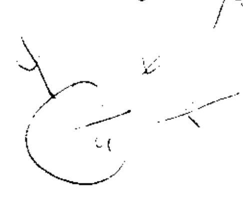
Steady-state solutions to Fick equations

$$\frac{\partial p}{\partial t} = 0 \quad \text{(steady state)}$$

$$\nabla^2 p = 0 \qquad \text{Laplace Eq}$$

Diffusion to spherical absorber



$r = a$ $\qquad p(r=a) = 0$

$p(r=\infty) = p_0$

⇒ with radial symmetry

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dp}{dr}\right) = 0$$

Solution like in electrodyn.

see Jackson (3.5) $\qquad u = \frac{p}{r}$

$$p = A\, r^{l} + B\, r^{-l-1} \qquad l = 0$$

boundary conditions

$p = A + \frac{B}{r}$ $\qquad r \to \infty \Rightarrow A = p_0$

$r \to a \qquad B = -a p_0$

$$p(r) = p_0\left(1 - \frac{a}{r}\right)$$

Flux

$$J = -D\frac{dp}{dr} = -D\, p_0 \frac{a}{r^2}$$

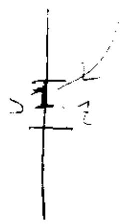
Current = flux × area

$$I = D\, p_0 \frac{a}{a^2} \cdot 4\pi a^2 = 4\pi D a p_0$$

Diffusion to disk-like absorber

$p = p_0$ at $r = \infty$

$p = 0$ at disk



cylindrical coordinates

$I = 4 D s p_0$

proportional to s not s^2!

area $\propto s^2$

conc gradient $\propto \frac{1}{s}$

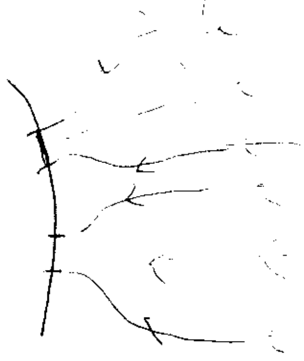
flux × area $\propto$ conc gradient × area $\propto s$

Cell absorption

N disk-like absorbers on surface of a sphere with radius a

N small $\quad I \sim 4 D N s p_0$

N large $\quad I \sim 4\pi D a p_0$



lines of flux radial for $r > a + \delta a$ i.e. constant p but converge on disks