Modelling of Frequency Sweeping with the HAGIS code

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Structure of Talk

• What is frequency sweeping?
  – Experimental evidence
  – Theoretical understanding

• Numerical modelling
  – Description of the HAGIS code
  – Simulations of frequency sweeping

• Summary
Experimental Observations

- **Frequency sweeping in MAST #5568**

  Chirping modes exhibit simultaneous upwards and downwards frequency sweeping.

  More experimental details in talk by M. Gryaznavich this afternoon.

  Frequency sweep $\frac{\delta \omega}{\omega_0} \sim 20\%$
JET Observations

- Shear optimised D-T pulse
- TAE modes during current ramp phase

Frequency sweep $\delta \omega / \omega_0 \sim 5\%$
Frequency Sweeping

- Universality in nonlinear response of resonant particles to low amplitude wave
  
  [Berk, Breizman, Pekker (1997)]

- Particle distribution satisfies a 1-dimensional equation (two phase-space coordinates)

- Constants of motion for wave

  \[ E(r,t) = C(t) \, E(r,\theta,n\phi - \omega_0 t) \]

  - Magnetic moment, \( \mu \) (if \( \omega_0 \ll \omega_c \) and \( L_\omega > \rho_i \))
  - Energy in rotating frame, \( H' = H - (\omega_0/n) \, P_\zeta \) (if \( 1/C \, dC/dt \ll \omega_0 \))
Wave-Particle Interaction

Define: \( \Omega_l(P_\phi, H', \mu) = n\langle \omega_\phi \rangle - l\langle \omega_\theta \rangle \)

As \( P_\phi \) changes due to interaction at fixed \( H', \mu \)

\[
P_\phi - P_{\phi,l} = \left. \frac{\partial P_\phi}{\partial \Omega_l} \right|_{H', \mu} \left[ \Omega_l(P_\phi) - \omega(t) \right]
\]

Equations of particle motion for fixed \( H', \mu \)

\[
\frac{d\xi}{dt} = \Omega_l - \omega_0, \quad \frac{d\Omega_l}{dt} = -\omega_{bl}^2(t) \sin \xi
\]

Hence,

\[
\frac{d^2\xi}{dt^2} + \omega_{bl}^2(t) \sin \xi = 0
\]

“Pendulum equation”

Trapping frequency, \( \omega_{bl}(t) \propto |E|^{1/2} F(H', \mu) \)

F is a phase space dependent form factor
Nonlinear Trapping in TAE

- Trapping frequency is related to TAE amplitude
  \[ \omega_{b,l}(t) \propto |\delta B|^{1/2} \]
- Frequency sweep is related to trapping frequency [Berk et al., (1997)]
  \[ \delta \omega \propto \omega_b^{3/2} t^{1/2} \]
- Amplitude related to frequency sweep
  \[ \Rightarrow \delta B \propto \left( \frac{\delta \omega^2}{t} \right)^{2/3} \]

Aim

• Use experimentally observed rate of frequency sweeping to determine wave amplitude

• In general, numerical modelling is needed to establish the form factor that relates $\delta \omega$ and $\delta B$

• Validate HAGIS for model case

• Employ HAGIS to establish $\delta B$ in general case
  - General geometry (including tight-aspect ratio)
  - Mode structure: global mode analysis
The HAGI S Code

[Pinches, Thesis (1996)]

Simon Pinches, 8th IAEA Technical Meeting on Energetic Particles, San Diego
Code Overview

• Straight field-line equilibrium
  – Boozer coordinates

• Hamiltonian description of particle motion [White & Chance 1984]

• Fast ion distribution function
  – $\delta f$ method

• Evolution of waves
  – Wave eigenfunctions computed by CASTOR
Equilibrium Representation

- Coordinates $\psi_p, \theta, \zeta$ chosen to produce straight field lines

$\mathbf{B} = \delta(\psi_p, \theta) \nabla \psi_p + I(\psi_p) \nabla \theta + g(\psi_p) \nabla \zeta,$

