Basic Control Theory.

Often discussed in Frequency dom

$$\mathcal{L}\,[y(t)] = y(s) = \int_0^\infty y(t)\, e$$

Laplace Transform

Why? Easier than FT for no
LT defined even for un

$$\mathcal{L}\left[\frac{d^n y}{dt^n}\right] = s^n\, y(s) \qquad n\text{-t} \rightarrow$$

$$\mathcal{L}\left[\int y(t)\,dt\right] = \frac{1}{s}\, y(s)$$

$$G * H = \int_0^\infty G(\tau)\, H(t-\tau)\, d\tau$$

$$\rightarrow \mathcal{L}\,[G * H] = G(s)\, H(s)$$

Transfer Function

$$u(s) \;\circ\!\longrightarrow \boxed{G(s)} \longrightarrow\!\circ\; y(s)$$

$$G(s) = \frac{y(s)}{u(s)}$$ transfer functi

Example: $\dot{y}(t) = -\omega_0\, y(t) + \omega$

LT $\quad s\, y(s) = -\omega_0\, y(s) + \omega$

$$G(s) = \frac{1}{1 + s/\omega_0}$$

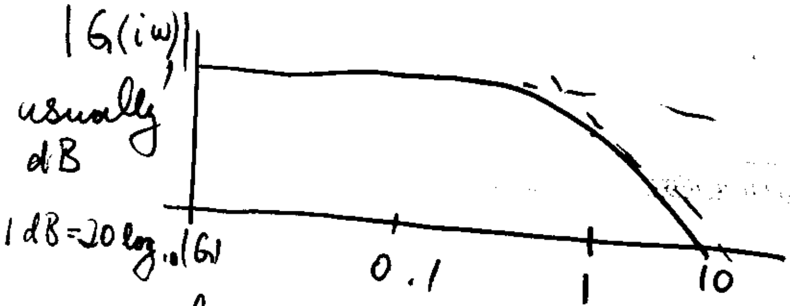
What can we learn from G

Frequency dependence look at

$$G(i\omega) = \frac{1}{1+i\omega/\omega_0} \qquad \chi(\omega$$

$$|G(i\omega)| = \frac{1}{\sqrt{1+\omega^2/\omega_0^2}}$$

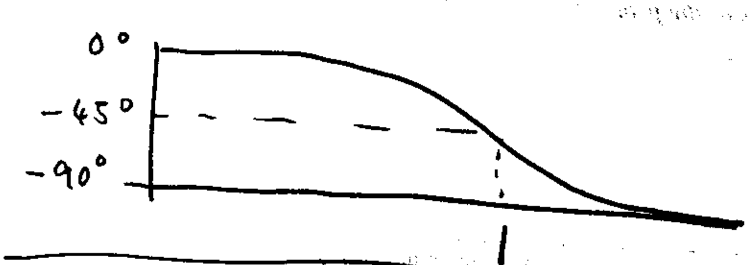
$$\arg G(i\omega) = -\operatorname{atan}\frac{\omega}{\omega_0} \qquad \text{Pha}$$



Bode

ω/ω_0

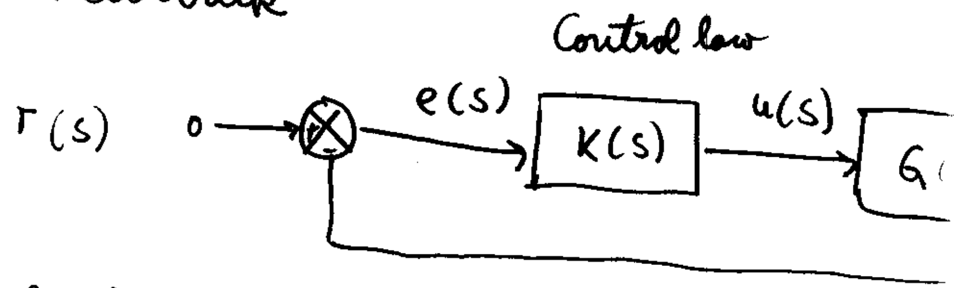
low-pass filter with $\omega_0 = 2\pi f_0$



