the FT of a product of two functions is the convolution of the FT of each individual function

$FT[sinc^2(q_c\, t)] = FT[sinc(q_c\, t)] * FT[sinc(q_c\, t)]$,

with the latter two functions being rectangular functions as shown in Data 1.

The convolution of two rectangular functions is a triangular function.

Convince yourself of this!

(b) Yes, with bandwidth $2q_c$

(c) this and the other aliasing graphics follow the example presented in the lecture

Problem 2

(a) with the substitution $z= \pi[y-(x-L/2)]$, the convolution integral is in a form corresponding to the hint
(b) This is resulting in a trapezoidal function with slope 1/2 from -3/2L to –L/2, the value 1/2 from –L/2 to L/2 and slope -1/2 from L/2 to 3/2 L

Problem 3

Use the scaling theorem and $FT[sinc(t)] = rect[\omega]$ to solve this

Problem 4

You need to use that the infinite integral over the product of an odd and an even function is zero. Express the Fourier term as $\cos 2\pi\omega x + i \sin 2\pi\omega x$ and exploit that cos is even and sin odd.

Problem 5

Again, just a graphical demonstration of aliasing

Problem set for Lects 3-4

There are several unclear points in the problem set and I apologize for that. I had taken the set from an earlier exam and assumed that the problems were all well posed. The key issue is the sloppy use of ‘transfer function’. In every case where the set refers to the transfer function, it should have been ‘point spread function’. One could have suspected that from the functional form, which is always in the time/space domain and not the Fourier domain.

Problem 1

(a) $o(x) = p(x) * i(x) = FT\ [p(x)]\ i(x)$. Use scaling theorem and $FT[rect(x)] = sinc[q]$ to obtain FT

(b) Just like above, but with different i(x)

Problem 2

In taking the Fourier integral, the first part of the sum (with the δ function) reduces to 2π $J_0(0)$, the second part with the exponential function can be solved using the second part of the hint

(a) MTF = |P(rho)|/P(0) This is the general definition of the MTF. I had illustrated in the lecture that this definition can be interpreted as a contrast using a function with 3 Fourier components. However, this is true generally as all functions can be expressed in Fourier space. The same arguments applying to a function with 3 Fourier components apply to a function with an infinite number of Fourier components.

(c) Inverse Filter = $FT^{-1}[1/FT[p(r)]$.

Problem 3

In this problem, the function p(r) that needs to be transformed is rotationally symmetric. One can therefore use polar coordinates. The FT in two dimensions is

$$F_{[\![2]\!]}\{p(r)\} = \int_0^{\infty} r dr \int_0^{2\pi} p(r)\, e^{-2\pi i \rho r \cos(\theta)}\, d\theta$$

In the hint, the cos(t) term should be included in the exponent.