$\mathbf{B} = \nabla \psi \wedge \nabla \theta - \nabla \psi_p \wedge \nabla \zeta,$

$\Rightarrow \mathbf{A} = \psi \nabla \theta - \psi_p \nabla \zeta.$
Particle Description

Exact particle Lagrangian, \( \mathcal{L}_{\text{exact}} = \sum_{ep} \frac{1}{2} m V^2 + e V \cdot A - e \phi \)
is gyro-averaged and written in the form,

\[
\mathcal{L}_{ep} = \sum_{j=1}^{n_p} P_{\theta j} \dot{\theta}_j + P_{\zeta j} \dot{\zeta}_j - \mathcal{H}_j
\]

with

\[
\mathcal{H}_j = \frac{1}{2} m_j v_{||j}^2 + \mu_j B_j + e_j \phi_j
\]

leading to \( 4 \times n_p \) equations

[White & Chance 1984]
Fast Particle Orbits

- I CRH ions in JET deep shear reversal
  - On axis heating\(^\dagger\):
    \[ \Lambda = \mu B_0 / E = 1 \]
  - \( E = 500 \text{ keV} \)

- Produces predominately potato orbits

\(^\dagger\)J. Hedin, Thesis 1999
Distribution Function

- Represented by a finite number of markers
- Markers represent deviation from initial distribution function - so-called $\delta f$ method
  - Dramatically reduces numerical noise

$$
\begin{align*}
  f &= f_0(\mathcal{E}, P_\zeta; \mu) + \delta f(\Gamma^{(p)}, t) \\
  \frac{df}{dt} &= 0 \Rightarrow \delta f = -\dot{P}_\zeta \frac{\partial f_0}{\partial P_\zeta} - \dot{\mathcal{E}} \frac{\partial f_0}{\partial \mathcal{E}} \\
  \int f g d\Gamma^{(p)} &\longleftrightarrow \int f_0 g d\Gamma^{(p)} + \sum_{j=1}^{n_p} \delta n_j g_j \\
  \text{where} \quad \delta n_j(t) &\equiv \delta f_j(t) \Delta \Gamma_j^{(p)}(t)
\end{align*}
$$

Parker & Lee, 1993
Denton & Kotschenreuther 1995
Wave Equations

- Linear eigenstructure assumed invariant
- Introduce slowly varying amplitude and phase:
  \[ \tilde{\phi}_k = A_k(t) \sum_m \tilde{\phi}_{km}(\psi)e^{i(n_k\zeta_m - m\theta - \omega_k t - \alpha_k(t))} \]
- Gives wave equations as:
  \[
  \dot{\chi}_k = \frac{1}{2E_k} \sum_{j=1}^{n_p} \delta f_j \Delta \Gamma_j^{(p)} \sum_m (k_m v_j - \omega_k) S_{jkm} + \chi_k \gamma_d, \\
  \dot{\gamma}_k = -\frac{1}{2E_k} \sum_{j=1}^{n_p} \delta f_j \Delta \Gamma_j^{(p)} \sum_m (k_m v_j - \omega_k) C_{jkm} + \gamma_k \gamma_d,
  \]
- where
  \[
  \chi_k \equiv A_k \cos(\alpha_k), \quad C_{jkm} \equiv \text{Re}[\tilde{\phi}_{km}(\psi_j)e^{i\Theta_{jkm}}] \\
  \gamma_k \equiv A_k \sin(\alpha_k), \quad S_{jkm} \equiv \text{Im}[\tilde{\phi}_{km}(\psi_j)e^{i\Theta_{jkm}}] \\
  \Theta_{jkm} \equiv n_k \zeta_j - m\theta_j - \omega_k t
  \]

Additional mode damping rate, \( \gamma_d \)
**HAGIS Code Performance**

- **HAGIS code parallelises very well**
  - relatively low level of inter-processor communication traffic

Wall-clock time to calculate TAE linear growthrate in ITER-like case
Self-Consistent Frequency Sweeping

