Non-perturbative Simulation of Resonant Modes*

N. N. Gorelenkov¹, C. Z. Cheng¹, M. J. Mantsinen², S. E. Sharapov³

6th October 2003

¹ Princeton Plasma Physics Laboratory, Princeton, NJ, 08543, USA;
² Helsinki University of Technology, Finland;
³ Euroatom/UKAEA Fusion Assoc., Culham Science Centre, Abingdon, Oxfordshire, OX14 3DB, UK

8th IAEA TCM on Energetic Particles
6-8 October 2003, San Diego, CA

*This research was supported in part by the U.S. Department of Energy under the contract DE-AC02-76CH03073 and in part by the European Fusion Development Agreement.
Motivation to develop the nonperturbative numerical code

Experimental motivation

Numerical procedure

Results and comparison with theory
**Introduction**

- Resonant (or Energetic Particle (EPM)) Modes are often observed in tokamak experiments in the presence of hot energetic ions (fishbones, BAEs, R-TAEs).
- New types of low frequency $n = 1$ MHD activity were observed in JET discharges with high fast-ion energy content (Mantsinen00).
- New realistic numerical tools are required to address resonant, non-perturbative modes:
  - such as NOVA-KN (Kinetic Non-perturbative) hybrid code we are developing and apply
  - Careful phase space integration to evaluate effect of $\omega_d$ on the resonant mode drive is required.
  - Important to know linear dispersion relation for perturbative nonlinear theory application (such as Berk-Breizman TAE saturation theory).
  - Complex plasma geometries require numerical treatment of particle-wave interaction.
  - Continuum mechanism may play major role in damping of some modes.
Two types of plasma oscillations are seen:

- $60 - 80\, kHz$ and $10 - 20\, kHz$ (around $t \simeq 52.5\, sec$) with $n = 1$.
- Hot ion (ICRH H-minority) beta is large $\beta_H \sim 2\% \sim \beta_{pl}$.
- Both are chirping frequency - “fishbones”?
Numerical procedure in NOVA-KN

NOVA-KN - hybrid code, integro-differential problem:
Second order ideal MHD + hot ions, coupled through perturbed pressure (Cheng, '92, see also next talk by C.Z.Cheng):

\[ \delta p_\perp = m_h \int d\mathbf{v} \mu B \hat{g}, \quad \delta p_\parallel = m_h \int d\mathbf{v} 2 (\mathcal{E} - \mu B) \hat{g} \]

\( \hat{g} \) - nonadiabatic perturbed distribution function (Gorelenkov, '99):

\[
\hat{g} = \mathcal{E} \sum_{m,p} \frac{(\omega - \omega_*) G_{m,p}}{\omega - \omega_d - p\omega_b} \frac{\partial F_h}{\partial \mathcal{E}} e^{-i\omega t + i\omega \theta - in\varphi},
\]

(1)

\( G_{m,p} \) is the matrix of wave-particle interaction (Cheng '92).

- Present version uses ZOW for \( G \) matrix.
- Accurate estimate of drift \( \omega_d \) and bounce \( \omega_b \) hot ion frequencies in FOW.
- NOVA-KN solves integro-differential equations and finds mode frequency in complex plane.
Consider deeply trapped hot ions at \( r/a = 1/3 \) and \( E_{H0} = 800 \text{keV} \) (agrees with ORBIT).

Characteristic frequencies are determined by the ions with \( \nu / \nu_{H0} \approx 1.7 \) \((E_{H0} = 2.3 \text{MeV})\), where the perturbed pressure integrand is peaked \( v^6 e^{-v^2} \). ⇒

1. \( \Omega_d \equiv \omega_d / \omega_{A0} \sim 0.7 \) \((\sim 0.35 \text{ for ICRH H-minority, } \sim 60 \text{kHz})\) and
2. \( \Omega_b \equiv \omega_b / \omega_{A0} \sim 2 \), \((\omega_{A0}/2\pi = 160 \text{kHz})\).
Alfvén continuum

\[ q = 1 \text{ surface is at } r/a \equiv \sqrt{\psi/\psi_0} = 0.6. \]

- Any mode with non-zero frequency is reflected from the continuum.
- Drive should be strong enough to overcome the continuum damping.
Two branches are found with NOVA-KN

- **Lower frequency** mode is a continuation of the ideal mode. It is strongly stabilized by hot ions at $\beta_{H0}/\beta_{pl} > 0.2$.
- **Higher frequency** mode (higher frequency fishbone-resonant mode) has frequency comparable to the measured at H-minority temperature $800\,keV$. 

![Graph showing branch points and relationship between $\omega/\Omega$ and $\beta_{H0}/\beta_{pl}$]
Shown are the radial structures of the dominant poloidal harmonics ($m = 1$).

- Ideal mode is strongly driven $\Rightarrow$ mode structure is effected.
- Resonant mode is reflected from the continuum due to finite frequency.
  - Predicted growth rate is probably too low for such nonperturbative mode.
Comparison with theory

Two types (branches) of fishbones are known Chen, et.al. ’84, Coppi, et.al. ’86, ’88, Porcelli ‘01: \( i [\omega (\omega - \omega_d)]^{1/2} = \lambda_{\text{mhd}} + \lambda_H \).

- Low frequency \( \omega \sim \omega_d \), if \( \omega_d \ll \omega_d H \sim 1:250 \) and
  \[
  \beta_H \lesssim \beta_{\text{min}} \approx \pi \left[ \beta^2 - (\beta^{\text{MHD}})^2 \right]
  \]

- High frequency \( \omega \sim \omega_d H \), if \( \beta_H \gtrsim \beta_{\text{max}} \approx \epsilon \omega_d H / \omega_A \pi \).

NOVA-KN predicts \( \omega / \omega_d H \sim 1/100 \) for low frequency branch.
H-minority distribution function (factorizing?) may be a key factor for \( \omega_I \).
Summary

A non-perturbative code NOVA-KN is being developed to account for realistic calculations of the non-perturbative resonant modes with FOW effects. It is fast and useful tool to study physics of particle-wave interaction.

- More new physics needs to be included in computations such as: nonisotropic distribution function of hot ions, nonisotropic plasma equilibrium, radial coupling due to FOW.

- Comparison with JET experiments shows that the possible explanation for the $10 - 70kH\omega$ frequency range MHD $n = 1$ oscillations is the excitation of the two types of modes.
  - At low frequency $f = 10 - 20kH\omega$ the unstable mode is hot ion modified ideal MHD mode excited when beta of hot ions is not so strong, but frequency in experiment is higher.
  - At higher frequencies $f = 50 - 70kH\omega$ NOVA-KN predicts instability of the resonant modes having frequency equal to characteristic toroidal precession frequency of H-minority ions.
Expected hot ion energy is $800\text{keV}$ in the discharge studied, which is slightly higher than typically observed $500\text{keV}$, but agrees with the preliminary results by V. Kiptily.