Singularity (Pole) at $s = -\omega_0$

real negative → exp. decay

Feedback



Goal : make y like r

$e(t) = r(t) - y(t)$ error sig

$y(s) = K(s)\,G(s)\,e(s)$

$y(S) = \dfrac{K(s)\,G(s)}{1 + K(s)\,G(s)}\, r(s) = \dfrac{L}{1+}$

$L(s) := K(S)\,G(s)$ (loop) gain $\quad$ o

$T = \dfrac{KG}{1+KG}$ $\quad$ closed loop dynam

negative feedback $\quad e = r - y$

with prop. control law $\quad u = k$

Example $\quad G(s) = \dfrac{G_0}{1+s\omega_0}$

$K(s) = K_p$

$T(s) = \dfrac{K_p G_0}{K_p G_0 + 1 + s/\omega_0} = \dfrac{K_p G_0}{K_p G_0 + 1}$

new low-pass filter

new gain $K_p G_0 / K_p G_0 + 1$

new cutoff frequency $\quad \omega' = \omega_0 ($

Why Feedback ?

Consider open-loop $\quad y_\infty = K_p$ (
vs.
closed-loop $\quad y_\infty = \dfrac{K_p G_0}{K_p G_0 + 1}$

In open loop, one could set K

whereas in closed-loop get $y_\infty =$

$y_\infty \cong r_\infty$ for $K_p \gg \frac{1}{G_0}$

But ! if ~~D~~ gain G_0 drifts, open
is off

→ <u>Parametric sensitivity S</u>

of a quantity Q w. respect to a
a parameter P

$$S \equiv \frac{P}{Q}\,\frac{dQ}{dP}$$

$$S_{open} = \frac{G_0}{K_p G_0}\,\frac{d}{G_0}\left(K_p G_0\right) = 1$$

$$S_{closed} = \frac{1}{1+K_p G_0}$$

For $K_p G_0 \gg 1$
and thus much
to fluctuations i

Sensor Noise and Output disturba

Sensor noise $e = r - y - \xi$

Output disturbance d

$$y = KG e + \text{(}$$

$$y(s) = \frac{KG}{1+KG}\left[r(s) - \xi(s)\right] + \frac{\ }{1+}$$

controls to $r - \xi$

$$e_0(s) = r(s) - y(s) = S(s)\left[r(s\right.$$

e_0 : tracking error

$$T(s) = \frac{KG}{1+KG} = 1 - S \quad \text{compl.}$$

If S small, disturbances are

$\rightarrow$ $T = 1 - S$ large : sensor

Time Domain,

$$u(t) \longrightarrow G \longrightarrow y(t)$$

$$\dot{\vec{x}} = A\vec{x} + \vec{b}u \qquad u = -k^T \vec{x}$$

$$y = \vec{c}^T \vec{x}$$

Problem : Choose feedback vecto
so ~~that~~ $\dot{\vec{x}} = \vec{A}'\vec{x}$

$\vec{A}' = A$ -

desir

Transfer Fct.

$$x = \underbrace{[sI - A]^{-1}\vec{b}}_{\cancel{G(s)}} u$$

$$G(s) = \vec{c}^T (sI - A)^{-1} \vec{b}$$

Coordinate change does not affect

Some subtleties

$$\dot{x}_1 = \lambda_1 x_1 + b_1 u$$

$$\dot{x}_2 = \lambda_2 x_2 + b_2 u$$

$$y = c_1 x_1 + c_2 x_2$$

If any $b_j = 0$ then the system is not c

If any $c_j = 0$ " " " " " o

General controlability condition

look at $\vec{A}^i \vec{b}$ $i = 0$

If $M_b \equiv (\vec{b} \;\; A\vec{b} \;\; A^2 b \;\; \ldots$

is invertable, then the syst