- **Equilibrium:**
  - $a/R_0 = 0.3$
  - $q_0 = 1.1$
  - $E_0 = 3.5$ MeV
Linear Growthrate

- \( \gamma_d/\omega_0 = 0, \langle \beta_f \rangle = 3 \times 10^{-4} \)

Mode saturates at \( \delta B/B \sim 10^{-3} \)

\( \gamma_d/\omega_0 = 2.7\% \)

\( n_p = 52,500 \)
...with additional damping

- $\gamma_d/\omega_0 = 2\%$, $\langle \beta_f \rangle = 3 \times 10^{-4}$

Mode saturates at much lower level, $\delta B/B \sim 10^{-4}$

$n_p = 210,000$
Frequency Sweeping

- Fourier spectrum of evolving mode

\[ \delta \omega = 0.44 \gamma_L^{3/2} t^{1/2} \]

Frequency sweep \[ \delta \omega / \omega_0 \approx 10\% \]
Linear Growth Rate

- $\gamma_d/\omega_0 = 0$, $\langle \beta_f \rangle = 7.5 \times 10^{-5}$

Mode saturates at $\delta B/B \sim 4 \times 10^{-5}$

$n_p = 52,500$

$\gamma_d/\omega_0 = 0.45\%$
...with additional damping

- $\gamma_d/\omega_0 = 0.4\%, \langle \beta_f \rangle = 7.5 \times 10^{-5}$

Mode saturates at much lower level, $\delta B/B \sim 3 \times 10^{-6}$

$n_p = 210,000$
Frequency Sweeping

- Fourier spectrum of evolving mode

\[ \delta \omega = 0.44 \gamma_L^{3/2} t^{1/2} \]
Fast Ion Redistribution

Resonant energy changes as mode sweeps in frequency.
• Obtain factor relating $\omega_b$ and $\delta B$

$E_b = 40$ keV
$a/R_0 = 0.7$
$B_0 = 0.5$ T
$R_0 = 0.77$ m

Global
$n=1$ TAE

Monotonic
$q$-profile
Particle Trapping in MAST

- Particles trapped in TAE wave
  - All particles have same
    \[ H' = E - \frac{\omega}{n} P_\zeta, \]
    \[ = 20 \text{ keV} \]
  - TAE amplitude:
    \[ \delta B/B = 10^{-3} \]
Scaling of Nonlinear Bounce Frequency

- Monotonic q profile
- \( H' = 20 \text{ keV} \)

\[
\omega_b = 1.156 \times 10^6 \sqrt{\frac{\delta B}{B}}
\]
Scaling of Nonlinear Bounce Frequency

- Reversed shear
- $H' = 20$ keV

$\omega_b = 7.2 \times 10^5 \sqrt{\frac{\delta B}{B}}$ for $\frac{\delta B}{B} < 2 \times 10^{-4}$
Mode Amplitudes

• For monotonic q-profiles we now know:

\[ \omega_b = C_1 \left( \frac{\delta B}{B} \right)^{1/2} \]

where \( C_1 = 1.156 \times 10^6 \)

• For a single resonance,

\[ \delta \omega = C_2 \omega_b^{3/2} t^{1/2} \]

where \( C_2 = \frac{\pi}{2\sqrt{2}} \approx 1 \)

• Therefore,

\[ \frac{\delta B}{B} = \frac{1}{C_1^2} \left( \frac{\delta \omega^2}{C_2^2 t} \right)^{2/3} \]
TAE Amplitude in MAST

\[ \frac{\delta B}{B} = \frac{1}{(1.156 \times 10^6)^2} \left( \frac{32 \delta f^2}{\delta t} \right)^{2/3} \]

\[ = 4 \times 10^{-4} \]

- \( df = 18 \text{ kHz} \)
- \( dt = 0.8 \text{ ms} \)
Conclusions

• Frequency sweeping has been modelled using the HAGIS code
  – Benchmarked against analytic theory

• The amplitude of a frequency sweeping mode in MAST has been calculated to be $\frac{\delta B}{B} = 4 \times 10^{-4}$